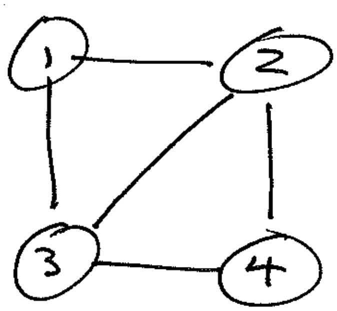
cse 101 - 02 4-13-23

Defn

let $x \in V(G)$. The degree of $x$ is the # of edges incident with $x$



$deg(2) = 3$

$deg(4) = 2$

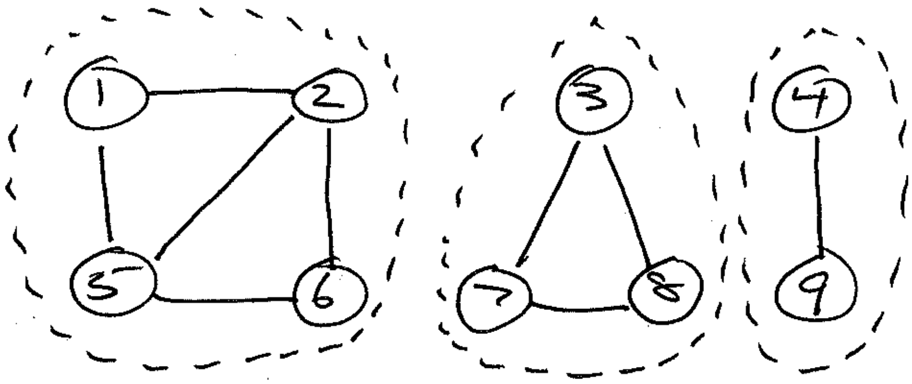
Lemma (handshake)

$$\sum_{x \in V(G)} deg(x) = 2|E(G)|$$

<u>Defn</u>

A graph $G$ is called <u>connected</u> iff for all $x, y \in V(G)$, $G$ contains an $x$-$y$ path.

<u>Ex.</u>



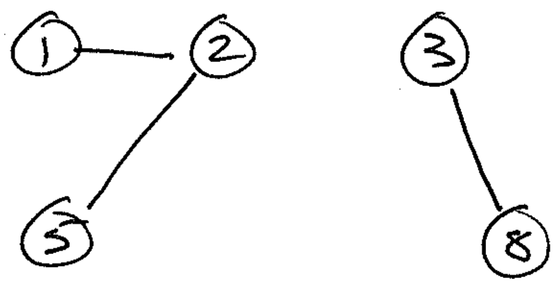

disconnected.

<u>Defn</u>

A <u>subgraph</u> of a graph $G$ is a graph $H$ such that

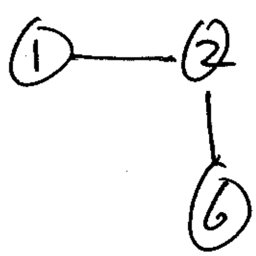
$$V(H) \subseteq V(G) \text{ and } E(H) \subseteq E(G)$$

Ex. $(\{1, 2, 5, 3, 8\}, \{12, 25, 38\})$



disconnected

Ex $(\{1, 2, 6\}, \{12, 26\})$



connected.

Ex. $(\{4\}, \emptyset)$

connected

Ex. $(\{1, 3, 4\}, \{12, 37, 49\})$

not a graph, so not a subgraph

## Defn

- A subgraph $H$ of $G$ is called a <u>connected component</u> of $G$ iff

(1) $H$ is connected.

and

(2) $H$ is maximal w.r.t. (1)

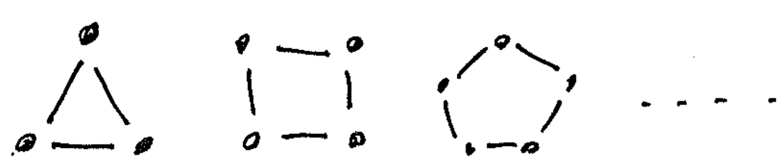
## Defn

(Also called <u>forest</u>)

A graph $G$ is called <u>acyclic</u> iff it contains no cycles

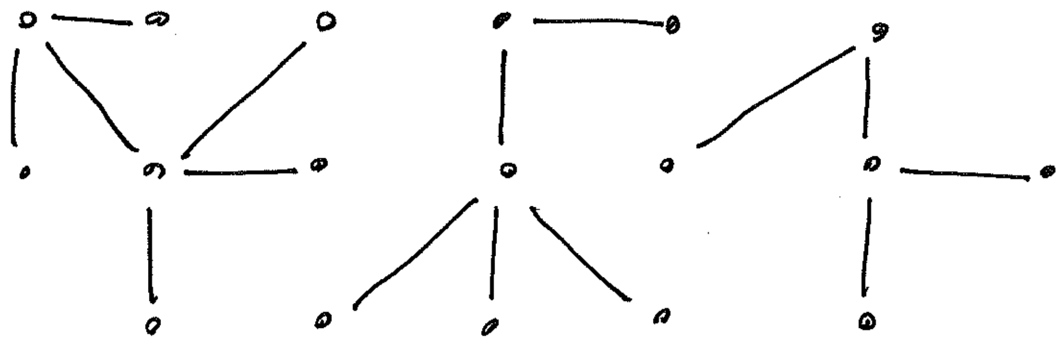
non-trivial closed path

trivial Path



cycles . . . . . . .

**Ex.**



| | | | |
|---|---|---|---|
| #V | 7 | 6 | 5 |
| #E | 6 | 5 | 4 |

**Defn**

a Graph $T$ is called a <u>tree</u> iff it is acyclic and connected.

**lemma**

if $T$ is a tree with $n$ vertices, then $T$ has $n-1$ edges.

# Representations of graphs

- **Incidence Matrix**: $n \times m$ matrix : $I$

$V = \{x_1, x_2, \dots, x_n\}$

$E = \{e_1, e_2, \dots, e_m\}$

## undirected

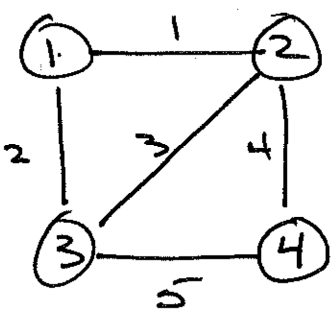
$$I_{ij} = \begin{cases} 1 & x_i \text{ is incident with } e_j \\ 0 & \text{otherwise} \end{cases}$$

## directed

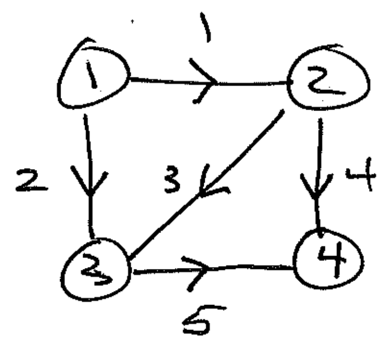
$$I_{ij} = \begin{cases} 1 & \text{if } \bigcirc \xrightarrow{e_j} x_i \\ -1 & \text{if } x_i \xrightarrow{e_j} \bigcirc \\ 0 & \text{otherwise} \end{cases}$$

Ex.

G



D



$$I = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \qquad I = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

- Adjacency Matrix: $n \times n$ matrix : $A$

$V = \{x_1, x_2, \ldots, x_n\}$

undirected

$$A_{ij} = \begin{cases} 1 & \text{if } x_i - x_j \\ 0 & \text{otherwise} \end{cases}$$

directed.

$$A_{ij} = \begin{cases} 1 & \text{if } x_i \to x_j \\ 0 & \text{otherwise} \end{cases}$$

Ex.

G

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

D

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Adjacency List: array of $^{n}$ lists

undirected

adj [i] = list of neighbors of $x_i$

directed

adj[i] = list of terminii of edges having $x_i$ as origin.

Ex.

| G | | D | |
|---|---|---|---|
| 1 : | 2 3 | 1 : | 2 3 |
| 2 : | 1 3 4 | 2 : | 3 4 |
| 3 : | 1 2 4 | 3 : | 4 |
| 4 : | 2 3 | 4 : | |

Convention: store vertices in adj lists by increasing vertex lables.

# Handout : Graph Algorithms

**Defn**

Given graph $G$ (directed or undirected) and $x, y \in V(G)$, the distance from $x$ to $y$ is

$$\delta(x,y) = \begin{cases} \text{min length of an } x\text{-}y \text{ path} \\ \text{if } y \text{ is reachable from } x. \\ \infty \text{ otherwise} \end{cases}$$

Single source shortest path (SSSP) Problem:

Given a graph $G$ and a source vertex $s \in V(G)$

(1) determine $\delta(s,x)$ for all $x \in V(G)$

and

(2) find, for each x that is reachable from x, a shortest s-x path, i.e. a path of length $\delta(s,x)$.

## Bredth First Search (BFS)

Vertex attributes: for $x \in V(G)$

color[x] : white, gray, black

distance[x] : $\delta(s,x)$ when complete

Parent[x] : predecessor of x along a shortest s-x path.

Also: FIFO queue Q.

See pseudo-code

cse101/Examples