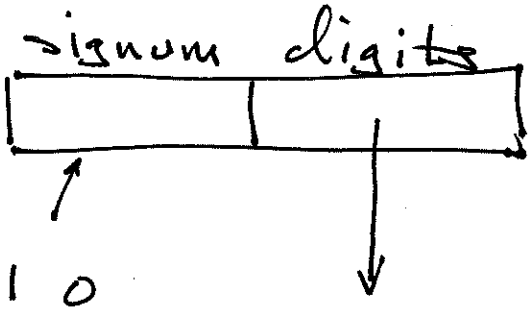
CSc 101 - 01 5-16-23

TaG:

BigInteger :



Ex. addition: $b = 100 = 10^2$, $p = 2$

1 1 1 0

(88 21 33) = 882,133

(65 91 79) = 659,179

(1 54 13 12) = 1,541,312

another way

(88 21 33)

(65 91 79)

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1 1 1 0

(153 112 112) vector sum

−100 −100 −100

(1 54 13 12) normalize

Ex. Subtraction $b = 100$, $p = 2$

$$\begin{array}{ccc} -1 & -1 & 0 \\ (88 & 21 & 33) = 882,133 \\ (65 & 91 & 79) = 659,179 \end{array}$$

$$(22 \quad 29 \quad 54) = 222,954$$

another day:

$$\begin{array}{ccc} (88 & 21 & 33) \\ (65 & 91 & 79) \end{array}$$

$$\begin{array}{ccc} -1 & -1 & 0 \\ (23 & -70 & -46) \\ & 100 & 100 \end{array}$$ vector diff.

$(22 \quad 29 \quad 54)$ normalize

Ex. subtraction $b = 100$, $p = 2$

$(65 \quad 91 \quad 79)$

$(88 \quad 21 \quad 33)$

$(-23 \quad 70 \quad 46)$ vector diff.

left most ↑
digit negative

$$\begin{array}{cccc} & -1 & & -1 \\ \boxed{-1} & (23 & -70 & -46) \\ & & 100 & 100 \end{array}$$

$(22 \quad 29 \quad 54)$ normalize

↓

return this from normalize.

Ex. normalize . $b=10$, $P=1$

$$\overset{-9}{(}\ 1\ ,\ -90\ ,\ 9\ ,\ 73\ ,\ 0\ ,\ -500\ \overset{0}{)}$$

$$\overset{1\quad 6\quad -5\quad -50}{\quad}$$

$$-89 \quad 15 \quad 68 \quad -50$$

$$90 \quad -10 \quad -60 \quad +50 \quad +500$$

$$(-8,\ 1,\ 5,\ 8,\ 0,\ 0)$$

$$\overset{-1\quad -1\quad -1}{\quad}\qquad\qquad\overset{0\quad 0}{\quad}$$

$\boxed{-1}$ $(\ 8\ ,\ -1\ ,\ -5\ ,\ -8\ ,\ 0\ ,\ 0\ )$

$$-2 \quad -6$$

$$10 \quad 10 \quad 10$$

$$(7,\ 8,\ 4,\ 2,\ 0,\ 0)$$

↓

Return sign

check! $1\cdot 10^5 + (-90)\cdot 10^4 + 9\cdot 10^3 + 73\cdot 10^2 + 0\cdot 10 + (-500)\cdot 1$

$= -784{,}200$

Ex. Multiplication : $b=10$, $p=1$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 123 \\ \times\ 456 \\ \hline .738 \\ .6150 \\ 49200 \\ \hline 56088 \end{array}$$

n digits

m digits

$$\begin{array}{r} (1\ 2\ 3) \\ (4\ 5\ 6) \\ \hline () \end{array}$$

		shift
empty		
(6 12 18)	scalar mult	0
	"add"	
(7 3 8)	normalize	
(5 10 15 0)	scalar mult	1
(5 17 18 8)	add	
(6 8 8 8)	normalize	
(4 8 12 0 0)	scalar mult	2
(4 14 20 8 8)	add	
(5 6 0 8 8)	normalize	

Runtime of multiplication

Runtime = (# digits in 1st) · (# digits in 2nd)

$= \Theta(m \cdot n)$

more detail:

digit multiplications $= m \cdot n$

digit additions $= 2mn + m^2$

Ex. $999 \times 999 = 998001$, $b = 10$

$$\begin{array}{r} 999 \\ 999 \\ \hline \end{array}$$

*empty ( )

		81	81	81	s.m.
		81	81	81	add
*		8	9	9	1 norm
		81	81	81	0 s.m.
	89	90	90	1	add
	9	8	9	0	1 norm
	81	81	81	0	0 s.m
	90	89	90	0	1 add
(9	9	8	0	0	1) norm