cse 101 - 02   6-2-22

- final exam: Tue June 7, 4-6 PM
- SETs: close Sun. June 5, 11:59PM
- Pa8: one last day: Thur 10 PM.

Runtime of BellmanFord:

let $n = |V|$, $m = |E|$

Initialize(G, s) : $\Theta(n)$

Relax costs : $\Theta(1)$

total : $(n-1) \cdot m \cdot \Theta(1) = \Theta(m(n-1)) = \Theta(mn)$

final loop : $\Theta(m)$

Total cost : $\Theta(n) + \Theta(m) + \Theta(mn) = \Theta(mn)$

Runtime of Dijkstra

$n = |V|$, $m = |E|$.

ExtractMin: $\Theta(\log n)$

total $= \Theta(n \log n)$

Relax: $\Theta(\log n)$

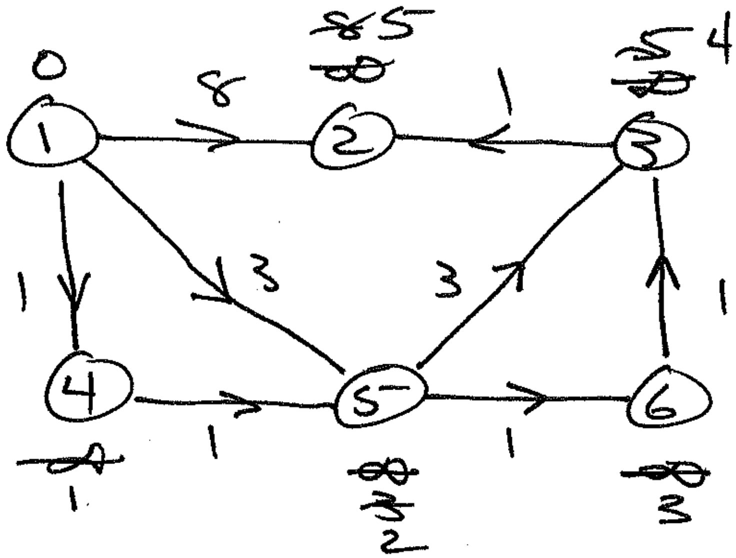
total $= \Theta(m \log n)$

Total cost $= \Theta(n \log n) + \Theta(m \log n)$

$= \Theta((n+m) \log n)$

Ex. Dijkstra: $s = 1$

G

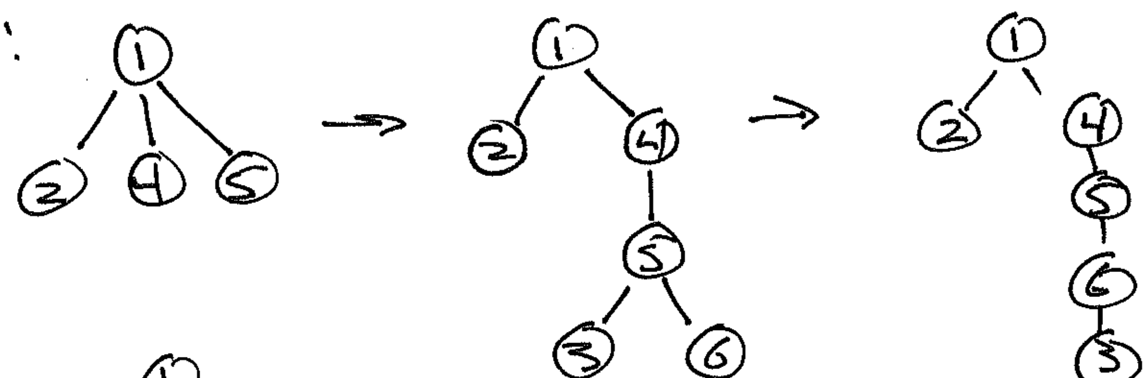


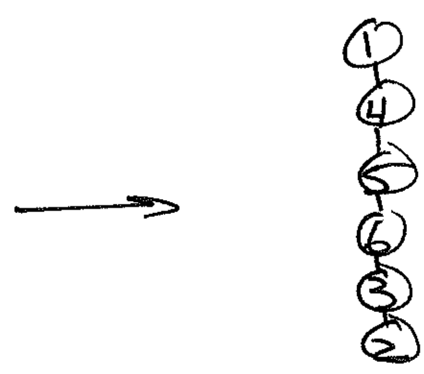
Extract order

1
4
5
6
3
2

P.Q. :	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d :	0	~~8~~ 5	~~5~~ 4	1	~~3~~ 2	3
p :	n	~~1~~ 3	~~5~~ 6	1	~~1~~ 4	5

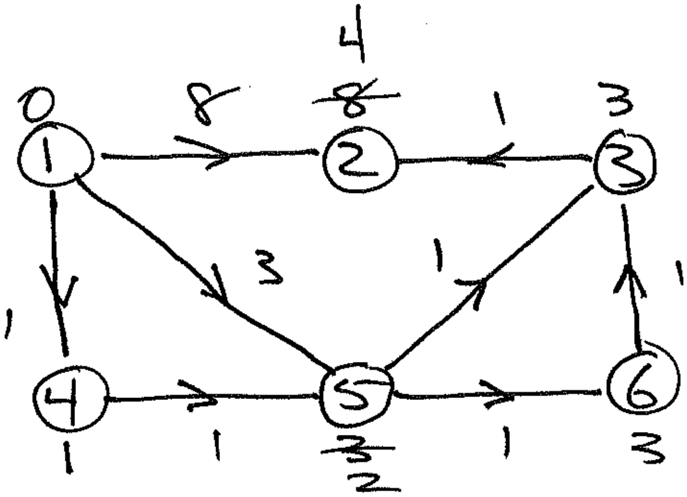
Tree:





<u>convention</u>: if two d-values are both minimum, then extract the one with lower label

EX

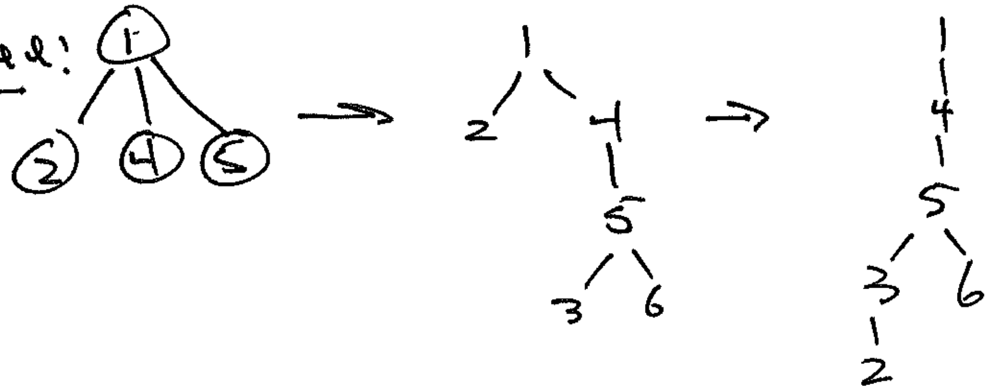


PQ:	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d:	0	~~8~~ 4	3	1	~~3~~ 2	3
p:	n	~~1~~ 3	5	1	~~1~~ 4	5

<u>remove order</u>

1
4
5
3
6
2

<u>Tree</u>:



Chap 23: MWST

Defn

Let $G = (V, E)$ be an (undirected) graph. A spanning Tree T in G is a subgraph satisfying

(1) T is a tree

(2) $V(T) = V(G)$

Thm: G is connected iff G contains a spanning tree.

$\Leftarrow$ obvious

$\Rightarrow$ (easy).

<u>Defn</u> Weighted

if G is an graph and H a subgraph, the weight of H is

$$w(H) = \sum_{e \in E(H)} w(e)$$

<u>Defn</u>

A <u>minimum weight spanning tree</u> (MWST) in G is a sp. tree T s.t.

$$w(T) \leq w(S)$$

for all sp. trees S in G.

Algorithms

- Prim
- Kruskal
- Dual-Kruskal

Prim(G)

1. $(W, F) = (\phi, \phi)$
2. Pick any $r \in V$, let $W = \{r\}$
3. while $|F| < n-1$
4. amongst all edges in $E-F$ with one end in W, and other in $V-W$, let e be one of min. weight
5. $F = F \cup \{e\}$
6. $W = W \cup \{$other end of $e\}$

when done: $T = (W, F)$ is a MWST.

Kruskal(G)

1. $F = \phi$
2. while $|F| < n-1$
3. amongst all edges whose addition to F would not create a cycle, let e be one of min. weight
4. $F = F \cup \{e\}$

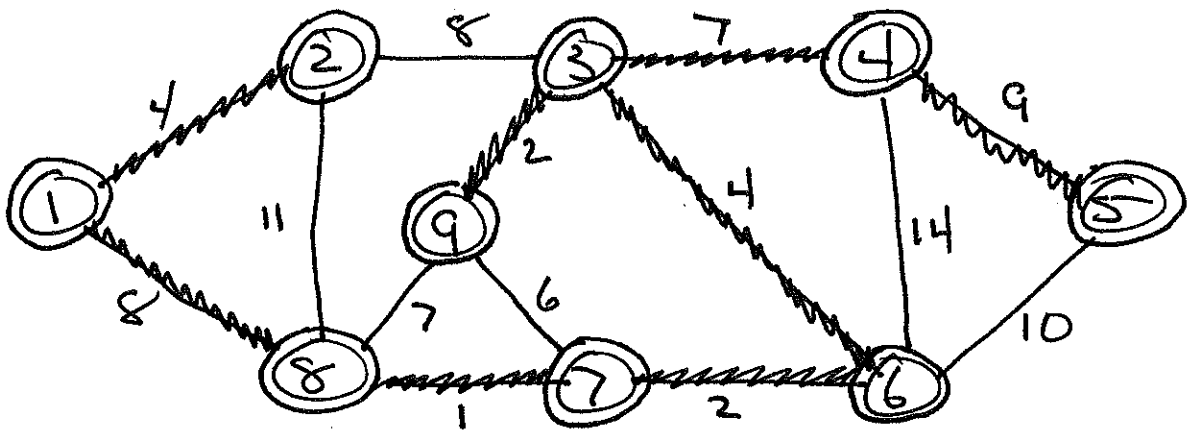
when done: $T = (V, F)$ is a mwst.

DualKruskal(G)

1. $F = E(G)$
2. while $|F| > n-1$
3. amongst all edges in F whose removal from F does not disconnect (F, V), let e be be one of max. weight
4. $F = F - \{e\}$.

when done: $T = (V, F)$ is a m.w.s.t.

Ex. (p. 632) Prim



$r = 3$

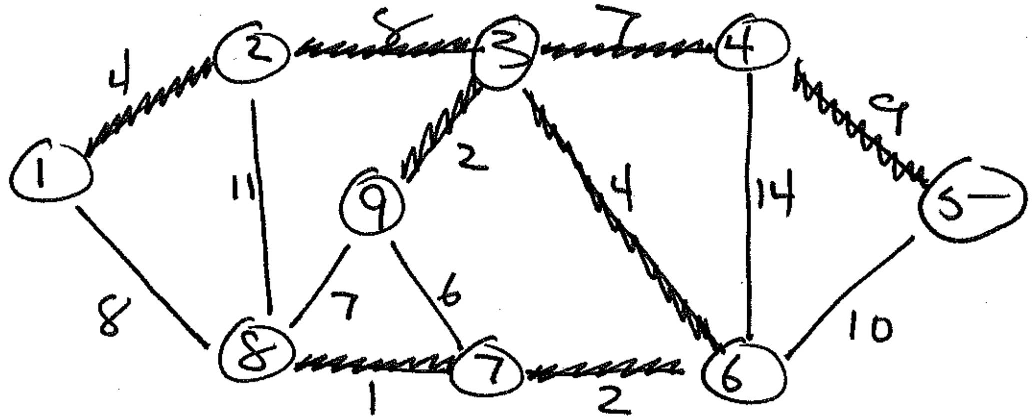
$W = \{3, 9, 6, 7, 8, 4, 1, 2, 5\}$

$F = \{39, 36, 76, 87, 34, 18, 12, 45\}$

$T = (W, F)$ is a mwst

$\text{weight}(T) = 37$

Ex. Kruskal



$F = \{87, 39, 67, 36, 12, 34, 23, 45\}$

$T = (V, F)$ is a mwst

$w(T) = 37$