

CSE 101-02 4-7-22

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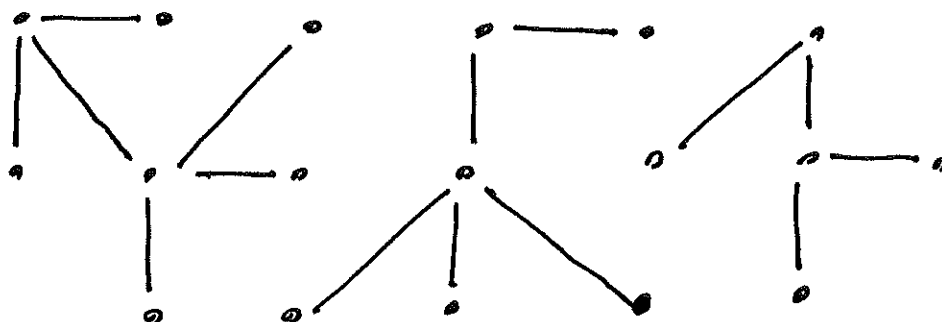
- Pal ext. 1 last day.

Defn

(or a forest)

A graph G is called acyclicⁿ iff
 G contains no cycles

Ex.



Vert:

7

6

5

edges:

6

5

4

Defn

a graph is called a tree iff
it is connected and acyclic.

Theorem

If T is a tree with n vertices
and m edges, then $m = n - 1$.

How to represent a graph (or digraph)

- adjacency matrix.
 - incidence matrix.
- } see handout.

• Adjacency list representation.

This is an array of lists: $adj[]$

whose

$$V(G) = \{1, 2, 3, \dots, n\}$$

$$\text{len}(adj) = n.$$

– undirected graph:

$adj[i]$: list of vertices adjacent to i .

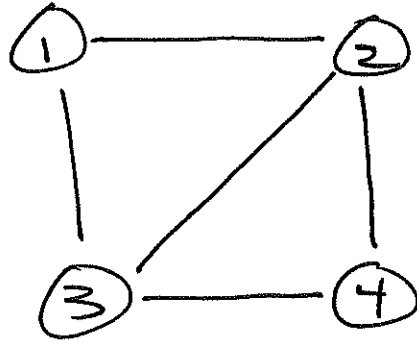
– directed graph:

$adj[i]$: list of terminii of edges having i as origin.

(Undirected)

Ex.

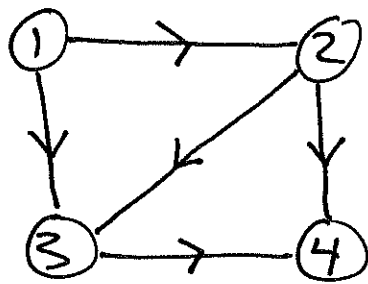
G



$$\text{adj}(G) = \left\{ \begin{array}{l} 1 : 2 \ 3 \\ 2 : 1 \ 3 \ 4 \\ 3 : 1 \ 2 \ 4 \\ 4 : 2 \ 3 \end{array} \right.$$

Ex. (directed)

G



$$\text{adj}(G) = \left\{ \begin{array}{l} 1 : 2 \ 3 \\ 2 : 3 \ 4 \\ 3 : 4 \\ 4 : \end{array} \right.$$

Handout: Graph algorithms

- we defined $\delta(x, y)$

- Single Source Shortest Paths (SSSP)
Problem:

Given a graph G (dir. or undir.),
and a fixed $s \in V(G)$ called source:

(1) determine $\delta(s, x)$ for all $x \in V(G)$.

(2) for all $x \in V(G)$ with $\delta(s, x) < \infty$,

determine a shortest s - x
Path in G .

Breadth First Search (BFS)

each $x \in V$ has attributes:

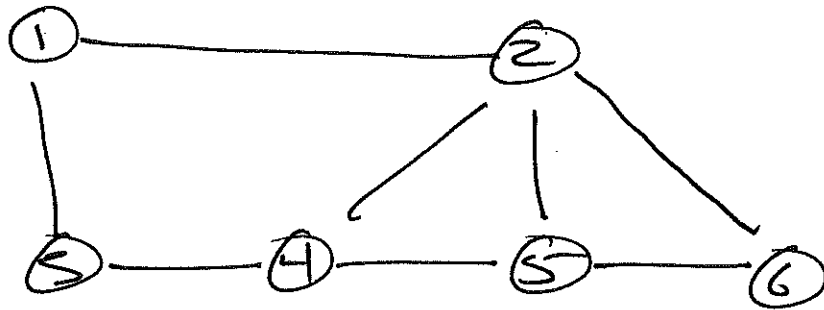
- $color[x]$: white: undiscovered
 gray: disc. but not all neigh.
 black: finished i.e. disc
 and all neighbors
 disc.
- $distance[x]$: $d(s, x)$
- $Parent[x]$: Predecessor of x
 in a shortest $s-x$
 Path.

BFS uses a FIFO queue Q .

See pseudo-code:

Ex.

$S = 3$

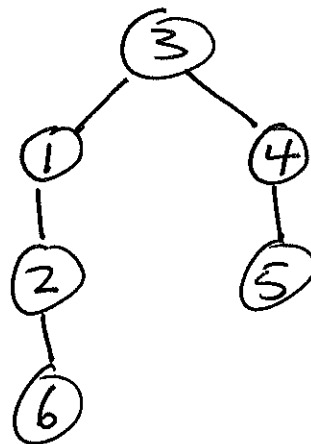


	adj	color	d	p
1	2 3	w g b	0 1	x 3
2	1 4 5 6	w g b	0 2	x 1
3	1 4	g b	0	n
4	2 3 5	w g b	0 1	x 3
5	2 4 6	w g b	0 2	x 4
6	2 5	w g b	0 3	x 2

Q: ~~2~~ ~~x~~ ~~4~~ ~~5~~ ~~6~~

BFS Tree :

Predecessor Subgraph:



depth =
dist

0

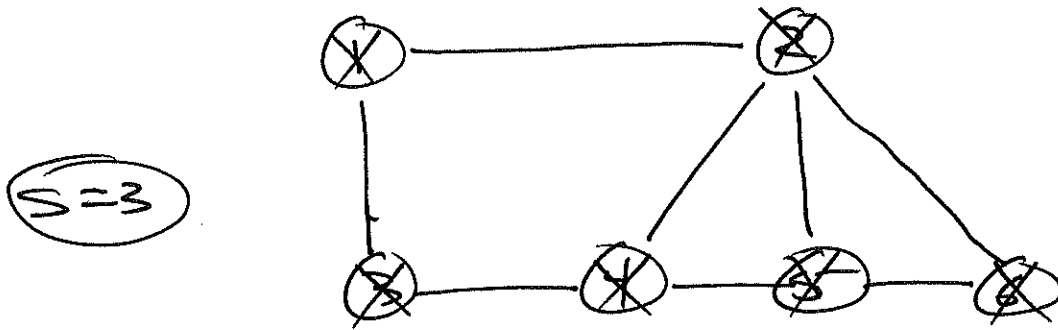
1

2

3

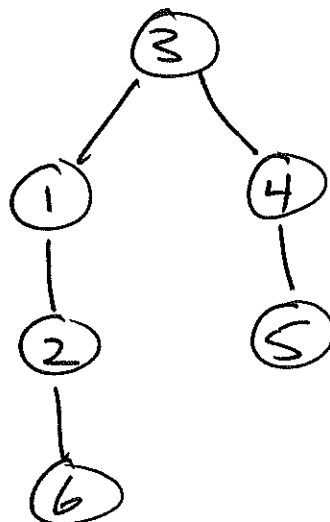
Ex. Same graph, Same source,

Quick way



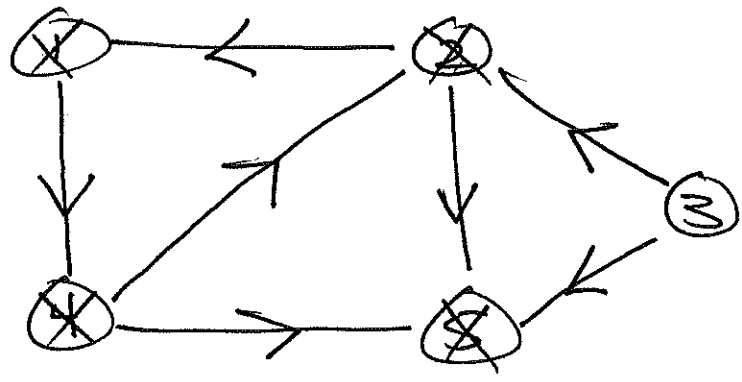
Q: ~~1~~ ~~2~~ ~~4~~ ~~7~~ ~~5~~ ~~6~~

BFS Tree:



Ex.

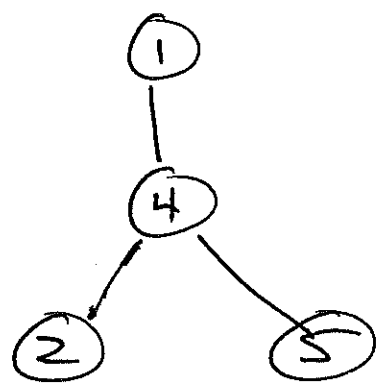
S=1



Q: $x \neq y \neq z$

dist

Tree:



0

1

2

	adj	color	d	p
1	4	b	0	n
2	1 5	b	2	4
3	2 5	w	∞	n
4	2 5	b	1	1
5		b	2	4

Defn

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Predecessor subgraph

$$V_p = \{ x \in V(G) \mid P[x] \neq \text{nil} \} \cup \{s\}$$

$$E_p = \{ \underbrace{(P[x], x)} \mid P[x] \neq \text{nil} \}$$

- ordered pair if directed
- unordered pair if undirected

How to extract shortest paths

from the Parent[] array ?

Pseudo-code: PrintPath(G, s, x)