CSE 101-02 4-26-22

• Pa4: Questions

helper fcn: dot(List A, List B)
returns double

Ex.

A: (10, .) (50, .) (80, .) (100, .)

B: (10, .) (30, .) (50, .) (90, .)

mult & add to an accum. sum

mult & add

Matrix operation:

changeEntry(M, i, j, x)

<u>case</u>: $M_{ij} = 0$, $x = 0$

do nothing.

<u>case</u>: $M_{ij} = 0$, $x \neq 0$

do a list insert

<u>case</u>: $M_{ij} \neq 0$, $x \neq 0$

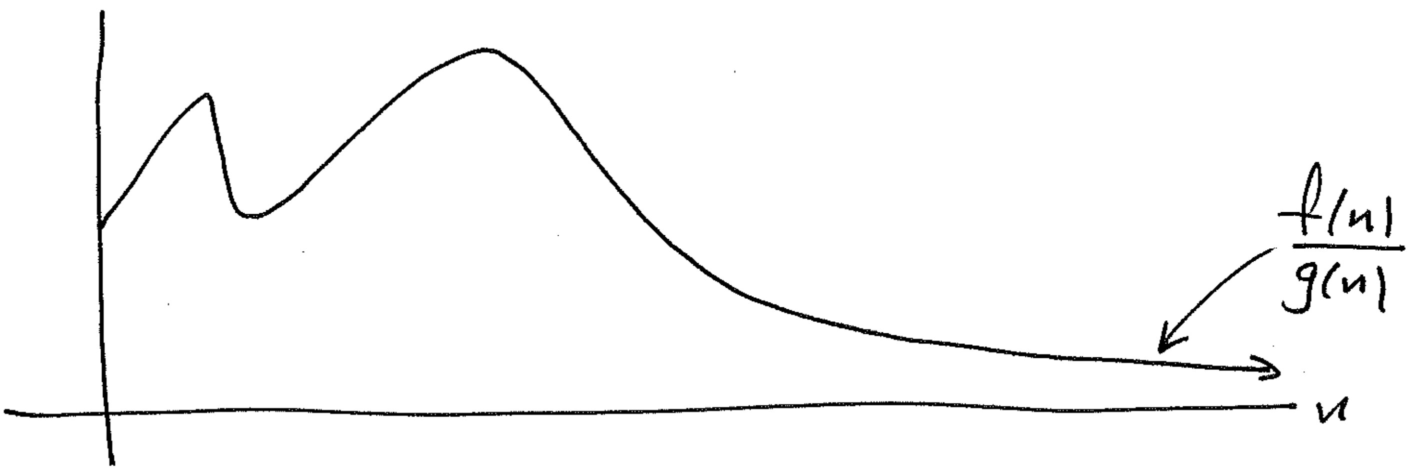
overwrite existing matrix entry (Val)

<u>case</u> $M_{ij} \neq 0$, $x = 0$

do a list delete

Defn

$$f(n) = o(g(n)) \text{ iff } \lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right) = 0$$



Ex: $\ln(n) = o(n)$

Exercise: show $(\ln(n))^{k_1} = o(n^{k_2})$

for any $k_1, k_2 > 0$.

Ex. $f(n) = n^k$, $g(n) = e^n$ $(k \geq 0)$

$$\lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty} \left(\frac{n^k}{e^n}\right)$$

$$= \lim_{n \to \infty} \left(\frac{k n^{k-1}}{e^n} \right)$$

$$\vdots$$

$$= 0$$

after $\lceil k \rceil$
# of applications
of l'Hopital's rule

Conclusion: $n^k = o(e^n)$

<u>Exercise</u>: show $n^k = o(b^n)$ for

any $k \geq 0$, and $b > 1$.

Ex. $f(n) = n^\alpha$, $g(n) = n^\beta$ $\quad 0 \le \alpha < \beta$

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(n^{\alpha-\beta}\right) = 0$$

since $\alpha - \beta < 0$

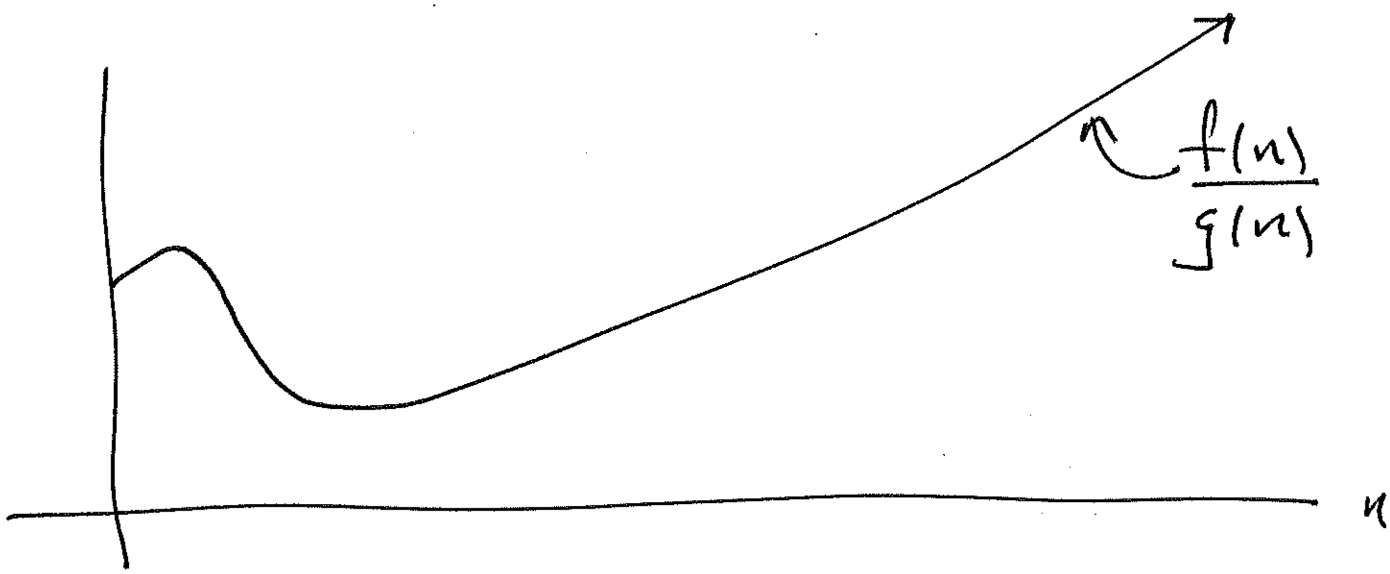
$\therefore \; n^\alpha = o(n^\beta)$ if $\alpha < \beta$.

Defn

$f(n) = \omega(g(n))$ iff $\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \infty$

means: $f(n)$ grows strictly faster than $g(n)$.

say: $g(n)$ is a strict asymptotic lower bound for $f(n)$.



Thus

The limit of a reciprocal is the reciprocal of the limit. (also $\frac{1}{0} = \infty$ and $\frac{1}{\infty} = 0$.)

so: $f(n) = o(g(n))$ iff $\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = 0$

iff $\lim_{n \to \infty} \left(\frac{g(n)}{f(n)} \right) = \infty$

iff $g(n) = \omega(f(n))$.

so last 3 examples give rise to 3 more.

<u>note</u>:

- $f(n) = o(g(n))$ iff $g(n) = \omega(f(n))$

and

- $f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

Also note:

if $\frac{f(n)}{g(n)} \to 0$, then $\frac{f(n)}{g(n)} \leq B$ for $n \geq n_0$

all $B > 0$ all n_0

Thus if $f(n) = o(g(n))$, then $f(n) = O(g(n))$

But the converse is false

Similarly

if $f(n) = \omega(g(n))$, then $f(n) = \Omega(g(n))$

but not converseley.

Exercise: Justify this.

Analogy:

$f(n) = o(g(n)) \xleftrightarrow{\text{is analogous to}} x < y$

$f(n) = \omega(g(n))$ $\quad x > y$

$f(n) = O(g(n))$ $\quad x \leq y$

$f(n) = \Omega(g(n))$ $\quad x \geq y$

$f(n) = \Theta(g(n))$ $\quad x = y$

<u>note</u>: Just as $x < y \Rightarrow x \ngtr y$

we have

$$f(n) = o(g(n)) \Rightarrow f(n) \neq \Omega(g(n)).$$

<u>why?</u> if $\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$, then

$\frac{f(n)}{g(n)}$ can't be bounded below.

<u>Exercises</u>:

(1) $n^\alpha = \begin{cases} o(n^\beta) & \text{iff } \alpha < \beta \\ \Theta(n^\beta) & \text{iff } \alpha = \beta \\ \omega(n^\beta) & \text{iff } \alpha > \beta \end{cases}$

$$\frac{n^\alpha}{n^\beta} = n^{\alpha-\beta} \longrightarrow \begin{cases} 0 & \alpha < \beta \\ 1 & \alpha = \beta \\ \infty & \alpha > \beta \end{cases}$$

(2) $$a^n = \begin{cases} o(b^n) & \text{iff } a < b \\ \Theta(b^n) & \text{iff } a = b \\ \omega(b^n) & \text{iff } a > b \end{cases}$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \longrightarrow \begin{cases} 0 & a < b \\ 1 & a = b \\ \infty & a > b \end{cases}$$

(3) $$c \cdot f(n) = O(f(n)) \quad \text{for } c > 0$$

$$c \cdot f(n) = \Omega(f(n)) \quad ''$$

$$c \cdot f(n) = \Theta(f(n)) \quad ''$$

if $f(n) = o(g(n))$, then $c \cdot f(n) = o(g(n))$

if ... ω .. .. $c \cdot f(n) = \omega(g(n))$

(4) for any a, b with $a, b > 1$

$$\log_b(n) = \Theta(\log_a(n)).$$

<u>why?</u>

$$\log_b(n) = \text{const} \cdot \log_a(n)$$

$\uparrow$

$\log_b(a)$ (maybe $\log_a(b)$?)