CSE 101-01 6-2-22

- final exam: Wed June 8, 12:00-2:00 PM
- SETs: Due Sun June 5, 11:59 PM
- Pa8: ext. to Thur. 10 PM.

Runtime of Bellman Ford: $n = |V|$, $m = |E|$.

Initialize: $\Theta(n)$

Relax(-,-): $\Theta(1)$

$$\text{total} = (n-1)\cdot m\cdot\Theta(1) = \Theta((n-1)\cdot m) = \Theta(nm)$$

final loop: $\Theta(m)$

$$\text{Total cost} = \Theta(n) + \Theta(m) + \Theta(nm) = \Theta(nm)$$

Runtime of Dijkstra:

Initialize : $\Theta(n)$

P.Q. constructor : $\Theta(n)$

Extract min : $\Theta(\log n)$

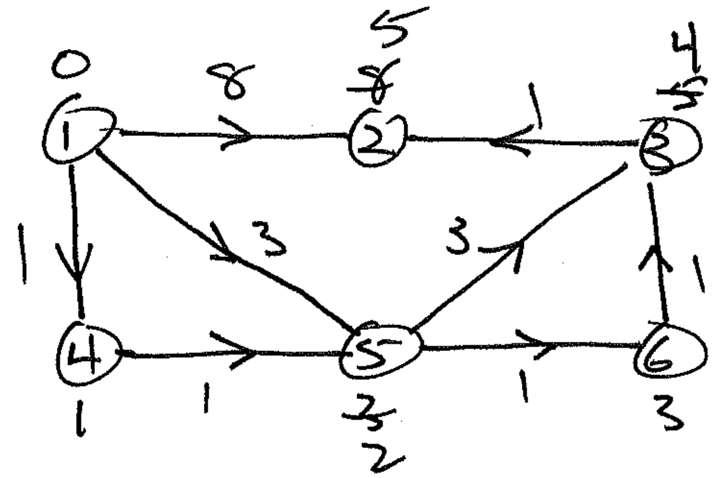
total : $\Theta(n \log n)$

Relax : $\Theta(\log n)$

total : $\Theta(m \log n)$

$$\text{Total cost} = \Theta(n) + \Theta(n \log n) + \Theta(m \log n)$$
$$= \Theta((n+m)\log n).$$

Ex. Dijkstra s = 1

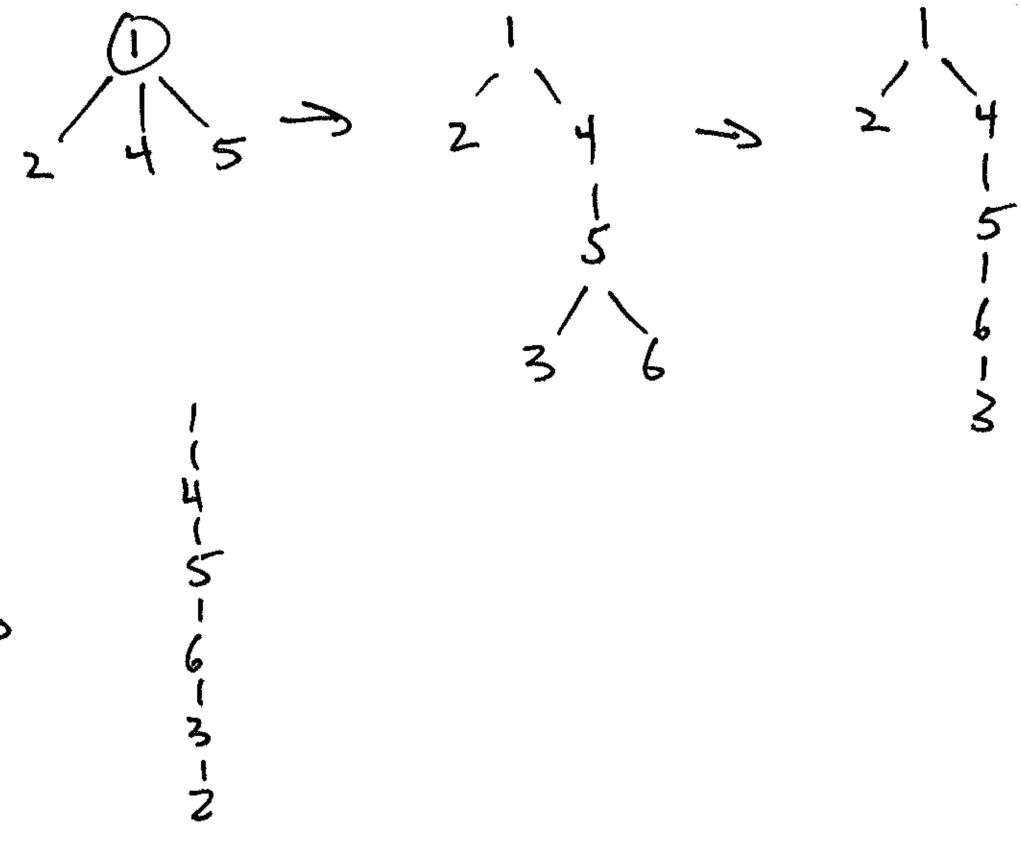


P.Q.	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d	0	~~8~~ 5	~~5~~ 4	1	~~3~~ 2	3
p	n	~~1~~ 3	~~5~~ 6	1	~~1~~ 4	5

Extraction order

1
4
5
6
3
2

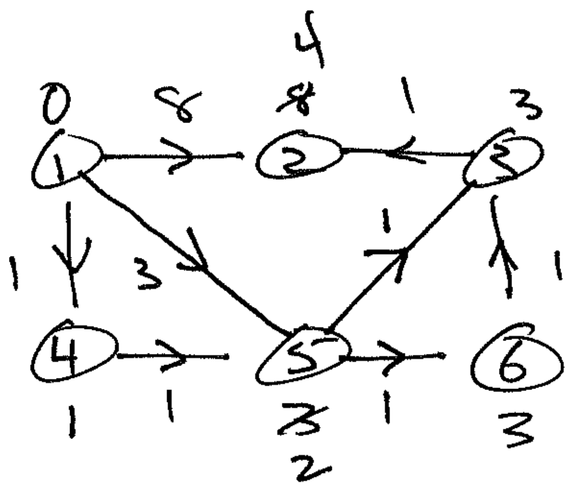
Tree!



<u>Convention</u> if two vertices have the same minimum d-value, extract the one with lower label.

<u>Ex.</u>

$\boxed{s=1}$

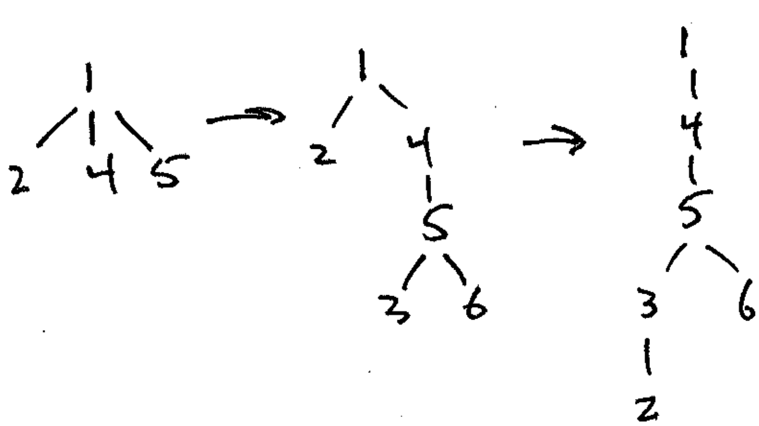


<u>extract order</u>

1
4
5
3
6
2

P.Q.:	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d :	0	~~8~~4	3	1	~~3~~2	3
p :	n	~~1~~3	5	1	~~1~~4	5

<u>tree:</u>



Chap 23: MWST

Defn

let $G = (V, E)$ be a graph (undirected).
A spanning tree in G is a subgraph T satisfying

(1) T is a tree

(2) $V(T) = V(G)$

Thm

G is connected iff G contains a spanning tree.

$\Rightarrow$

$\Leftarrow$ (obvious)

Defn

If $G = (V, E, w)$ is a weighted graph, a <u>minimum weight spanning tree</u> (MWST) is a sp. tree T such that

$$w(T) \leq w(S)$$

for all sp. trees S in G.

<u>note:</u>

$$w(T) = \sum_{e \in E(T)} w(e)$$

3 algorithms

- Prim
- Kruskal
- DualKruskal

Prim(G) (let $n = |V(G)|$)

1. $W = \emptyset$, $F = \emptyset$
2. Pick any $r \in V(G)$, set $W = \{r\}$
3. while $|F| < n-1$
4. amongst all edges in $E-F$ with one end in W and the other in $V-W$, let e be one of min weight
5. $F = F \cup \{e\}$
6. $W = W \cup \{\text{other end}\}$

<u>Thm</u>: when done $T = (\overset{\overset{V}{\|}}{W}, F)$ is a MWST.

<u>Kruskal(G)</u>

1. $F = \emptyset$
2. while $|F| < n-1$
3. amongst all edges whose addition to F would not create a cycle in F, let e be one of minimum weight
4. $F = F \cup \{e\}$

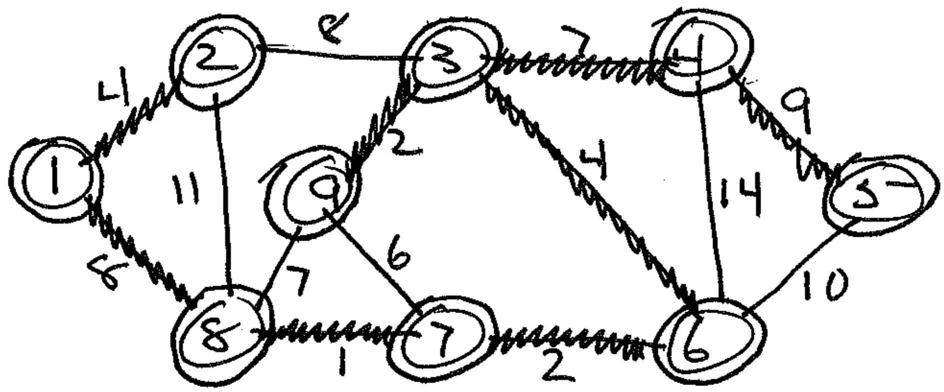
<u>Thm</u> when done: $T = (V, F)$ is a MWST in G.

DualKruskal(G)

1. $F = E(G)$
2. while $|F| > n-1$
3. amongst all edges in F whose removal from F does not disconnect (V, F), let e be one of max weight
4. $F = F - \{e\}$

Thm: when done: $T = (V, F)$ is a MWST in G.

Ex. (P. 632)



Prim: $r=3$

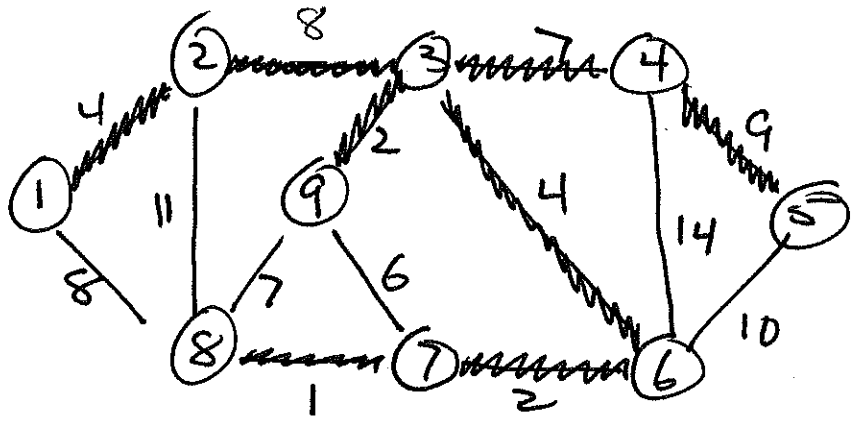
$W = \{3, 9, 6, 7, 8, 4, 1, 2, 5\}$

$F = \{39, 36, 67, 78, 34, 18, 12, 45\}$

$T = (W, F)$

$w(T) = 37$

Ex. Kruskal



$F = \{ 87, 67, 39, 36, 12, 34, 23, 45 \}$

$T = (V, F)$

$w(T) = 37$