CSE 101-01   5-31-22

- SETs : closes Sun. 11:59 pm

  incentive: if response rate ≥ 85%, then will add 0.5% to all scores.

- final exam:

  101-01 : Wed June 8, 12-2 PM

<u>Priority Queues</u> (max)

implemented as (max) Heap

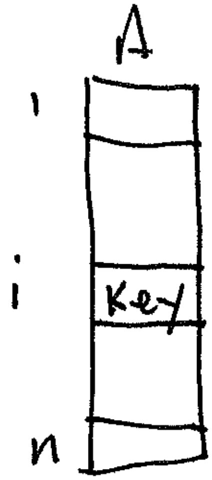
- HeapInsert(A, k)

  cost : $\Theta(\log n)$

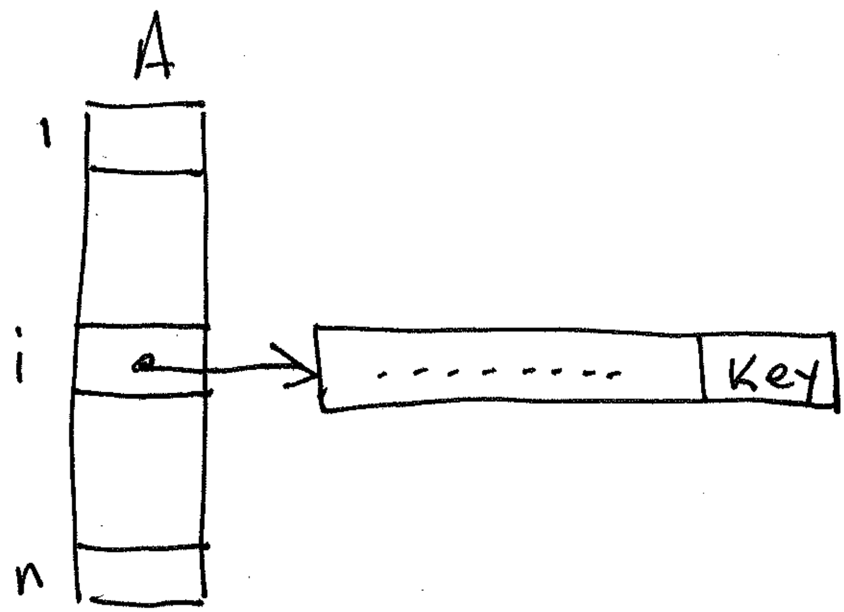
Exercise

re-write all algorithms (heap/P.Q.) for min heap/min P.Q.

our Picture:



general Picture:



Exercise: re-write everything in general Picture.

SSSP in weighted graph (ch. 24)

Defn

A weighted Graph (directed or undirected) is a Graph $G=(V,E)$ with a weight function $w: E \to \mathbb{R}$.

Defn

if P is a u-v path in G:

$$P: u = x_0, x_1, x_2, \ldots, x_k = v$$

then the weight of P, $w(P)$ is

$$w(P) = \sum_{i=1}^{k} w(x_{i-1}, x_i)$$

Defn

Shortest path weight from u to v is

$$\delta(u,v) = \begin{cases} \text{weight of a min weight } u\text{-}v \text{ path} \\ \text{if } v \text{ reachable from } u. \\ \\ \infty \text{ otherwise}. \end{cases}$$

A shortest u-v path is a u-v path P s.t. $w(P) = \delta(u,v)$.

SSSP problem:

Given a weighted graph $G = (V, E, w)$ and a source vertex $s \in V$, find $\delta(s,x)$ for all $x \in V$. Also, if $\delta(s,x) < \infty$, determine a shortest s-x path.

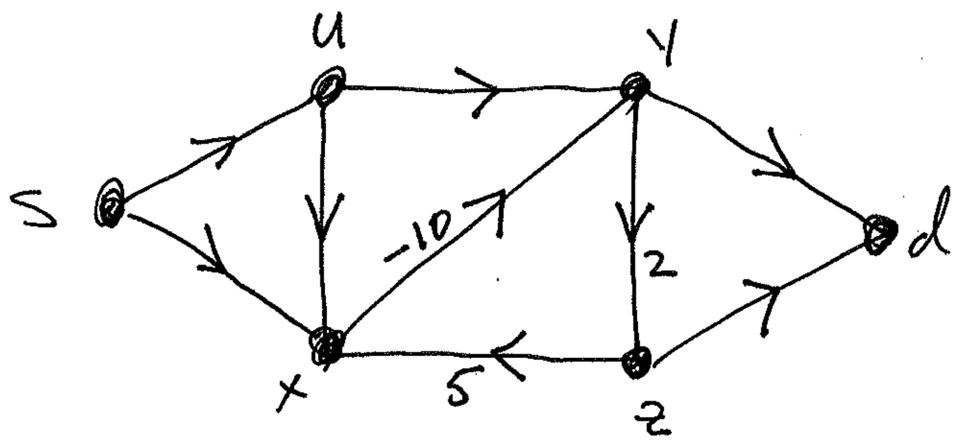
Assume G is a digraph.

2 Algorithms

- Dijkstra
- Bellman-Ford

Both algorithms actually find shortest walks. Problem occurs when there are negative weight cycles, reachable from source.

Ex.



$w(x,y,z,x) = -3$

Common Infrastructure:

$P[x]$: Parent, encodes a shortest Paths tree

$d[x]$: estimate $\delta(s,x)$

Predecessor subgraph : $G_p = (V_p, E_p)$

$$V_p = \{x \in V \mid P[x] \neq nil\} \cup \{s\}$$

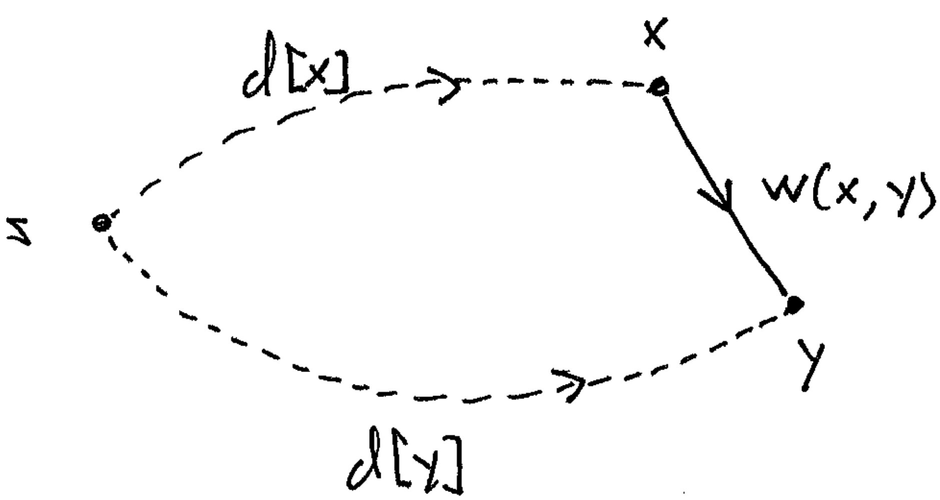
$$E_p = \{(P[x], x) \mid P[x] \neq nil\}$$

Use PrintPath(G, s, x) to find shortest Paths.

two functions

Initialize(G, s)

Relax(x, y)   pre: $y \in adj[x]$



<u>note</u>

- Relax(x, y) changes attributes of y only
- after Relax(x, y)

$$d[y] \le d[x] + w(x,y)$$

is true

<u>Lemma 1</u>

Let $x \in V$ and suppose after

$$\text{Initialize}(G, s)$$

is called, some sequence of calls to Relax(·, ·) results in $d[x]$ being finite. Then G contains an $s-x$ path of weight $d[x]$.

<u>Lemma 2</u>

After Initialize(G, s), the inequality

$$\delta(s, x) \leq d[x]$$

is maintained over <u>any</u> relaxation sequence.

Lemma 3 (Path Relaxation Property)

If

$$p: s = x_0, x_1, x_2, x_3, \ldots, x_k$$

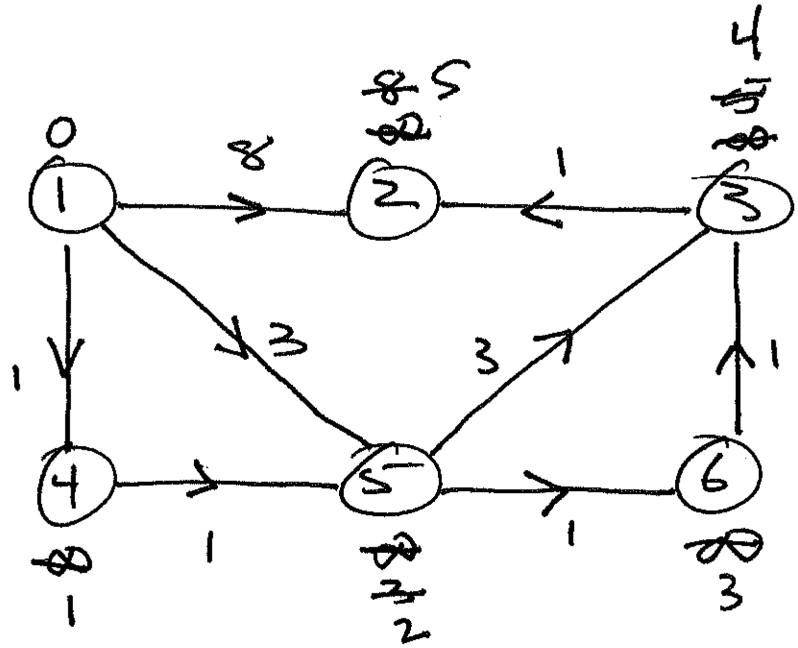
is a shortest s-x_k path, and the edges of p are relaxed in order

$$(x_0, x), (x_1, x_2), (x_2, x_3), \ldots, (x_{k-1}, x_k)$$

then $d[x_k] = \delta(s, x_k)$. Also, this is true regardless of any other relaxations that occur, even if interspersed with those above.

Ex. Dijkstra(G, 1)   s = 1

G



PQ:	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
d:	0	~~8~~ 5	~~5~~ 4	1	~~3~~ 2	3