

- mid2 : Thur 5/19
- Pa6 : ext. one day (Tue)
- Pa7 : Posted, due mon 5/23

Dictionary ADT (or Map)

a state is a set of ordered Pairs

(key, value)

i.e.

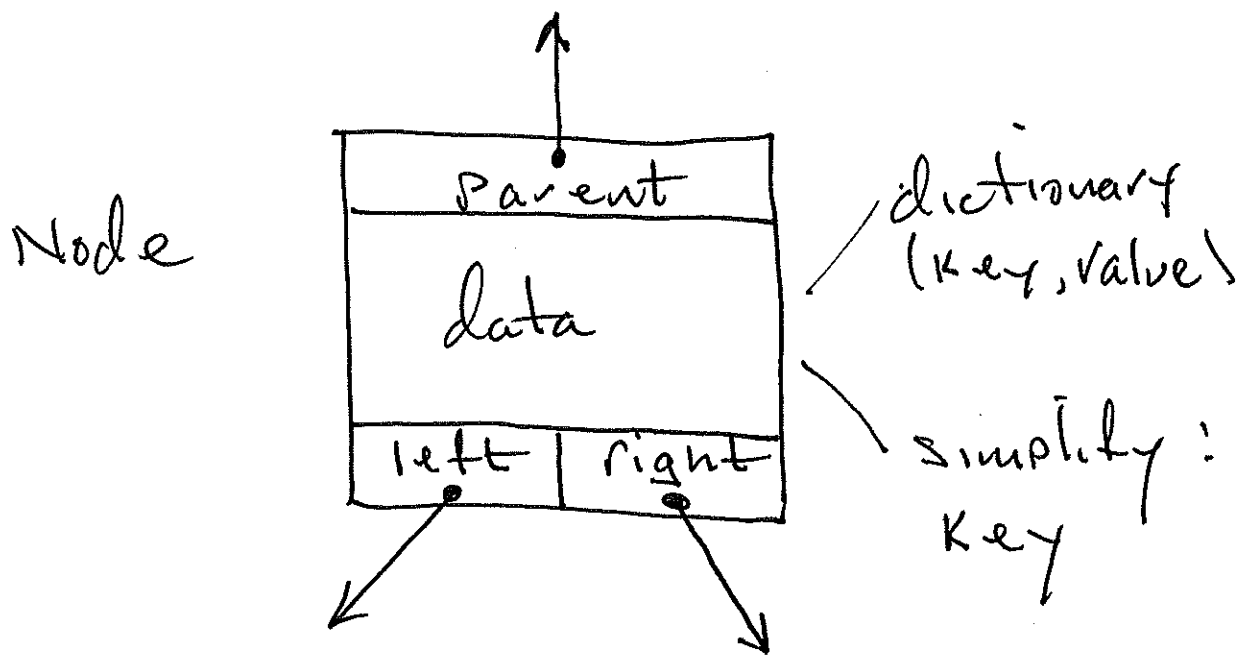
$$\{(k_1, v_1), (k_2, v_2), \dots, (k_n, v_n)\}$$

Option: Keys are unique.

main operations:

- $\text{insert}(k, v)$: adds (k, v) to a dictionary. Pre: $\text{lookup}(k) == \text{nil}$
- $\text{lookup}(k)$: returns v s.t. (k, v) is in dict. or nil if no such pair exists.
- $\text{delete}(k)$: removes (k, v) from dict. Pre: $\text{lookup}(k) \neq \text{nil}$

Binary Search Tree (BST)

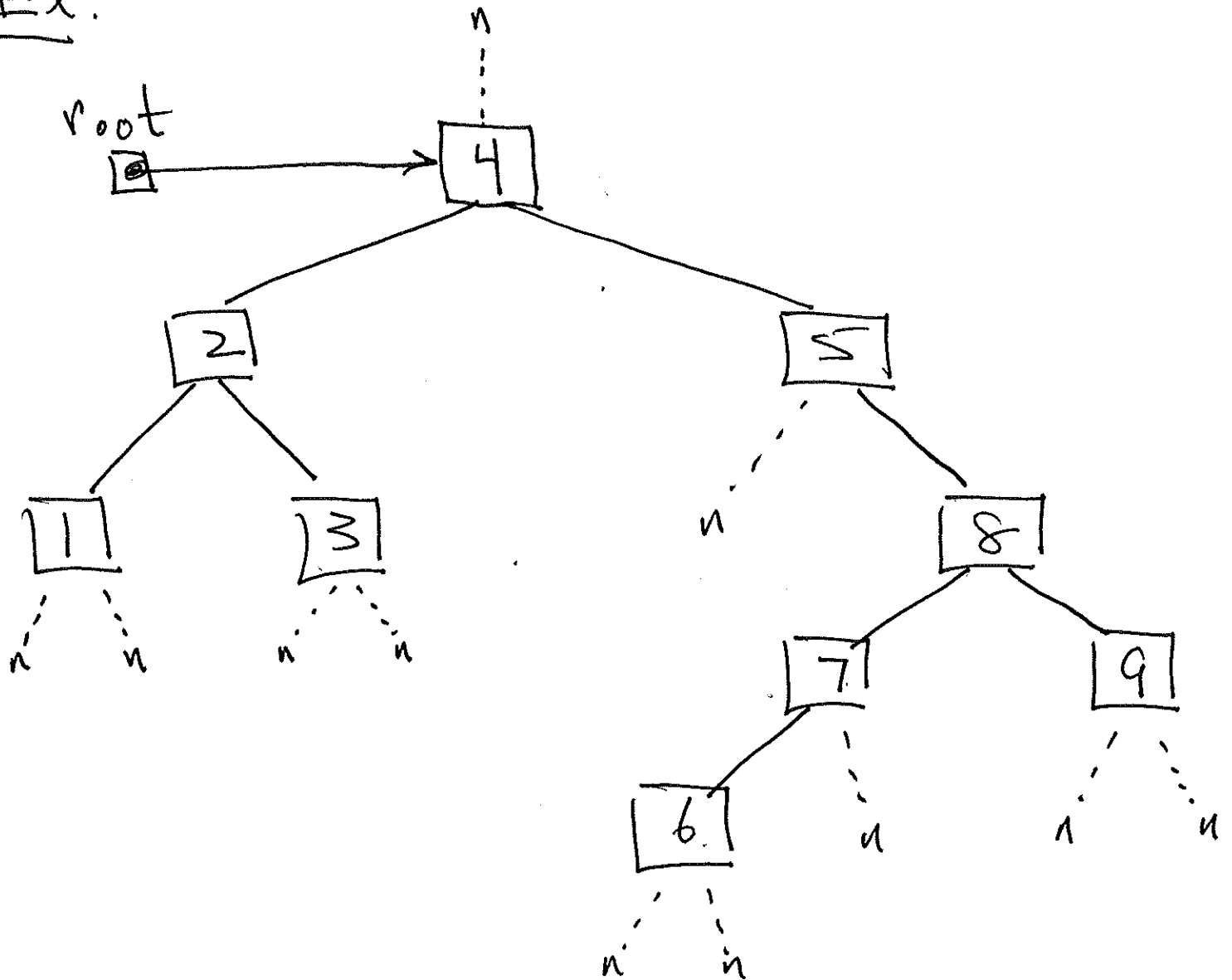


BST Properties

let x, y be nodes in a BST

(1) if y is in the left subtree of x
then $key[y] \leq key[x]$

(2) if y is in the right subtree of x
then $key[x] \leq key[y]$

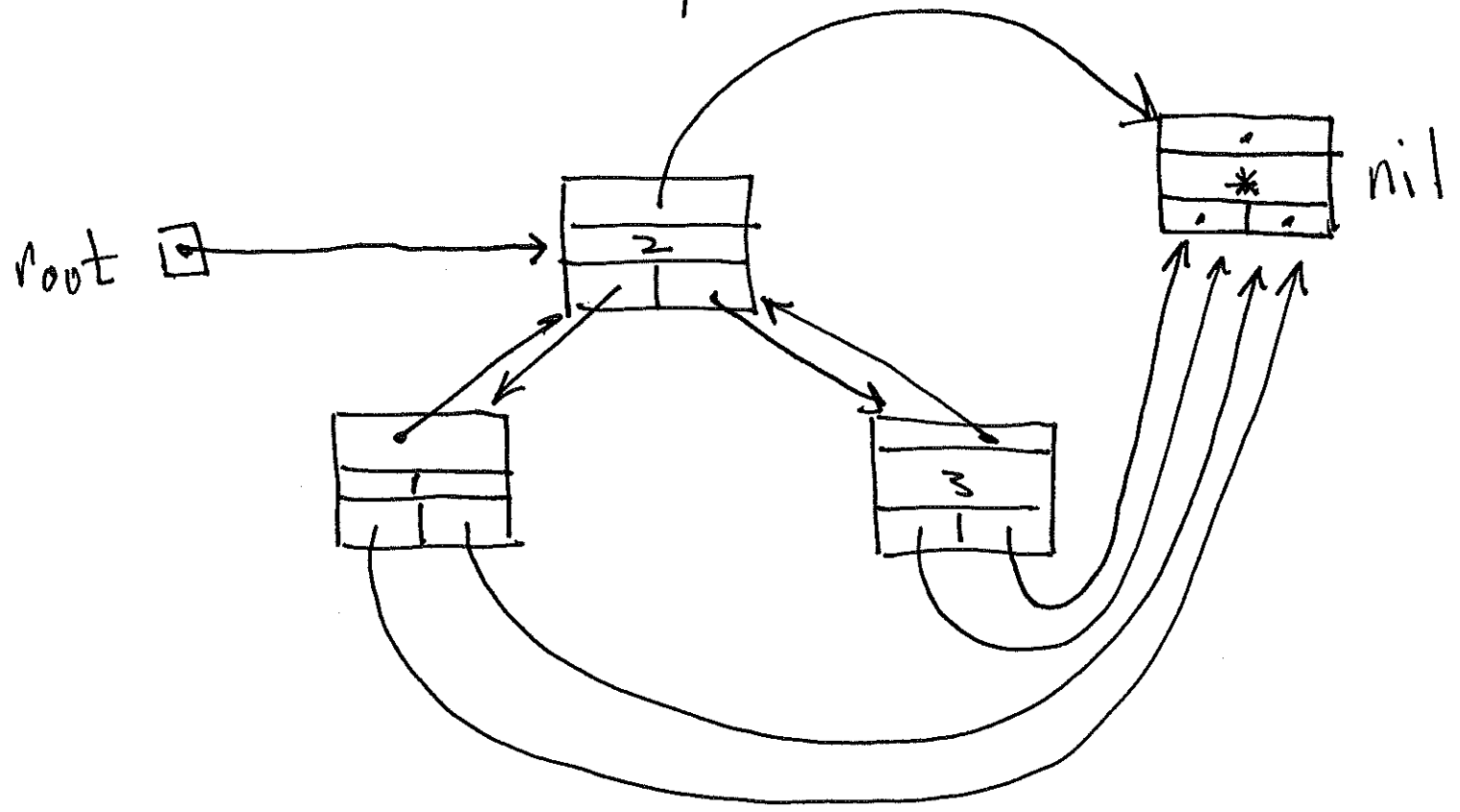
Ex.Defn

- a leaf is a node with no children
- an internal node is a non-leaf

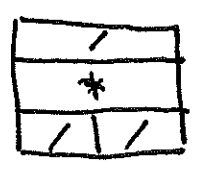
Implementation option:

have a dummy (or sentinal) node called nil in our dictionary fields.

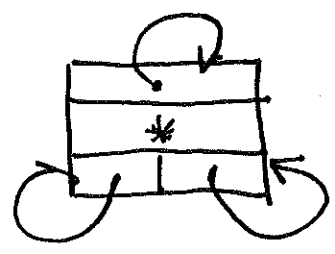
Ex. with dummy nil.



nil.
options



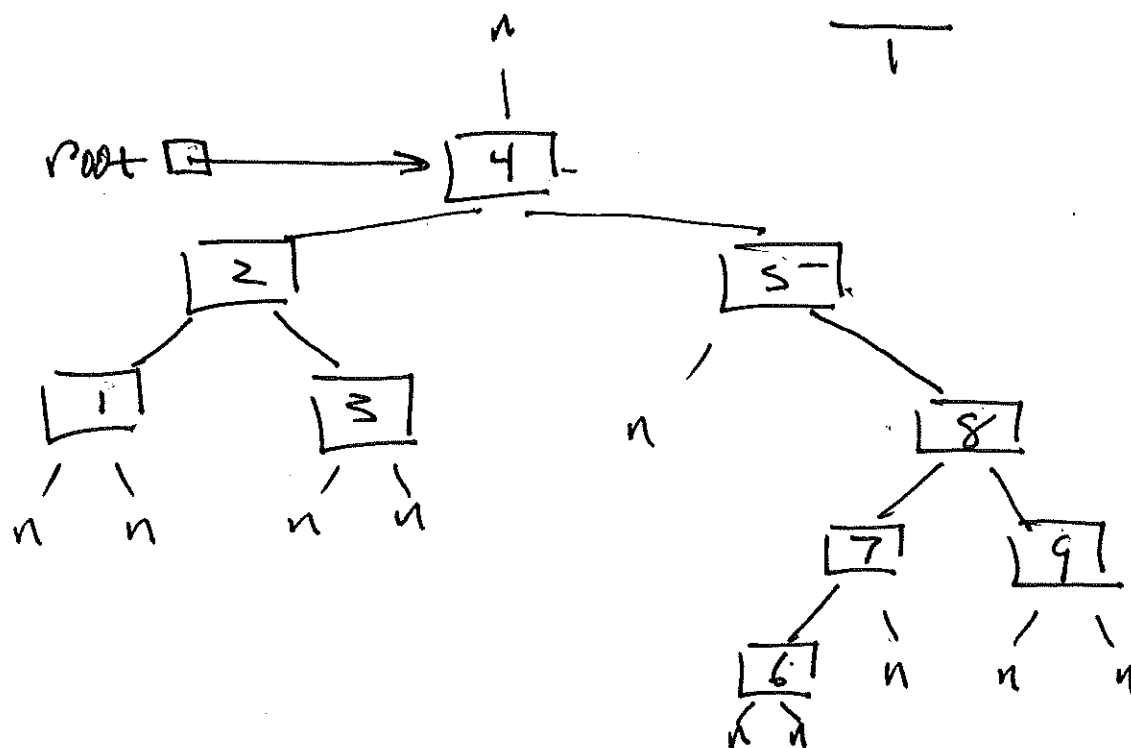
or



Tree Traversals

- InOrderTreeWalk(x) Node
- PreOrderTreeWalk(x)
- PostOrderTreeWalk(x)

Ex.



InOrderTreeWalk($root[T]$) :

1 2 3 4 5 6 7 8 9

PreOrderTreeWalk($\text{root}[T]$) :

4 2 1 3 5 8 7 6 9

PostOrderTreeWalk($\text{root}[T]$) :

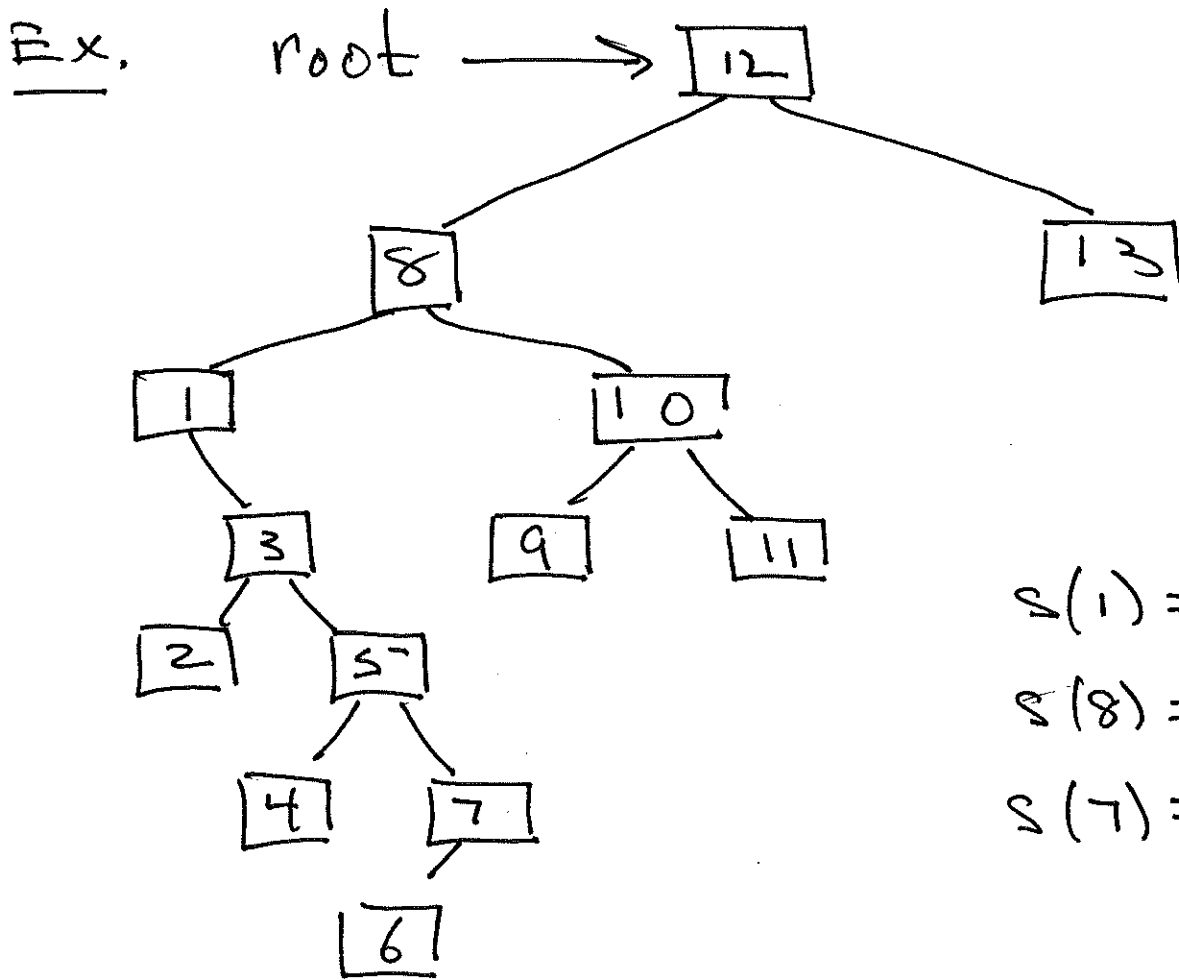
1 3 2 6 7 9 8 5 4

Queries

- TreeSearch(x, k)
- TreeMinimum(x)
- TreeMaximum(x)
- TreeSuccessor(x)
- TreePredecessor(x)

Defn

The successor of a node x is the next node after x to be processed in an I.D.T.W.



$$\left. \begin{array}{l} s(1) = 2 \\ s(8) = 9 \end{array} \right\} \text{Case 1}$$

$$s(7) = 8 \text{] Case 2}$$

I.D.T.W. : 1 2 3 4 5 6 7 8 9 10 11 12 13

Have 2 cases:

Case 1: x has a right child

Successor of x is the leftmost descendant of that child

Case 2: x has no right child

Successor of x is the lowest ancestor of x whose left child is also an ancestor of x .

Exercise:

- define Predecessor of x
- write TreePredecessor(x)

Routine

- Traversals: $\left. \begin{array}{l} \text{Pre} \\ \text{Post} \end{array} \right\} \text{cost} = \Theta(n)$

where $n = \# \text{ nodes}$.

- Queries:

Defn

$\text{height}(T) = \text{dist. from root to deepest leaf}$

$\text{height}(x) = \text{height of subtree rooted at } x.$

note: $\text{height}(x) \leq \text{height}(T)$