CSE 101-01 5-10-22

- Pa5 : Due tonight.
- Mid2 : Thur. 5/19

Pa6 : examples

Ex. subtraction $b = 100$, $p = 2$

$$
\begin{array}{r}
(\overset{-1}{8}\overset{}{8}\ \ 2\overset{-1}{1}\ \ 33) = 882{,}133 \\
(65\ \ 91\ \ 79) = 659{,}179 \\
\hline
(22\ \ 29\ \ 54) \quad\ 222{,}954
\end{array}
$$

another way:

$$\begin{pmatrix} 88 & 21 & 33 \end{pmatrix}$$
$$\begin{pmatrix} 65 & 91 & 79 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 23 & -70 & -46 \end{pmatrix}$$ vector diff

$$100 \quad 100$$

$$\begin{pmatrix} 22 & 29 & 54 \end{pmatrix}$$ normalize

Ex. same, other order

(65 91 79)

(88 21 33)

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leftmost digit negative → -23 70 46 vector diff

-1 -1 0

-1 23 -70 -46

Pull out (-1), then normalize 100 100

(22 29 54) normalize

return value = (-1)

note: helper fcns in BigInteger operate on Lists, so don't need to be members of BigInteger.

Ex. multiplication: $b = 10$, $p = 1$

$$\begin{array}{r} 123 \\ \times\ 456 \\ \hline 738 \\ 6150 \\ 49200 \\ \hline 56088 \end{array} \checkmark$$

(1 2 3) ← n digits

(4 5 6) ← m digits

		shift
(6 12 18)	scalar-mult	0
(7 3 8)	normalize	
(5 10 15 0)	scalar mult	1
(5 17 18 8)	add	
(6 8 8 8)	normalize	
(4 8 12 0 0)	scalar-mult	2
(4 14 20 8 8)	add	
(5 6 0 8 8)	normalize	

Runtime = (# digits in 1st) · (# digits in 2nd)

$= \Theta(m \cdot n)$

more detail:

(# digit mult) $= m \cdot n$

(# digit additions) $= 2m \cdot n + m^2$

Ex. Multiplication $b=10$, $P=1$

$999 \times 999 = 998001$ ✓

$$\begin{array}{r} (9\ 9\ 9) \\ (\underline{9}\ \underline{9}\ \underline{9}) \\ \hline \end{array}$$

Prod: ( )

						shift
	81	81	81		s.m.	0
	81	81	81		add	
8	9	9	1		norm	
81	81	81	0		s.m	1
89	90	90	1		add	
9	8	9	0	1	norm	
81	81	81	0	0	s.m	2
90	89	90	0	1	add	
(9	9	8	0	0	1) norm	

Note:

intermediate calculations did not exceed $10^2 = 100$. In general, calculations dont exceed b^2 for base b.

Exercise

check that in base b:

$$\underbrace{(b-1, b-1, \ldots\ldots, b-1)}_{n}{}^2$$

$$= (\underbrace{b-1, b-1, \ldots, b-1}_{n-1}, b-2, \underbrace{0, 0, \ldots, 0}_{n-1}, 1)$$

Ex. general normalize: $b=10$, $p=1$

$$L = \begin{pmatrix} -9 & 1 & 6 & -5 & -50 & 0 \\ 1 & -90 & 9 & 73 & 0 & -500 \end{pmatrix}$$

$$\begin{matrix} -89 & 15 & 68 & -50 \\ +90 & -10 & -60 & +50 & +500 \end{matrix}$$

$$\begin{pmatrix} -8 & 1 & 5 & 8 & 0 & 0 \end{pmatrix}$$

-1

$$\begin{matrix} -1 & -1 & -1 \\ 8 & -1 & -5 & -8 & 0 & 0 \end{matrix}$$

$$\begin{matrix} -2 & -6 \\ 10 & 10 & 10 \end{matrix}$$

-1

$$\begin{pmatrix} 7 & 8 & 4 & 2 & 0 & 0 \end{pmatrix}$$

check that:

$$1 \cdot 10^5 + (-90) \cdot 10^4 + 9 \cdot 10^3 + 73 \cdot 10^2 + 0 \cdot 10 + (-500) \cdot 10^1$$

$$= -784,200$$

Pub: from string constructor

should accept:

"+12345"	"+0"
"-12345"	"-0"
"12345"	"0"

Ex. b=100, P=2

"1|23|45" → (01, 23, 45)

Ex. b=1000, P=3

"12|345" → (012, 345)