CSE 101-01 4-28-22

Pa4: ext. 2 days (only)

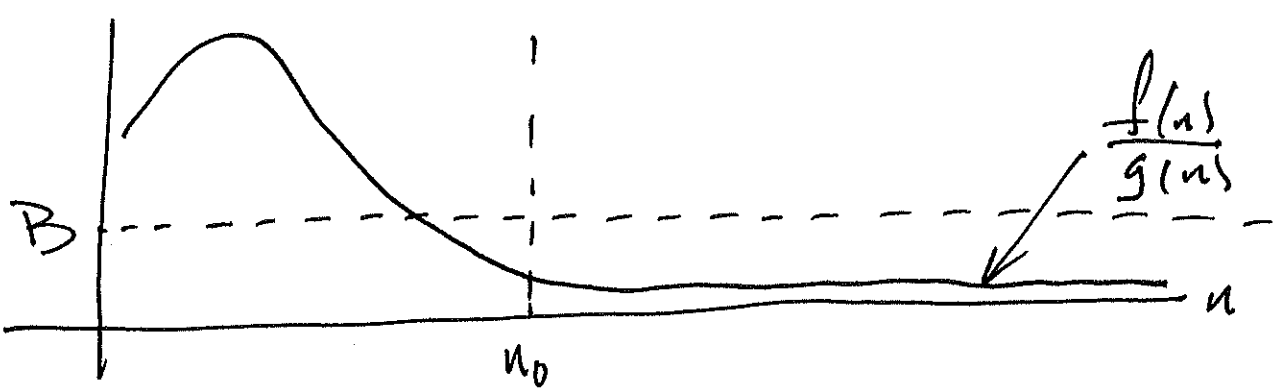
Handout: Runtime

analogy: $x < y \Rightarrow x \leq y$

similarly: $f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$ ✓

<u>why?</u>

$$f(n) = o(g(n)) \Rightarrow \lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$$



any $B > 0$ will work

$\therefore f(n) = O(g(n))$.

Exercise: explain same thing for ω vs. Ω.

Exercises

(1) for any $\alpha, \beta \in \mathbb{R}$

$$n^{\alpha} = \begin{cases} o(n^{\beta}) & \text{iff } \alpha < \beta \quad \checkmark \\ \Theta(n^{\beta}) & \text{iff } \alpha = \beta \quad \checkmark \\ \omega(n^{\beta}) & \text{iff } \alpha > \beta \quad \checkmark \end{cases}$$

$$\frac{n^{\alpha}}{n^{\beta}} = n^{\alpha - \beta} \xrightarrow[n \to \infty]{} \begin{cases} 0 & \text{iff } \alpha < \beta \\ 1 & \text{iff } \alpha = \beta \\ \infty & \text{iff } \alpha > \beta \end{cases}$$

(2) if $a, b \in \mathbb{R}$ with $a > 1, b > 1$,

then

$$a^n = \begin{cases} o(b^n) & \text{iff } a < b \quad \checkmark \\ \Theta(b^n) & \text{iff } a = b \quad \checkmark \\ \omega(b^n) & \text{iff } a > b \quad \checkmark \end{cases}$$

<u>why?</u>

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \xrightarrow[\text{as } n \to \infty]{} \begin{cases} 0 & \text{iff } a < b \\ 1 & \text{iff } a = b \\ \infty & \text{iff } a > b \end{cases}$$

(6) $f(n) + o(f(n)) = \Theta(f(n))$

<u>why?</u> let $h(n) = o(f(n))$ so that

$$\lim_{n \to \infty} \left(\frac{h(n)}{f(n)}\right) = 0$$

so that

$$\frac{f(n)+h(n)}{f(n)} = 1 + \frac{h(n)}{f(n)} \longrightarrow 1 \quad \text{as } n\to\infty$$

$\therefore \quad f(n)+h(n) = \Theta(f(n))$.

(7) compare $(\ln n)^2$ to $\ln(\ln\sqrt{n})$

$$\begin{aligned}\ln(\ln(\sqrt{n})) &= \ln(\ln(n^{1/2})) \\ &= \ln(\tfrac{1}{2}\ln n) \\ &= \ln(\tfrac{1}{2}) + \ln(\ln n) = \Theta(\ln(\ln(n)))\end{aligned}$$

$$\lim_{n\to\infty}\left(\frac{\ln(\ln n)}{(\ln n)^2}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{\ln n}\cdot\cancel{\frac{1}{n}}}{2\ln n\cdot\cancel{\frac{1}{n}}}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{2\cdot(\ln n)^2}\right) = 0$$

$\therefore \quad \ln(\ln\sqrt{n}) = o((\ln n)^2)$.

Ex. for $i = 1$ to n

for $j = 1$ to i

op

$$(\# \text{ ops}) = 1 + 2 + 3 + \cdots + n$$

$$= \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n = \Theta(n^2)$$

note:

let $S = 1+2+3+\ldots+(n-2)+(n-1)+n$

$S = n+(n-1)+(n-2)+\ldots+3+2+1$

$\therefore S+S = (n+1)+(n+1)+(n+1)+\ldots+(n+1)+(n+1)+(n+1)$

$2S = n(n+1)$

$\therefore S = \frac{n(n+1)}{2}$.

Ex SelectionSort(A)

```
n = length(A)
for i = 1 to (n-1)
    imin = i
    for j = i+1 to n
        if A[j] < A[imin]
            imin = j
    A[i] <-> A[imin]   // swap
```

$$\underbrace{A_1 \cdots\cdots A_{i-1}}_{\text{sorted}} \; \underbrace{A_i \; A_{i+1} \cdots A_n}_{\text{unsorted}}$$

Basic operation: comparison of array elements.

$(\#\text{comp}) = (n-1) + (n-2) + (n-3) + \cdots + 1$

$= 1 + 2 + 3 + \cdots\cdots + (n-1)$

$= \frac{(n-1)((n-\cancel{1})+\cancel{1})}{2} = \frac{n(n-1)}{2}$

$= \frac{1}{2}n^2 - \frac{1}{2}n = \Theta(n^2)$.

Ex. BFS: $n = |V(G)|$, $m = |E(G)|$

cost of initialization $= \Theta(n)$

" " Queue ops $= \Theta(n)$

" " scanning adj lists $= \Theta(m)$

total cost $= \Theta(n) + \Theta(n) + \Theta(m)$

$= \Theta(n+m)$

i.e. linear in # bytes for adj list rep.

Ex. DFS: $n = |V(G)|$, $m = |E(G)|$.

initialization: $\Theta(n)$

main loop (apart from Visit()) : $\Theta(n)$

calls to Visit (apart from for-loop) : $\Theta(n)$

Scanning adj lists : $\Theta(m)$

total cost $= \Theta(n) + \Theta(n) + \Theta(n) + \Theta(m)$

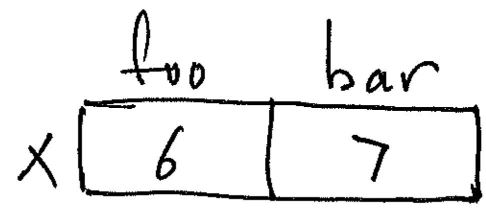
$$= \Theta(n+m),$$

i.e. linear in size of adj list representation.

Handout: ADT's in C++

struct in C:

```
struct Blah{ // no typedef
    int foo;
    int bar;
};
  :
struct Blah x;
x.foo = 6;
x.bar = 7;
```



struct in C++

```
struct Blah {
    int foo;
    int bar;        public by default

    Blah(int a, int b){
        foo = a;
        bar = b;
    }

};
:
Blah x(6, 7);
```

	foo	bar
x	6	7