cse 101-01 4-26-22

Pa4: Questions.

how to write dot(A, B)

Lists

Ex.

A : (10, ·) (100, ·) (200, ·) (300, ·)

B : (20, ·) (30, ·) (100, ·) (150, ·) (200, ·)

how to write changeEntry(M, i, j, x)

<u>case</u>: $M_{ij} = 0$, $x = 0$

do nothing

<u>case</u>: $M_{ij} = 0$, $x \neq 0$

do a list insert

<u>case</u>: $M_{ij} \neq 0$, $x \neq 0$

overwrite existing Entry (val)

<u>case</u>: $M_{ij} \neq 0$, $x = 0$

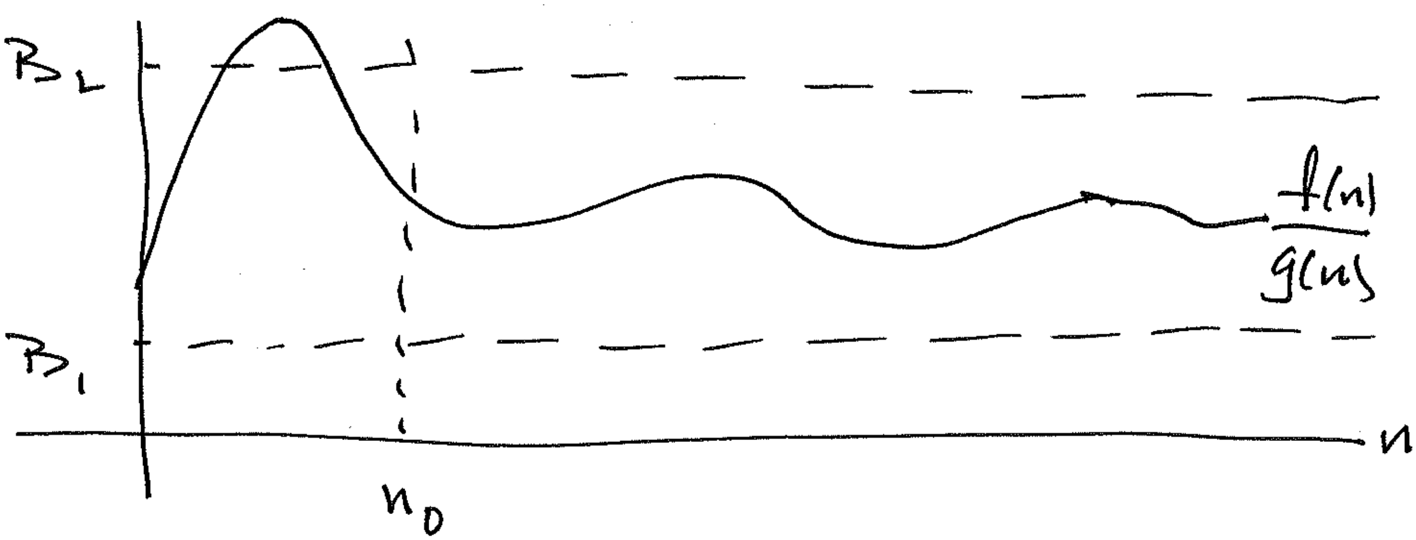
do a List delete

Handout: Runtime

Defn

$f(n) = \Theta(g(n))$ iff there exist pos. B_1, B_2, n_0 such that

$$B_1 \le \frac{f(n)}{g(n)} \le B_2 \quad \text{for all } n \ge n_0.$$



<u>means</u>: $f(n)$ and $g(n)$ have same growth rate.

<u>say</u>: $g(n)$ is a <u>tight</u> asymptotic bound for $f(n)$.

<u>EX.</u> $f(n) = 5n^2 + 11n - 24$, $g(n) = n^2$. Then check

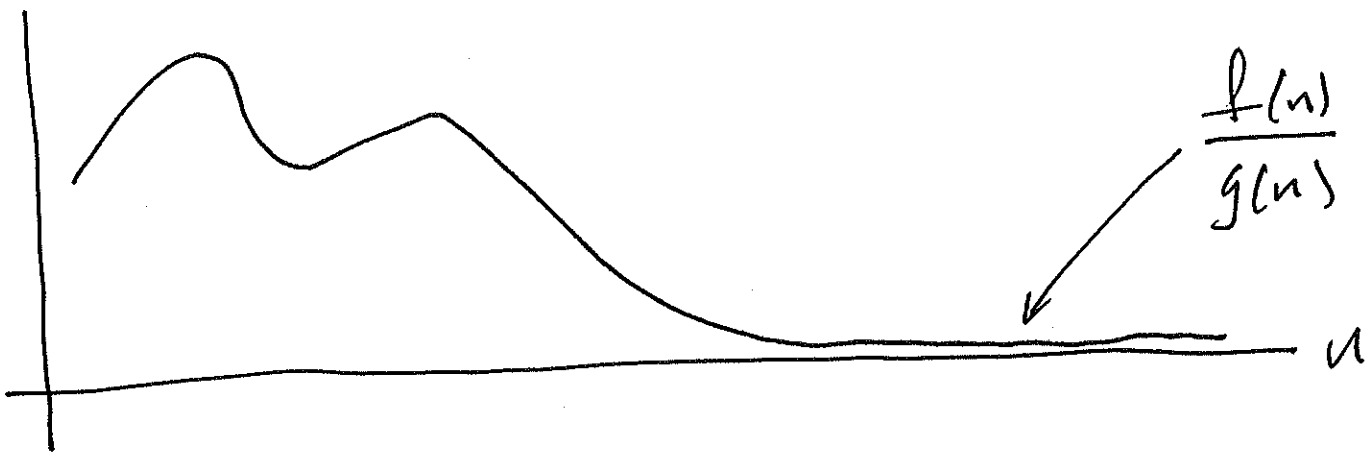
$$\overset{B_1}{4} \le \frac{5n^2 + 11n - 24}{n^2} \le \overset{B_2}{6}$$

for all $n \geq \overset{n_0}{8}$.

$\therefore \quad 5n^2 + 11n - 24 = \Theta(n^2)$

Defn

$f(n) = o(g(n))$ iff $\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$.



means: $f(n)$ grows strictly slower than $g(n)$

say: $g(n)$ is a strict asymptotic upper bound for $f(n)$.

Ex. $f(n) = \ln(n)$, $g(n) = n$

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(\frac{\ln n}{n}\right) = \lim_{n\to\infty}\left(\frac{1/n}{1}\right) = 0$$

$\therefore$ $\ln(n) = o(n)$.

Exercise: show $(\ln(n))^{k_1} = o(n^{k_2})$ for any $k_1, k_2 > 0$.

Ex. $f(n) = n^k$, $g(n) = e^n$. $(k \geq 0)$

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(\frac{n^k}{e^n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{kn^{k-1}}{e^n}\right)$$

$$\vdots$$

$$= 0$$

after $\lceil k \rceil$ applications of l'Hop. rule.

$\therefore \; n^k = o(e^n)$

Exercise: show $n^k = o(b^n)$ for any $k \geq 0$, $b > 1$.

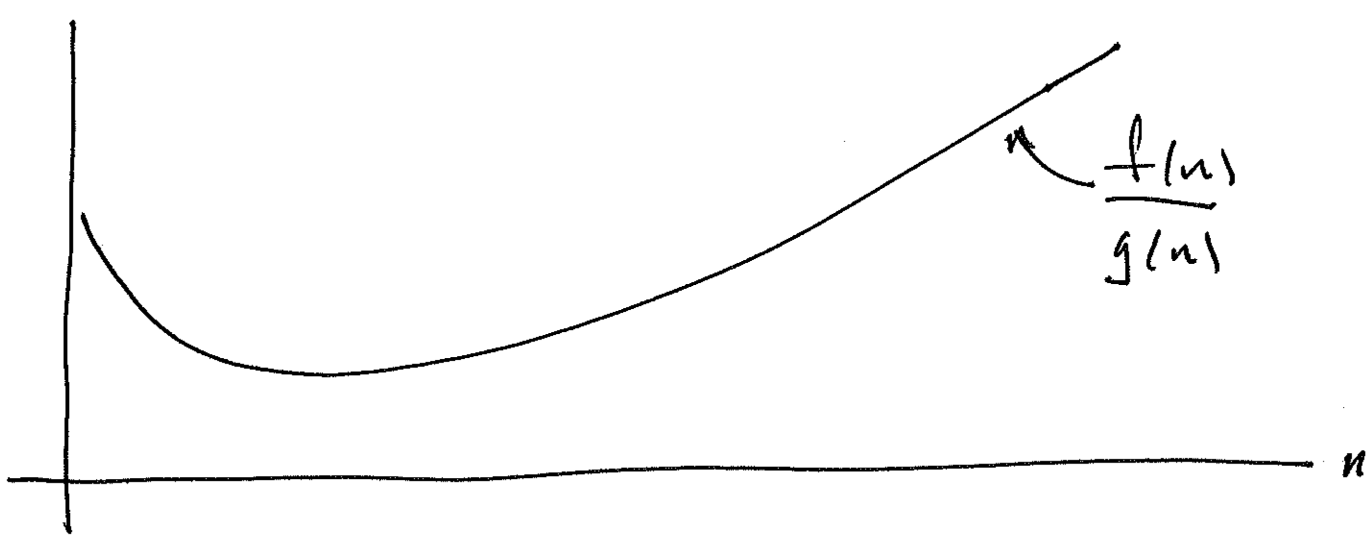
Ex. $f(n) = n^{\alpha}$, $g(n) = n^{\beta}$ where $0 \le \alpha < \beta$.

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(n^{\alpha-\beta}\right) = 0 \text{ since } \alpha-\beta<0.$$

$\therefore \quad n^{\alpha} = o(n^{\beta})$.

Defn

$f(n) = \omega(g(n))$ iff $\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \infty$.



Means: $f(n)$ grows strictly faster than $g(n)$.

Say: $g(n)$ is a strict asymptotic lower bound to $f(n)$.

Thm:
The limit of a reciprocal is the reciprocal of the limit. (recall $\frac{1}{0} = \infty$ and $\frac{1}{\infty} = 0$.)

Thus $f(n) = o(g(n))$

iff $\lim \left(\frac{f(n)}{g(n)} \right) = 0$

iff $\lim \left(\frac{g(n)}{f(n)} \right) = \infty$

iff $g(n) = \omega(f(n))$.

so the previous 3 examples for $o(\ )$ give rise to 3 examples for $\omega(\ )$.

<u>Ex.</u> $f(n) = a^n$, $g(n) = b^n$ where $1 \le a < b$.

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \lim_{n\to\infty}\left(\frac{a}{b}\right)^n = 0$$

$\therefore \quad a^n = o(b^n)$

$\therefore \quad b^n = \omega(a^n)$

<u>Analogy</u>: comparison of growth rates vs. comparison of numbers

$$f(n) = O(g(n)) \xleftrightarrow{\text{is analogous to}} x \le y$$

$$f(n) = \Omega(g(n)) \qquad x \ge y$$

$$f(n) = \Theta(g(n)) \qquad x = y$$

$$f(n) = o(g(n)) \qquad x < y$$

$$f(n) = \omega(g(n)) \qquad x > y$$

<u>Transitivity</u>: $x \le y$ and $y \le z$ then $x \le z$

$$f(n) = O(g(n)) \Rightarrow \frac{f(n)}{g(n)} \le B_1 \text{ for } n \ge n_1$$

$$g(n) = O(h(n)) \Rightarrow \frac{g(n)}{h(n)} \le B_2 \text{ for } n \ge n_2$$

Let $B_3 = B_1 \cdot B_2$ and $n_3 = \max(n_1, n_2)$

Then

$$\frac{f(n)}{h(n)} = \frac{f(n)}{g(n)} \cdot \frac{g(n)}{h(n)} \leq B_1 \cdot B_2 = B$$

for all $n \geq n_3$.