

CSE 101-01 4-14-22

11

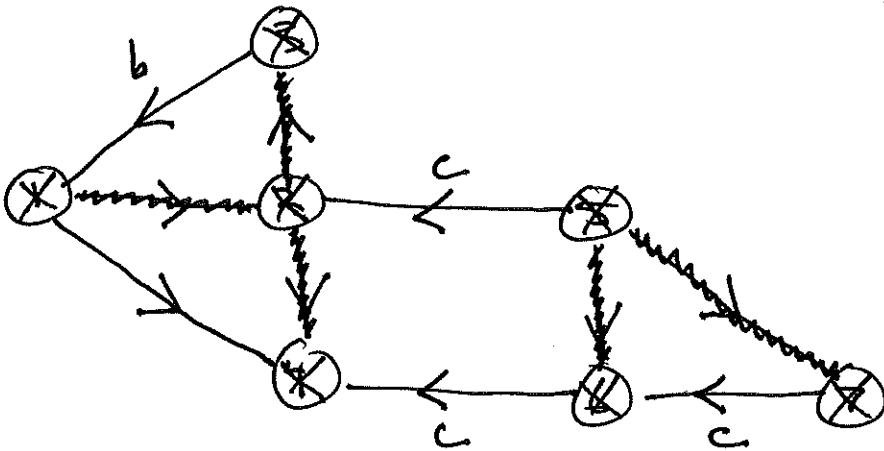
Paz: Ext. 1 last day

What structural information
does DFS give on a
digraph.

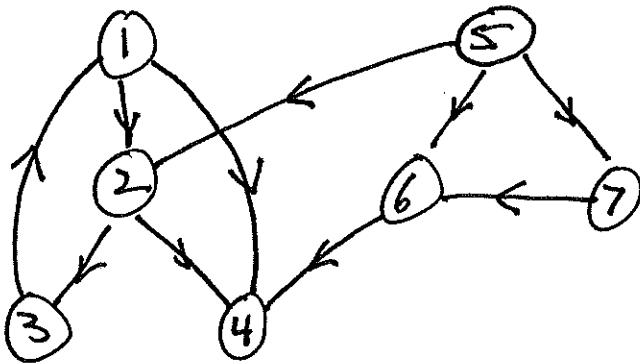
Edge Classification

- Tree: belong to Predecessor subgraph
- Back: join Descendant $\rightarrow$ ancestor
- Forward: join ancestor $\rightarrow$ descendant
- Cross: all else
 - cousin to cousin in some tree
 - tree to tree

Ex



forest



Tree:

$$(1,2), (2,3), (2,4), (5,6), (5,7)$$

Back:

 $(3, 1)$

Forward:

 $(1, 4)$

Cross:

$$C \rightarrow C : (7, 6)$$
$$t \rightarrow t': (5, 2), (6, 4)$$

Defn

A digraph G with no directed cycles is called a directed acyclic graph (DAG).

Lemma

a digraph G is acyclic iff $\text{DFS}(G)$ yields no back edges.

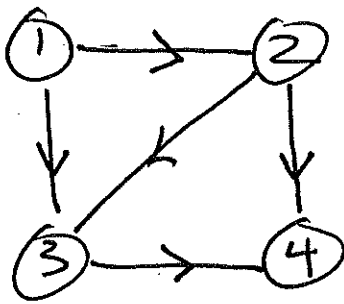
(i.e. dir. cycle $\equiv$ back edge)

Defn

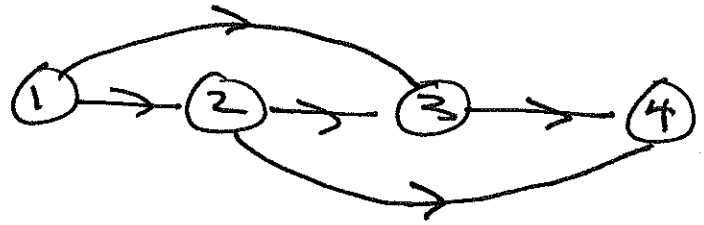
A Topological sort of a DAG G is a linear ordering of $V(G)$ such that:
if $(x, y) \in E(G)$ then x is before y in the ordering.

EX.

G



Topological Sort:

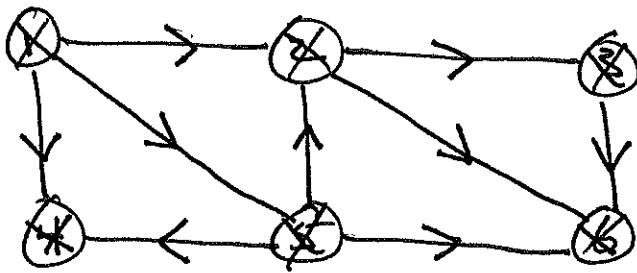


To find a T.S. of a DAG

- Run DFS(G), as vertices finish, push them onto a stack S.

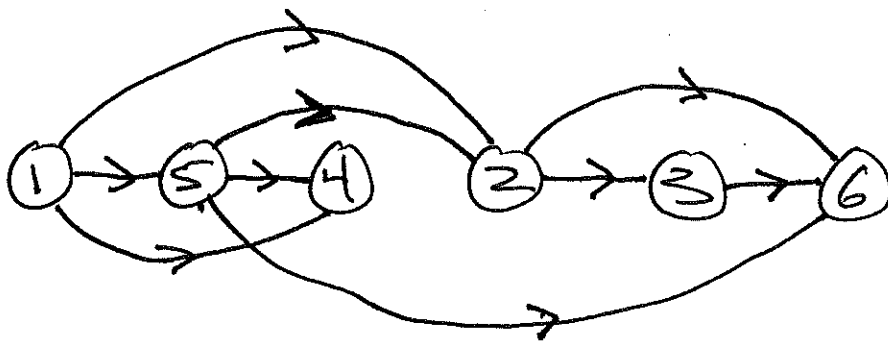
when complete, S is a T.S. of $V(G)$.

Ex.



stack

1
5
4
2
3
6



note: there are 3 other Topological sorts.

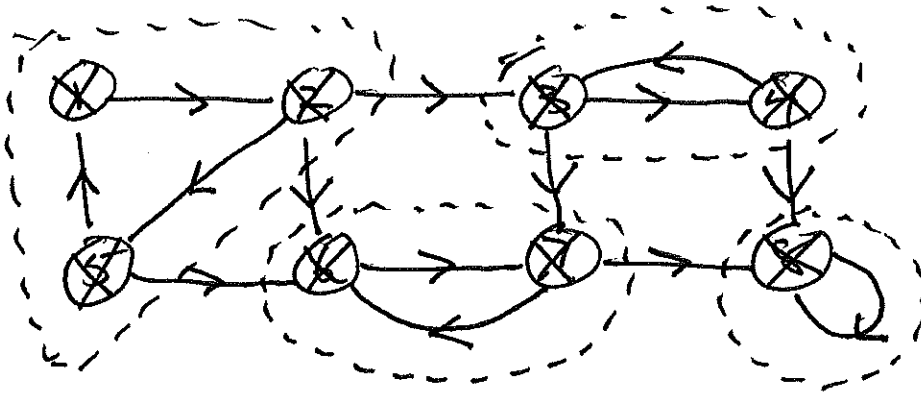
To find the S.C.C. of
A digraph G (not necessarily
a DAG):

- Run DFS(G), as vertices finish push them onto a stack S .
- compute the transpose G^T of G (i.e. reverse all directed edges in G .)
- Run DFS(G^T), this time executing main loop by decreasing finish times from 1st call, i.e. Pop vertices of stack S .

When complete, the trees in the DFS forest (2nd call) have vertex sets constituting the S.C.C.s of G .

Ex.

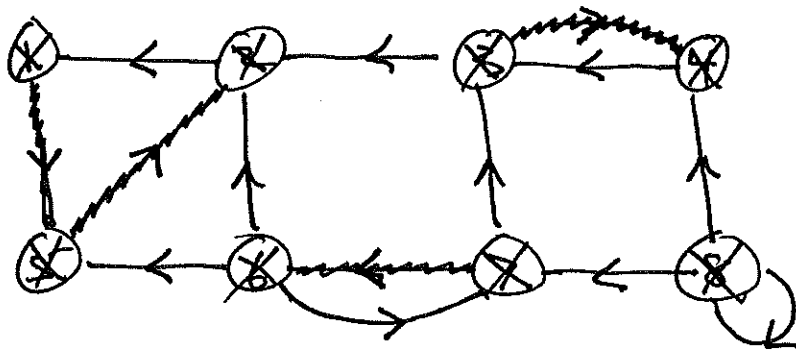
G :



stack

- 1 ✓
- 2 ✓
- 5 ✓
- 3 ✓
- 7 ✓
- 6 ✓
- 4 ✓
- 8 ✓

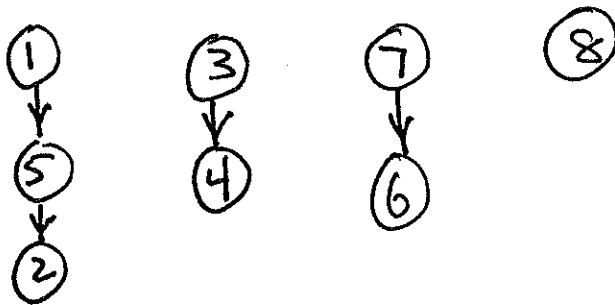
G^T :



stack

- 8
- 7
- 6
- 3
- 4
- 1
- 5
- 2

forest:



note: G and G^T have same S.C.C.s

Defn

The Component Graph (or Condensation Graph) of G , denoted G^{scc} is the digraph with

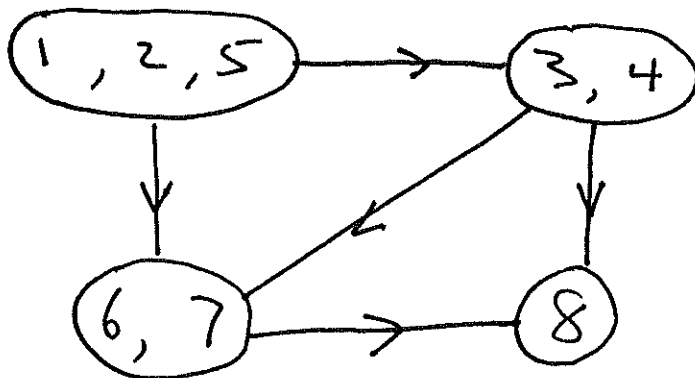
$V(G^{scc}) = \{\text{the s.c.c.s of } G\}$

$E(G^{scc}) : (C_i, C_j) \in E(G^{scc}) \text{ iff}$

$\exists x \in C_i, y \in C_j \text{ s.t. } (x, y) \in E(G)$

Ex.

G^{scc}



Top. sort of G^{scc} :

$C_1 : 1, 2, 5$

$C_2 : 3, 4$

$C_3 : 6, 7$

$C_4 : 8$

$$\left. \begin{array}{l} \text{note} \\ (G^{scc})^T \\ = (G^T)^{scc} \end{array} \right\}$$

To find a T.S. of G^{scc} : read
2nd stack bottom to top, using
n.l Parent vertices as separators,

Pa3:

$S: 1\ 2\ 3\ \dots\ n$

$DFS(G, S)$

$S: \text{dec. finish times}$

$DFS(G^T, S)$

$S: \text{dec. fin. time}$

use this to find

- ① SCCs of G
- ② T.S. of G^{SCC}