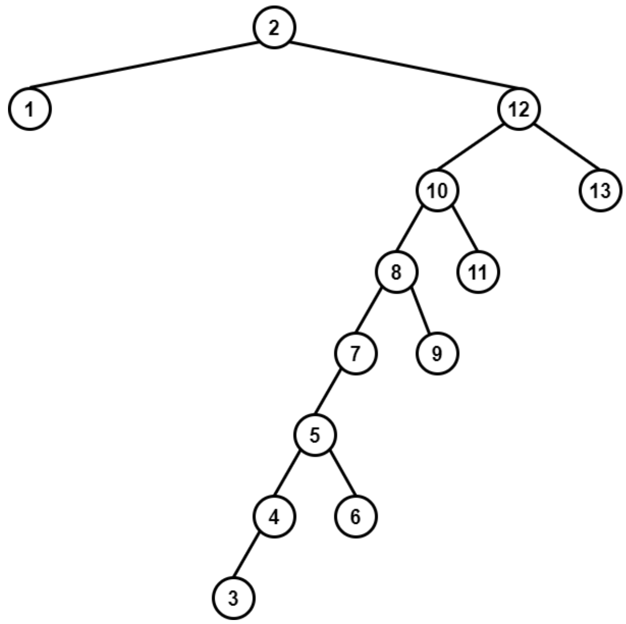
CSE 101
Spring 2021
Quiz 5 **Solutions**

1. (25 Points) Consider the Binary Search Tree pictured below.



Use the Theorem discussed in class (see notes from [5-18-21] page 9) to explain why it is **impossible** to assign colors Red and Black to the nodes in this tree so as to satisfy the RBT properties. (Note: be sure to include the nil leaves when calculating the height of this tree.)

Solution:
According to the stated theorem, we must have $h \leq 2\lg(n+1)$ for any RBT with n internal nodes and (absolute) height h. Observe that the tree above has $n = 13$ and $h = 8$ (counting the nil children of node 3 as the deepest leaves.) But then

$$2\lg(n+1) = 2\lg(14) < 2\lg(16) = 2 \cdot 4 = 8 = h$$

making the inequality in the theorem is false. Therefore above tree cannot be colored so as to satisfy the RBT properties. ■

2. (25 Points) Perform BuildHeap(A) on the following unordered array, making it into a max-heap. (The heap algorithms can be found here.) Observe that identical keys are accompanied by letters representing different satellite data. Thus the elements 2a and 2b have the same key, but are distinguishable elements in the max-heap.

A	2a	4	7	1a	2b	3	1b	5a	2c	6	8	5b

Show the state of array A after the call to BuildHeap(A).

Solution:

A	8	6	7	5a	4	5b	1b	1a	2c	2a	2b	3