CSE 101 6-3-21

- quizes solutions posted
- Final mon. 6/7
- SETs

SSSP weighted graph

Lemma 1

Let $x \in V(G)$. Suppose after Initialize(G, s), that some sequence of calls to Relax$(\cdot, \cdot)$ results in $d[x]$ being finite. Then G contains an s-x path of weight $d[x]$.

Lemma 2

After Initialize(G, s), the inequality

$$\delta(s, x) \leq d[x] \quad \text{(for all } x \in V(G)\text{)}$$

is maintained over any sequence of calls to Relax$(\cdot,\cdot)$.

Lemma 3 (Path relaxation property)

If $P: s = x_0, x_1, x_2, \ldots, x_k$ is a minimum weight $s \rightsquigarrow x_k$ path in G, and if the edges of P are relaxed in order:

$$(x_0, x_1), (x_1, x_2), (x_2, x_3), \ldots, (x_{k-1}, x_k)$$

then $d[x_k] = \delta(s, x_k)$. This is true regardless of any other relaxations that may occur, even if interleaved with the above relaxations.

Proof: in book

lemma 24.15, P. 673

- see Bellman-Ford

Runtime $n = |V(G)|$, $m = |E(G)|$

- Initialize: $\Theta(n)$
- Relax(·,·) cost $\Theta(1)$
  Each edge is relaxed $(n-1)$ times in first for loop.
  cost = $\Theta(m(n-1)) = \Theta(mn)$
- second for loop cost: $\Theta(m)$
- total cost = $\Theta(mn) + \Theta(m)$
  $= \Theta(mn)$

- see Dijkstra

Runtime:

Assume Priority Queue Q is implemented as a min-heap.

In this case, Relax(·,·) implies a call to DecreaseKey(·,·)

- cost of Relax: $\Theta(\log n)$
- Initialize: $\Theta(n)$
- ExtractMin: $\Theta(\log n)$

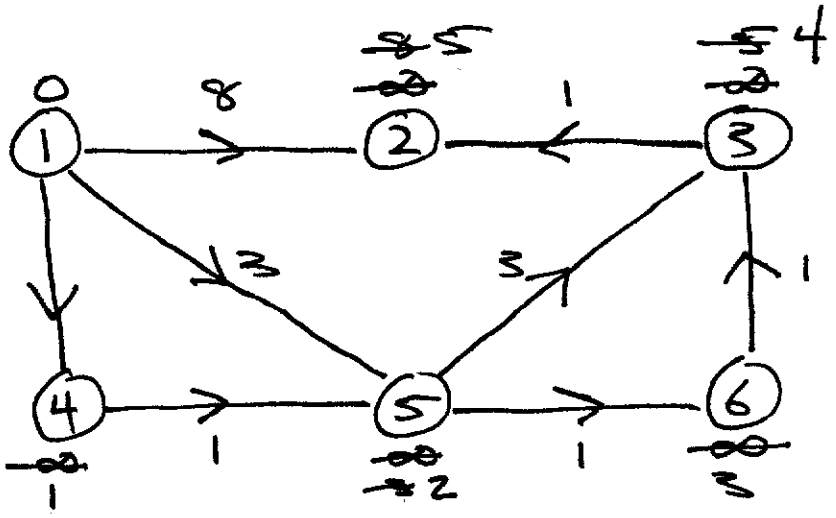
  total cost: $\Theta(n \log n)$
- Relax: $\Theta(\log n)$

  total cost: $\Theta(m \log n)$
- Runtime: $\Theta(n \log n) + \Theta(m \log n)$

  $= \Theta((n+m) \log n)$

Ex

s=1



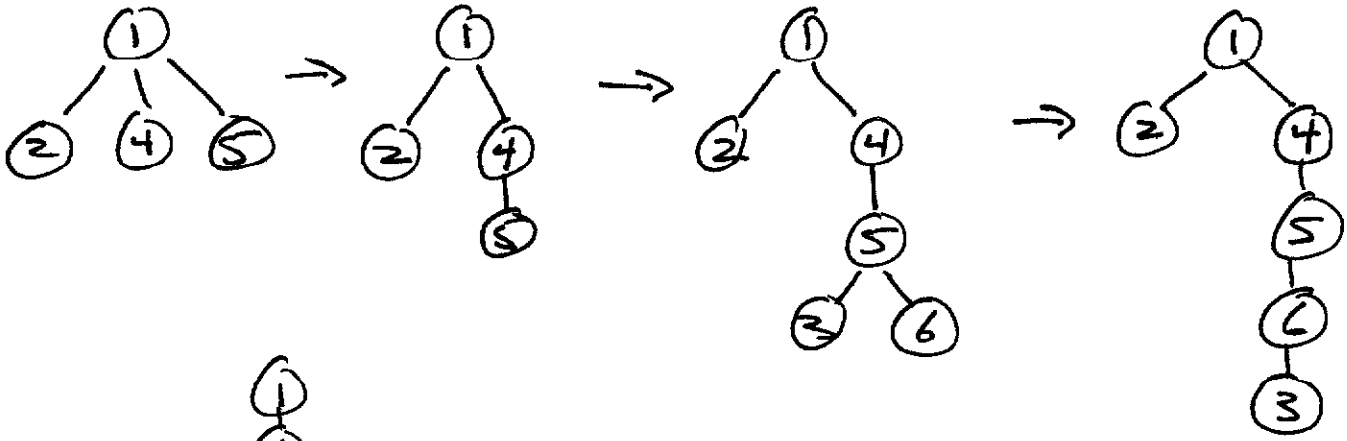
PQ: ~~x~~ ~~x~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~

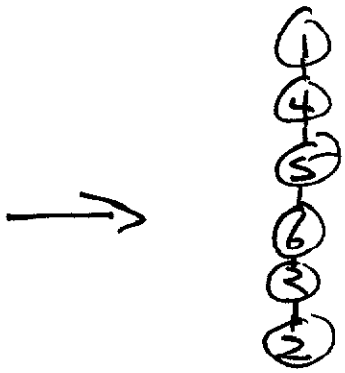
d: 0 ~~5~~ ~~4~~ 1 ~~2~~ 3

P: n ~~3~~ ~~6~~ 1 ~~4~~ 5

Removal order: 1, 4, 5, 6, 3, 2

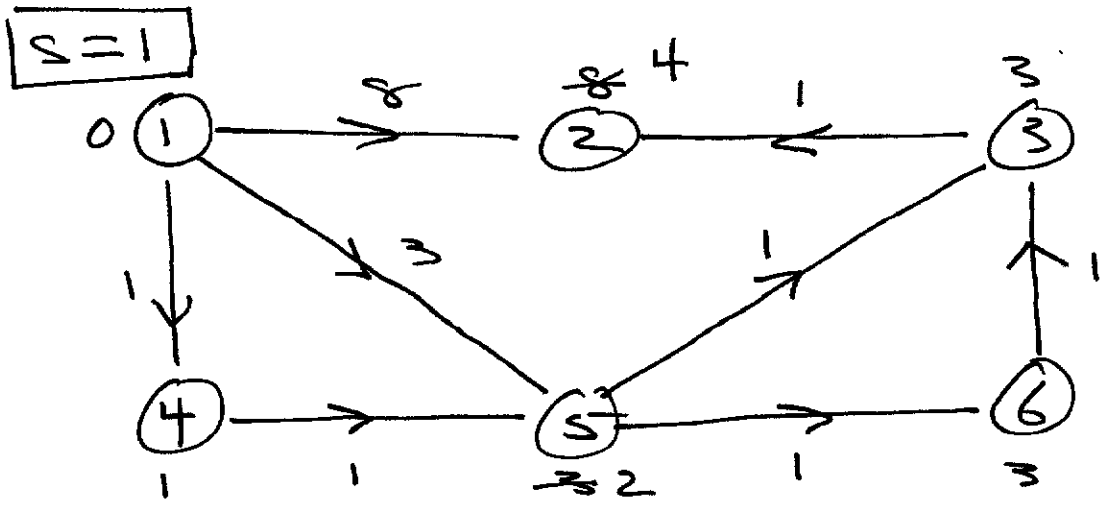
Tree:





Ex. same but 1 diff weight

Convention: If there are multiple minimum keys (d-values), extract the one with lowest label first.



P.Q: ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6

d: 0 ~~4~~ 3 1 ~~2~~ 3

p: n ~~3~~ 5 1 ~~4~~ 5

order of removal from P.Q:

1, 4, 5, 3, 6, 2

See:

CmPS 101/Winter 06/Pa5.pdf

from my personal homepage

Exercise: do this Project.