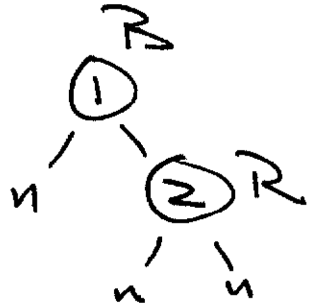
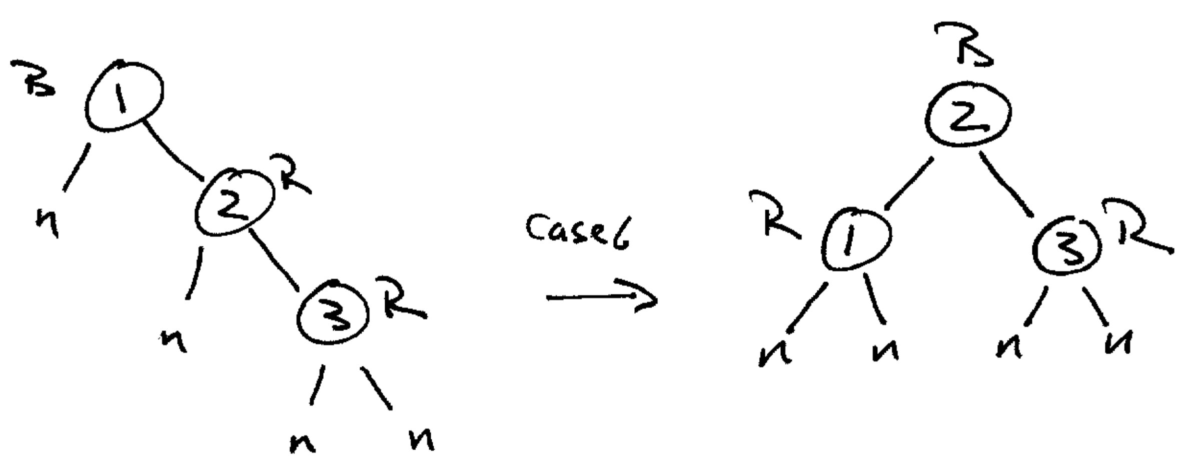
cse 101 5-25-21

RB-Insert ; RB-insert-fixup

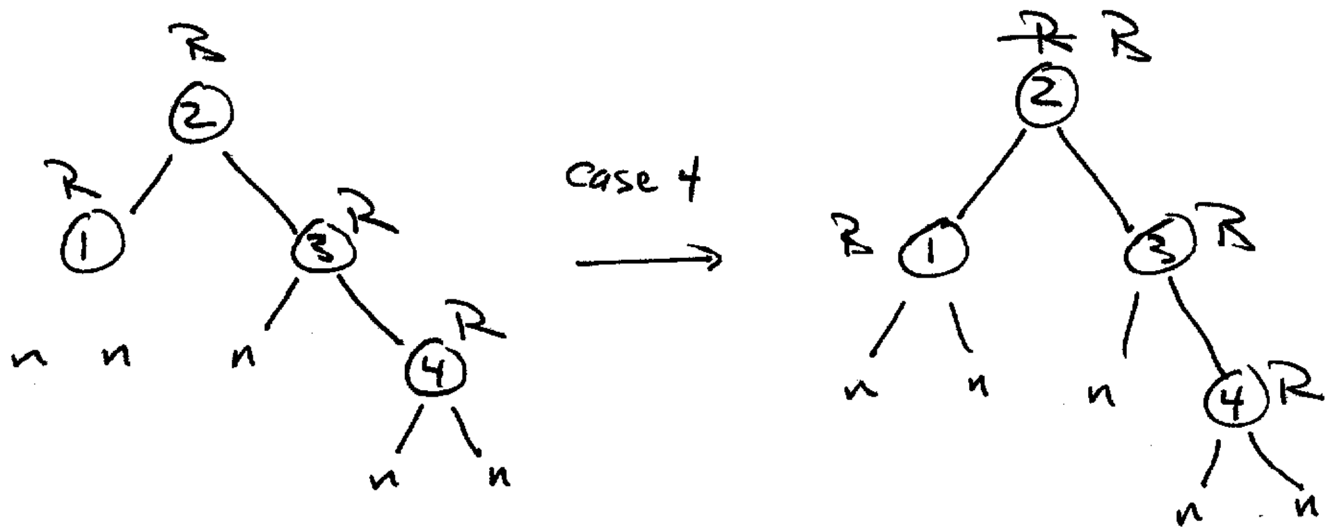
<u>Ex.</u> insert : 1, 2, 3, 4, 5 into an initially empty RBT.

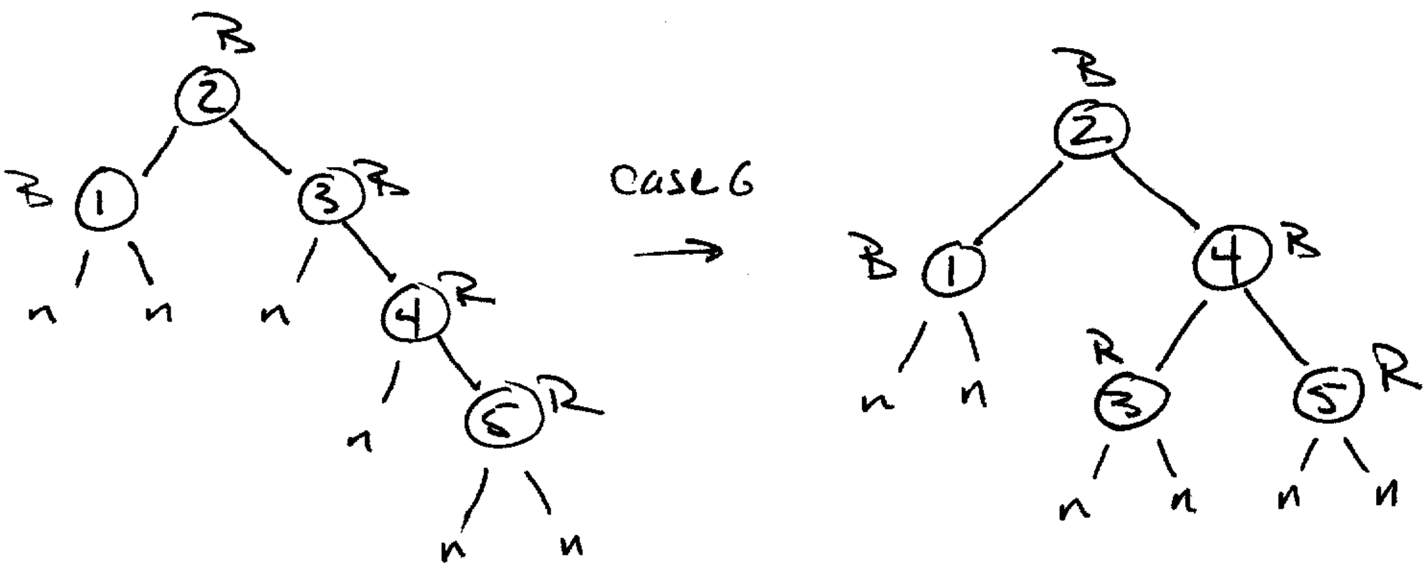


← no violation RBT prop.



RBT 4 violated



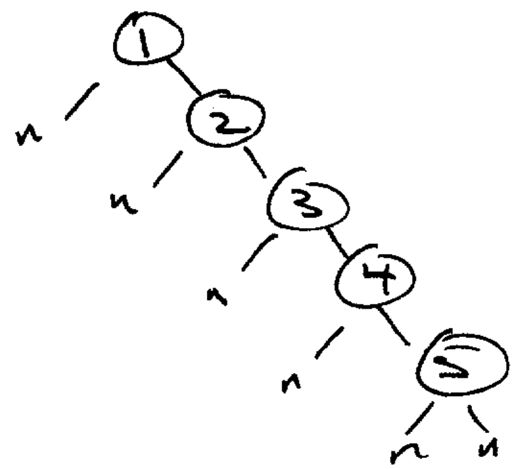


Exercise

- insert : 5, 4, 3, 2, 1
- other insertion orders

Problem (Quiz ?)

BST insert: 1, 2, 3, 4, 5



$n = 5$

$h = 5$

is it possible to assign colors to satisfy RBT Properties.

ans. No! why?

if it were then: $h \le 2\lg(n+1)$

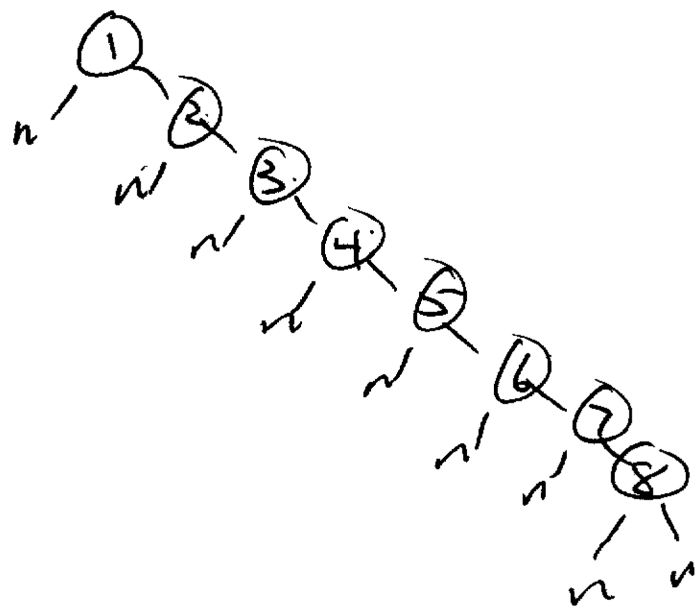
i.e. $5 \le 2\lg(6) = \lg(6^2) = \lg(36)$

$\therefore 2^5 \le 36 \quad \therefore 32 \le 36$

no violation of thm.

Better example

BST insert: 1, 2, 3, 4, 5, 6, 7, 8



$n = 8$
$h = 8$

? $h \le 2\lg(n+1)$ i.e. $8 \le 2\lg(9)$

? $8 \le \lg(9^2) = \lg(81)$

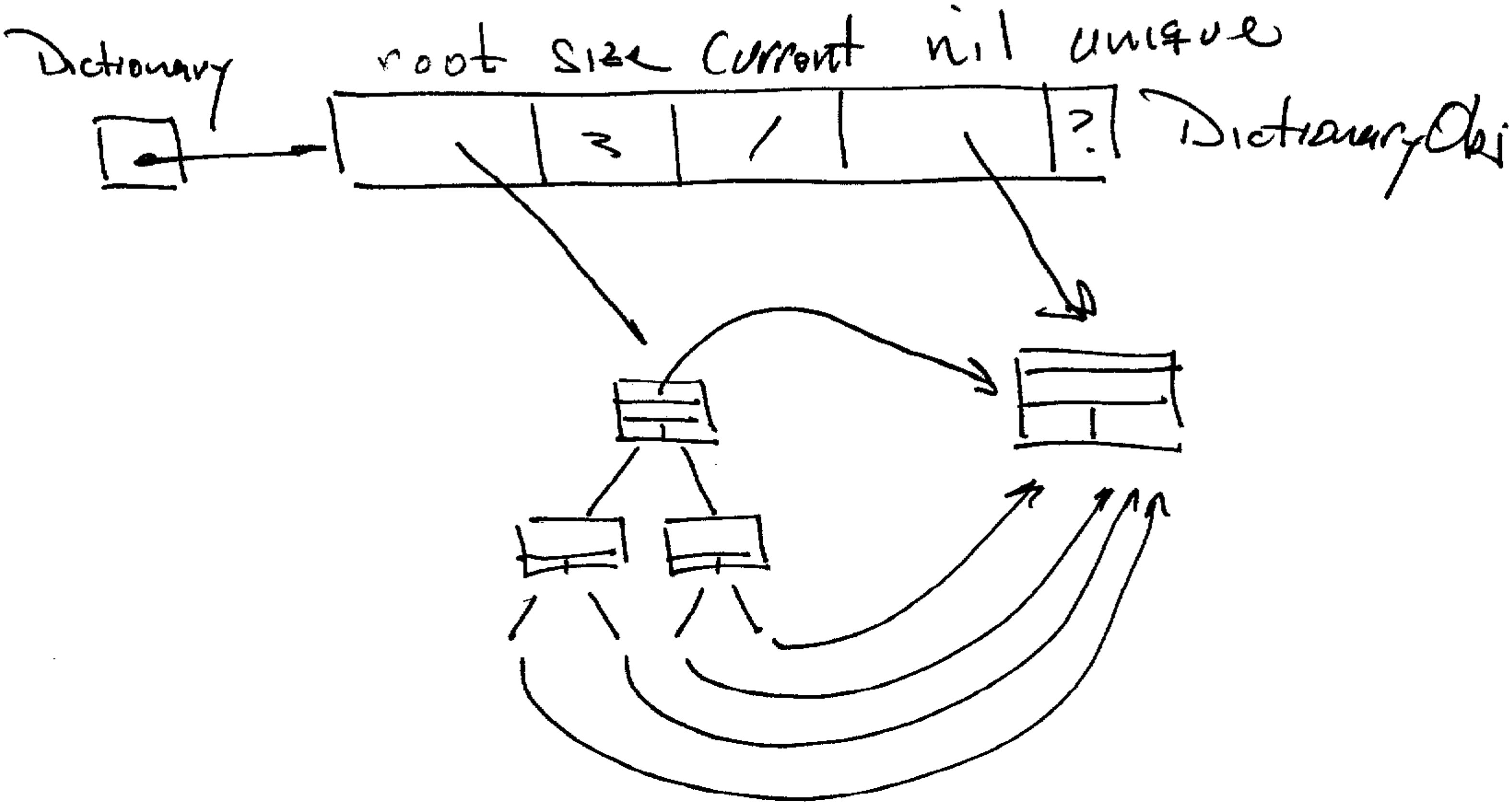
? $2^8 \le 81$

? $256 \le 81$ false!!

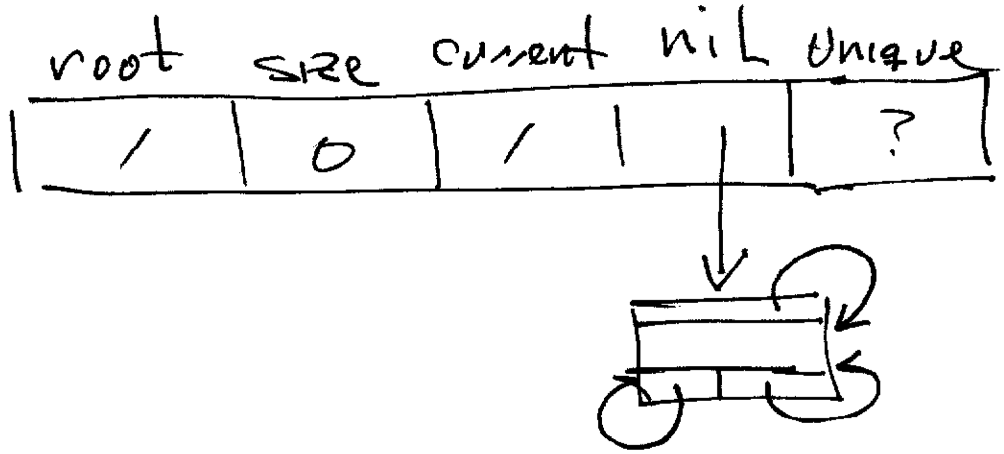
$\therefore$ It is not possible to assign colors to T making it a RBT.

Pac questions:

DictionaryObj



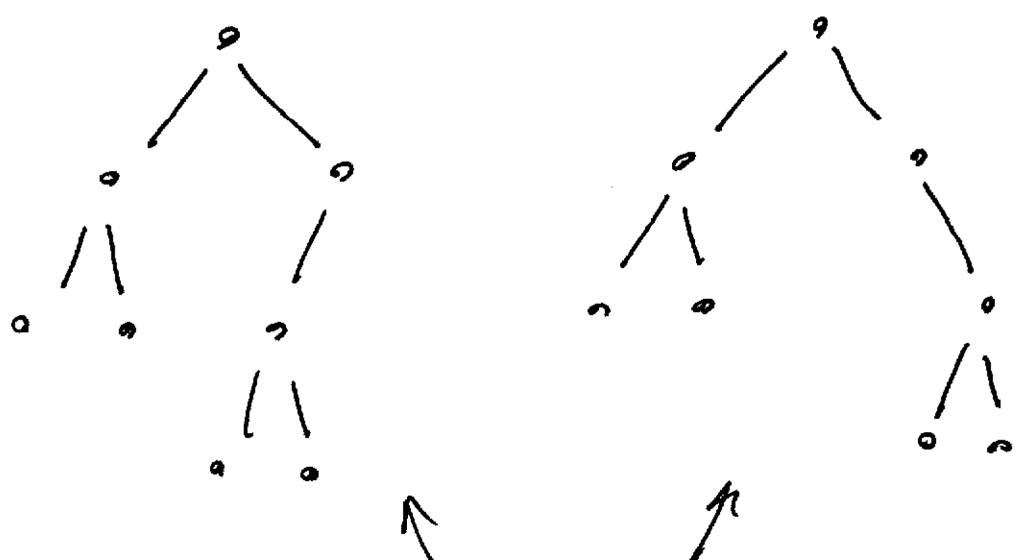
empty state



New Data Structure:

(Binary) heap — max heap / min heap.

Binary Trees



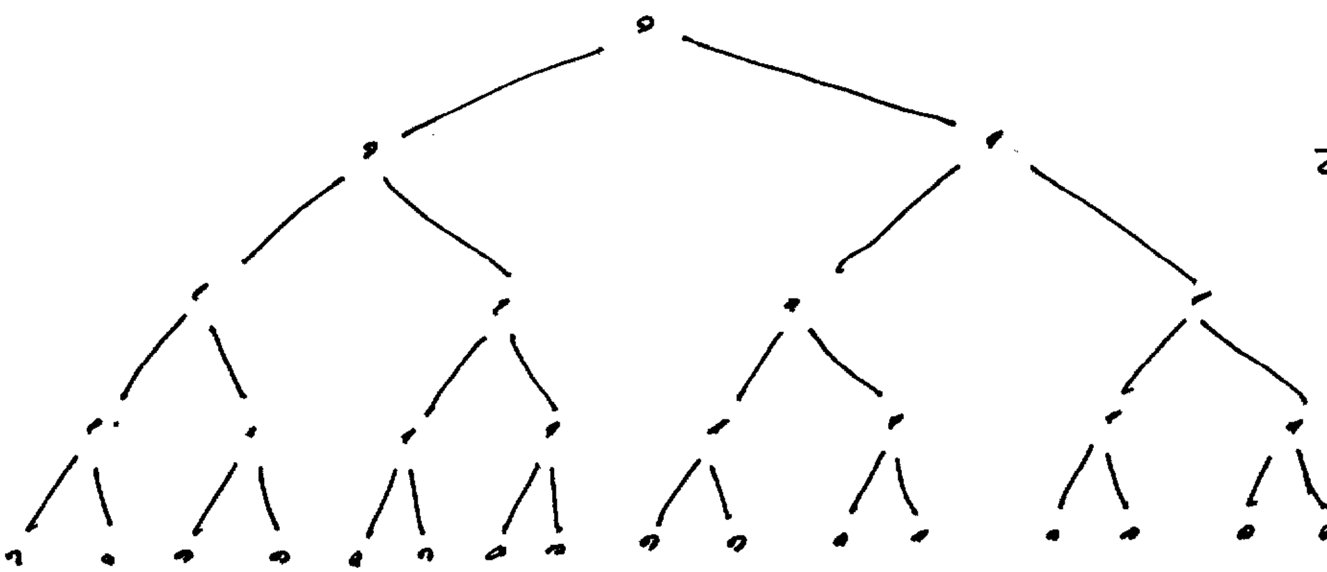
unequal binary tree.

Theorem

If T is a binary tree with height h and $n = \#\text{nodes}$, then

$$h \geq \lfloor \lg n \rfloor .$$

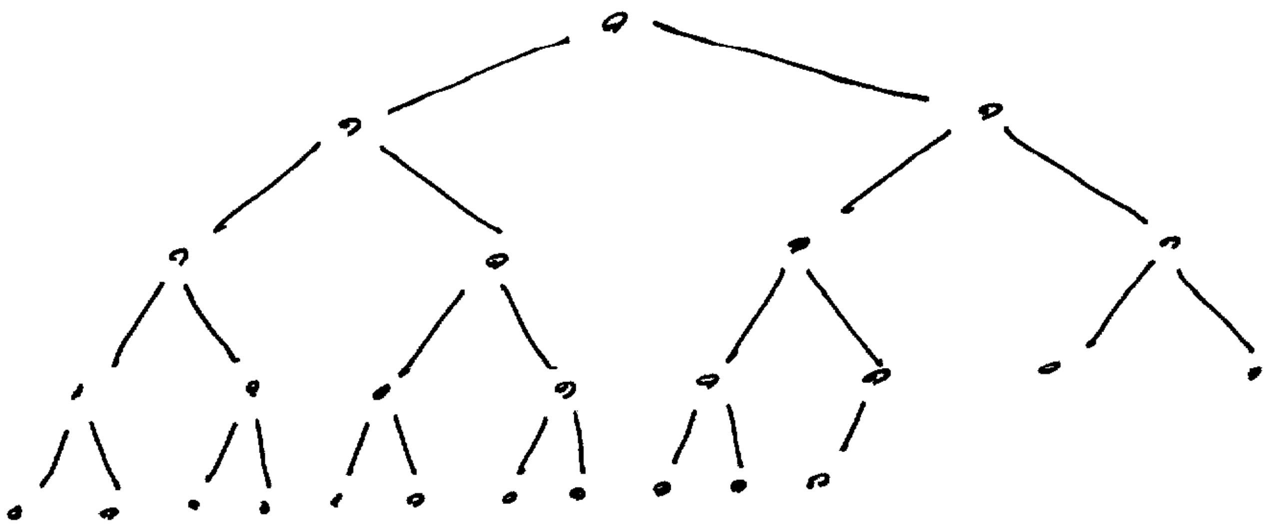
Complete Binary Tree (CBT)



nodes
$1 = 2^0$
$2 = 2^1$
$4 = 2^2$
$8 = 2^3$
$16 = 2^4$

$$n = \sum_{d=0}^{h} 2^d = \frac{2^{h+1} - 1}{2 - 1} = 2^{h+1} - 1$$

Almost complete binary tree (ACBT)



$$2^h - 1 < n \le 2^{h+1} - 1$$

$$\therefore \quad 2^h \le n < 2^{h+1}$$

$$\therefore \quad h \le \lg(n) < h+1$$

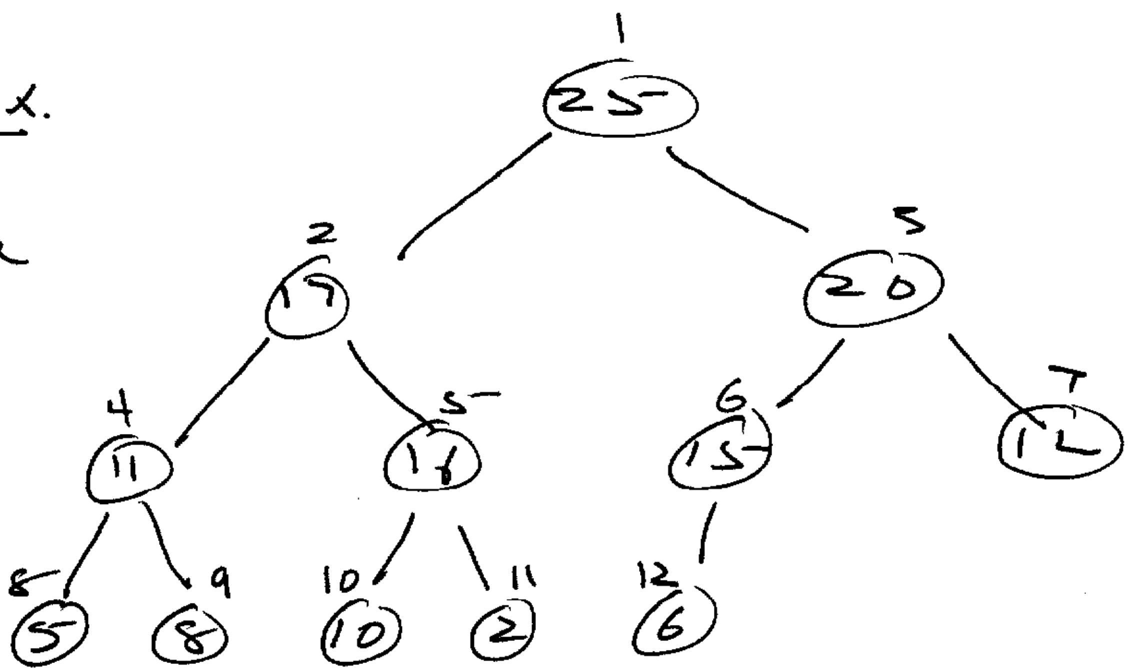
$$\therefore \quad \boxed{h = \lfloor \lg n \rfloor}$$

sec. 6.1 Heaps

The <u>Binary Heap</u> data struct. is an array object which represents an ACBT. each node corresponds to an array element

Ex.

Tree



Array

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
25	17	20	11	16	15	12	5	8	10	2	6	0	0	0	0