CSE 101 4-20-21

Defn

A <u>Topological Sort</u> of $V(G)$, where G is a digraph is a linear ordering of $V(G)$ such that: if $(x,y) \in E(G)$, then x is before y in the ordering:

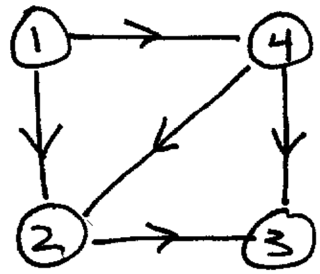
$x \leq y$

linear ordering

i.e. if we arrange $V(G)$ on a horiz. line (according to the lin. ordering) and draw in all edges, then all edges go left-to-right.

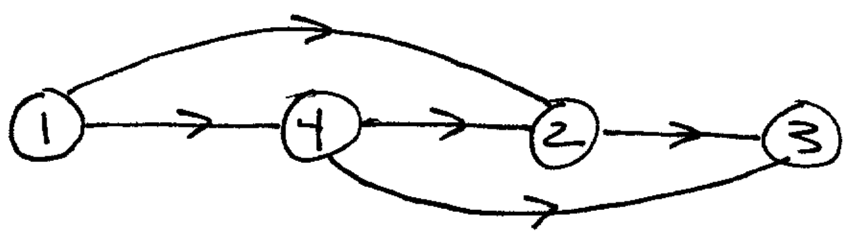
Ex.

G



Defn a digraph with no directed cycles is called directed acyclic graph (DAG)

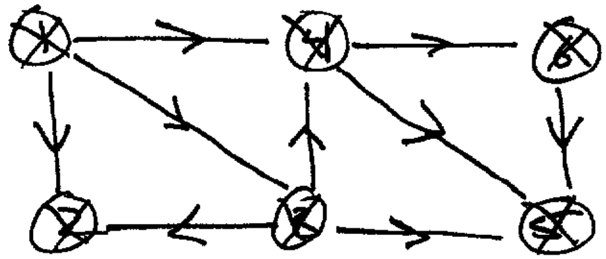
Topological sort:



To obtain a topological sort:

- Perform DFS(G). As vertices finish, push them onto a stack. When DFS is complete, the stack (top to bottom) is a topological sort (if G is a DAG).

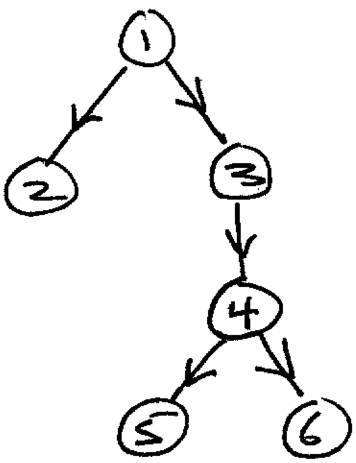
Ex



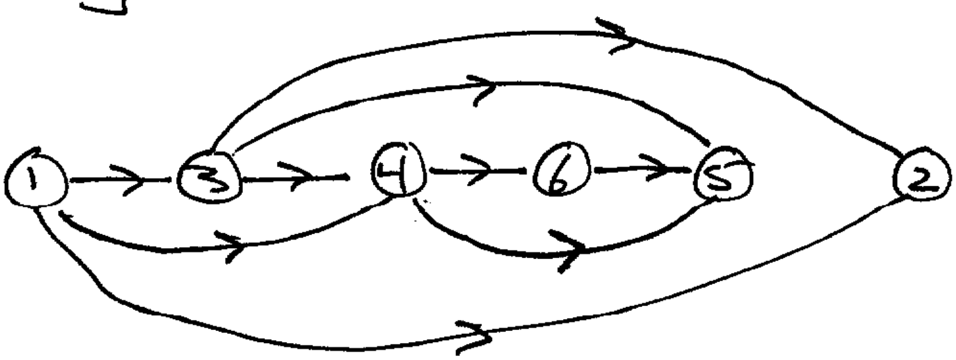
stack

1
3
4
6
5
2

forest



topological sort:



observe: digraph has exactly 4 top. sorts. Write down the other 3.

strongly connected components

let $G = (V, E)$ be a digraph.

we call $U \subseteq V$ a strongly conn. comp. (SCC) of G iff both

(1) U is strongly connected.

and

(2) U is maximal w.r.t. (1)

To find SCCs in a digraph:

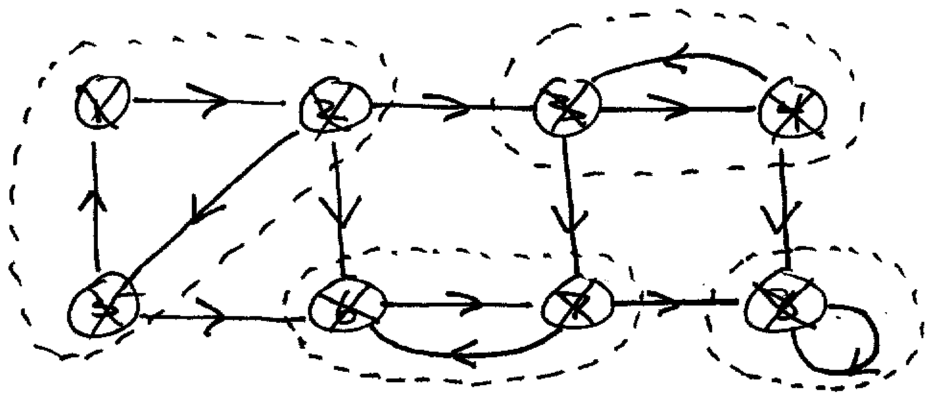
- call DFS(G). as vertices finish push them onto a stack
- compute the transpose G^T (reverse directions on all edges).
- call DFS(G^T), process vertices in main loop of DFS by decreasing finish times from 1st call, i.e., pop vertices off the stack.

Thm (22.16 on P. 556)

When this process is complete, then the trees in the DFS forest (from 2^{nd} call) span the SCCs of G.

Ex.

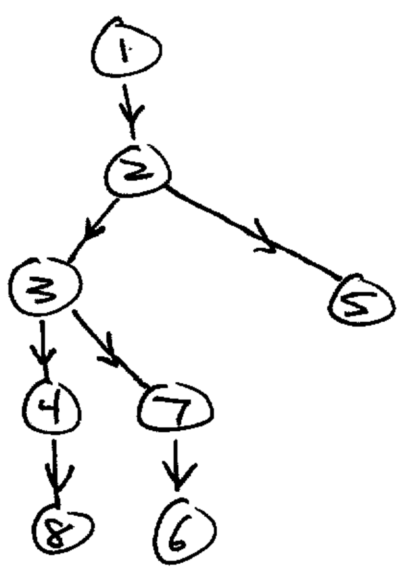
G



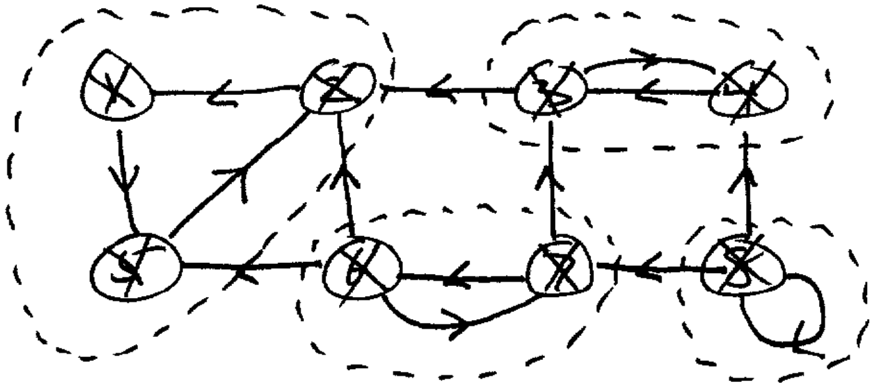
stack

1 ✓
2 ✓
5 ✓
3 ✓
7 ✓
6 ✓
4 ✓
8 ✓

forest

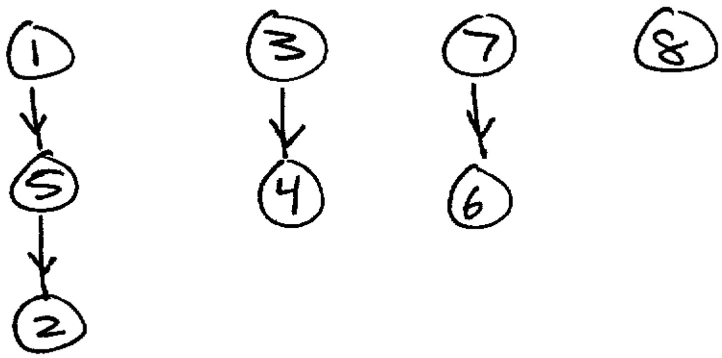


G^T



stack	p
8	n
7	n
6	7
3	n
4	3
1	n
5	1
2	5

forest



strong components of G

$$C_1 = \{1, 2, 5\}$$
$$C_2 = \{3, 4\}$$
$$C_3 = \{6, 7\}$$
$$C_4 = \{8\}$$

<u>Defn</u>

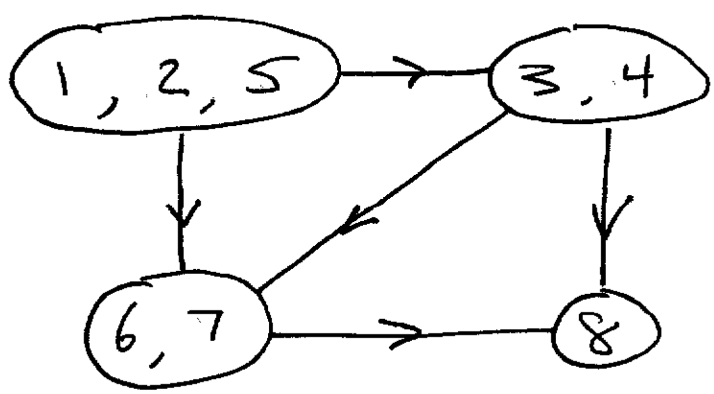
The <u>component graph</u> of a digraph G (also <u>condensation graph</u>), denoted G^{SCC}, is the digraph with

$$V^{SCC} = \{ \text{strong components of } G \}$$

$$E^{SCC} = \{ (C_i, C_j) \mid C_i, C_j \in V^{SCC} \text{ and } \exists x \in C_i, \exists y \in C_j \text{ such that } (x,y) \in E(G) \}$$

<u>Ex</u>

G^{SCC}



Rmk

- G and G^T have same SCC's.
  in ~~fact~~ $(G^{SCC})^T = (G^T)^{SCC}$
- G^{SCC} is necessarily acyclic, so it has a ~~topological~~ sort.
- we can obtain this by reading the 2^{nd} stack from bottom to top.