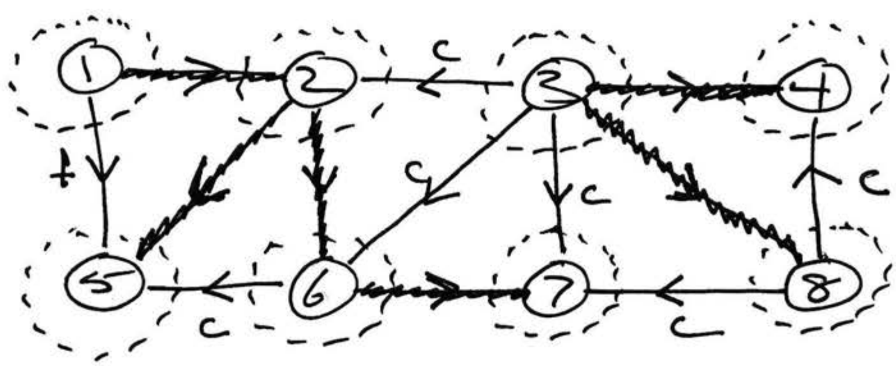
cse 101 4-15-21

quizz : next Tuesday

Pa2 : ext. 1 last day

Ex.



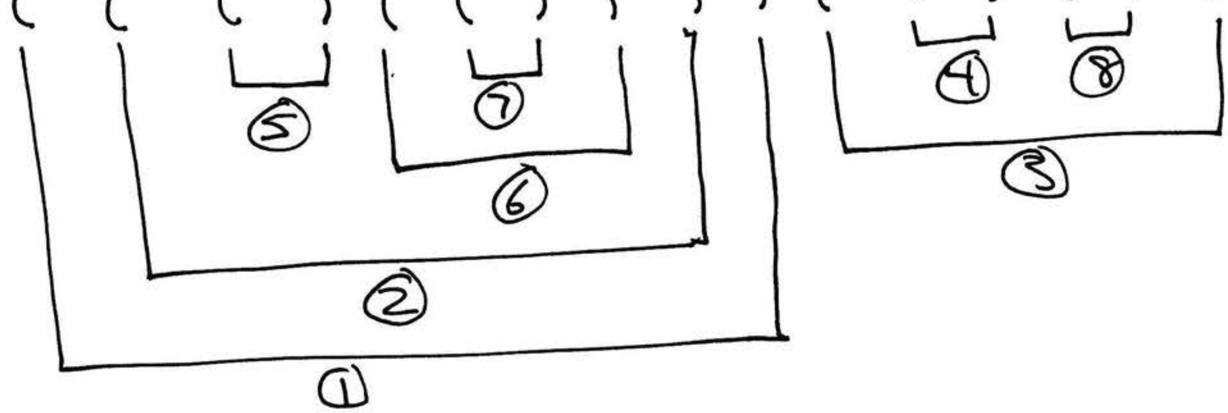

SCCs : {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}.

Parenthesis string:

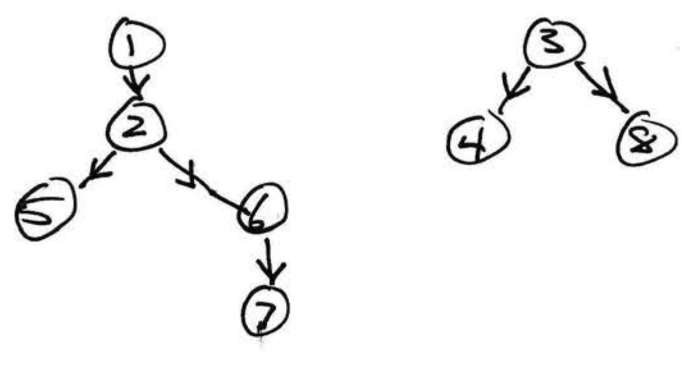
- Run DFS.
- for each discover event: '('
- for each finish event: ')'

Ex.

| time: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| string: | ( | ( | ( | ) | ( | ( | ) | ) | ) | ) | ( | ( | ) | ( | ) | ) |




forest



Theorem: Parenthesis

If $x, y \in V(G)$ and $d[x] < d[y]$, then exactly one of the following hold

(1) $d[x] < f[x] < d[y] < f[y]$

$$\underset{x}{(}\cdots\cdots\underset{x}{)} \qquad \underset{y}{(}\cdots\cdots\underset{y}{)}$$

gray time of $x$ — gray time of $y$

or (2) $d[x] < d[y] < f[y] < f[x]$

$$\underset{x}{(}\cdots\underset{y}{(}\cdots\cdots\underset{y}{)}\cdots\underset{x}{)}$$

Note: (2) says: $y$ is discovered while $x$ is gray, also: $y$ is a descendant of $x$ (in some tree). Thus (1) says: $y$ is not a descendant of $x$. (cousin in same tree, or other tree).

Theorem: White Path

let $x, y \in V(G)$. Then after DFS(G), y is a descendant of x if and only if at time $d[x]$, G contains an x-y path consisting entirely of white vertices.

equivalently:
(2) in Parenthesis Thm is logically equivalent to: at time $d[x]$, G contains a white x-y path.

Rmk:
Think of DFS as a movie, with each time stamp being a frame. a white x-y path in frame $d[x]$ exist iff y is a descendant of x at end of movie.

## Edge classification:

- Tree edge: member of $G_p$
- Back " : joins vertex to ancestor
- Forward " : joins ancestor to descendant (other than child)
- Cross " : cousin-to-cousin, or tree-to-tree

Ex. (last example)

Tree : (1, 2), (2,5), (2,6), (6,7), (3,4), (3,8)

Back :

Forward: (1,5)

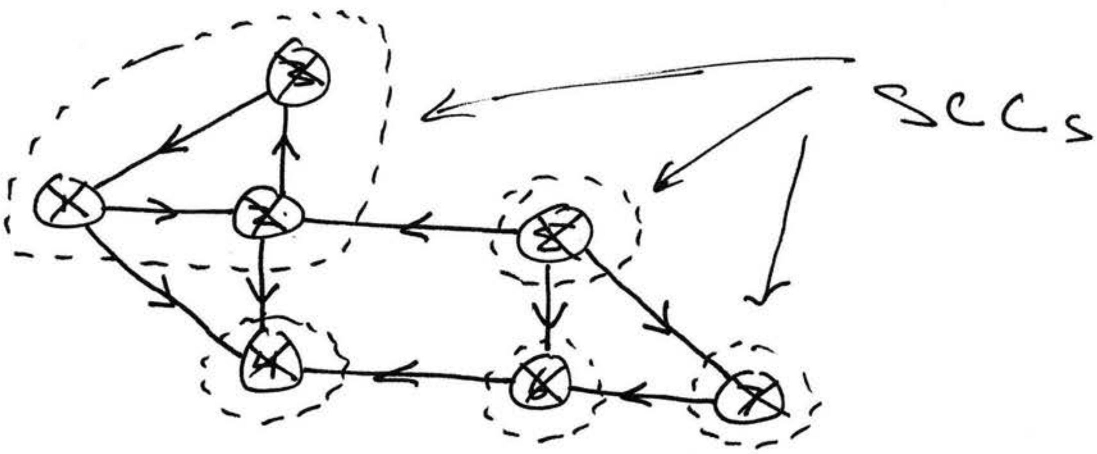
Cross : $\underbrace{(6,5), (8,4)}_{\text{cousin to cousin}}, \underbrace{(3,2), (3,6), (3,7), (8,7)}_{\text{tree to tree}}$

Note in an undirected graph, can't distinguish Back from Forward, so we call them Back edges. So we have just 3 categories.

Thm

In an undirected graph, there are no cross edges.

Ex.




forest

Tree: (1, 2), (2, 3), (2, 4), (5, 6), (5, 7)

Back: (3, 1)

Forward: (1, 4)

Cross: $\underbrace{(7, 6)}_{\text{cousin-cousin}}$, $\underbrace{(5, 2), (6, 4)}_{\text{tree-tree}}$

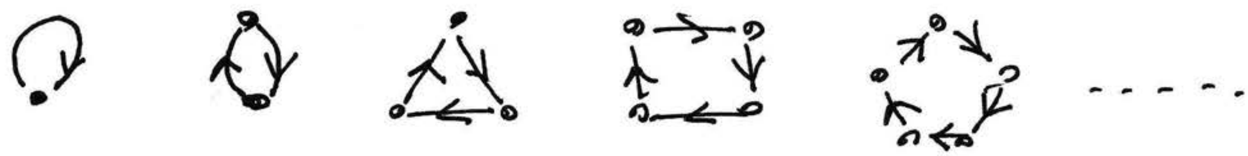
Exercises

- Run DFS on other examples.
- Modify DFS so it prints each edge with its classification (Problem 22.3-10 in CLRS.)

## Topological Sort

Let $G=(V,E)$ be a digraph.

directed cycles:



Length 1 2 3 4 5

**Defn**
A digraph is called acyclic iff it contains no directed cycles.

**Ex.**


acyclic not acyclic.

Theorem

a digraph G is acyclic iff DFS(G) yields no back edges.

equivalently

DFS(G) yields a back edge iff G contains a directed cycle.

what is a topological sort?

⋮

next time.