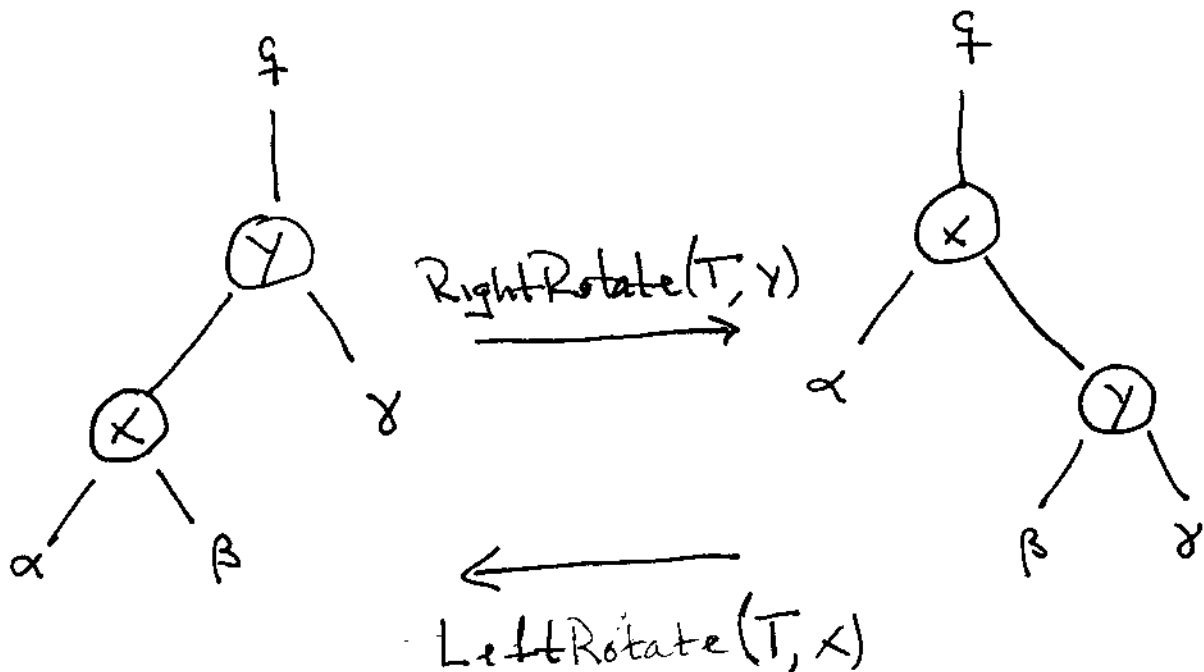


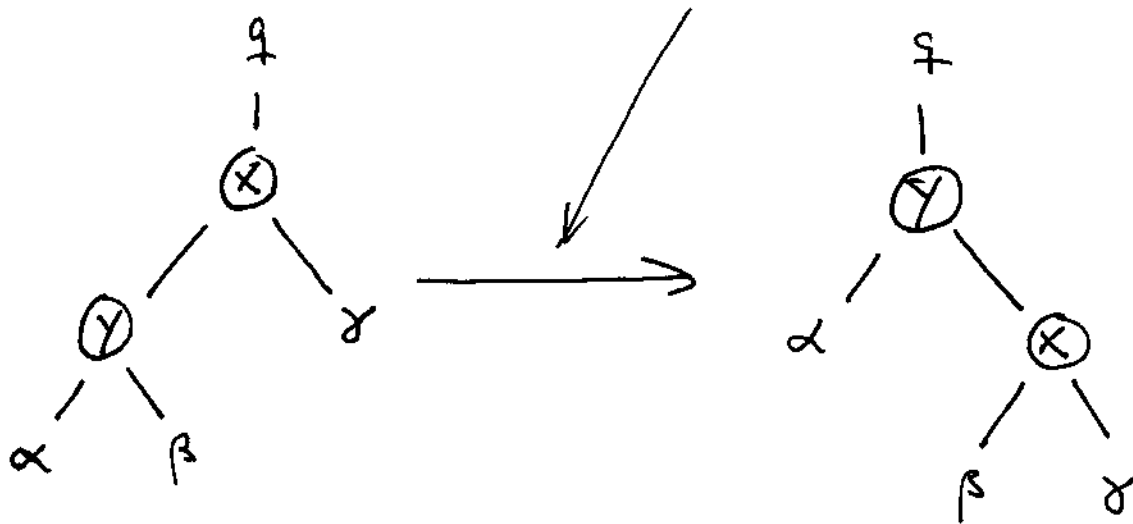
Pass: ext another day

13.2 Rotations

$\left. \begin{array}{l} \text{LeftRotate}(T, x) \\ \text{RightRotate}(T, x) \end{array} \right\} \text{ inverse}$



Book describes LeftRotate(T, x),
we'll discuss RightRotate(T, x)



note: Rotation does Preserve BST Properties

$$\text{keys}[\alpha] \leq \text{key}[y] \leq \text{keys}[\beta] \leq \text{key}[x] \leq \text{keys}[z]$$

note: Preconditions for

RightRotate(T, x) : $\text{left}[x] \neq \text{nil}$

LeftRotate(T, x) : $\text{right}[x] \neq \text{nil}$

summarize $\text{RightRotate}(T, x)$:

- fix β

$$\text{left}[x] = \text{right}[y]$$

$$\text{Parent}[\text{right}[y]] = x$$

- fix Parent q :

$$\text{Parent}[y] = \text{Parent}[x]$$

$$\text{Parent}[x] (\text{left or right child}) = y$$

- fix relationship between x & y

$$\text{right}[y] = x$$

$$\text{Parent}[x] = y$$

Runtime: $\Theta(1)$

13.3 Insertion

Recall RBT

1. all nodes R or B
2. nil (leaves) are B
3. root is B
4. R have only B children
5. $bh(x)$ is well-defined for all $x \in T$

BST insert:

Preserve 1, 2, 3

it new node is colored RED ,

$RBT \# 5$ also preserved.

(if insertion is at root, turn it
 $BLACK \Rightarrow RBT \# 3$ still satisfied)

Problem: RBT #4 may be violated

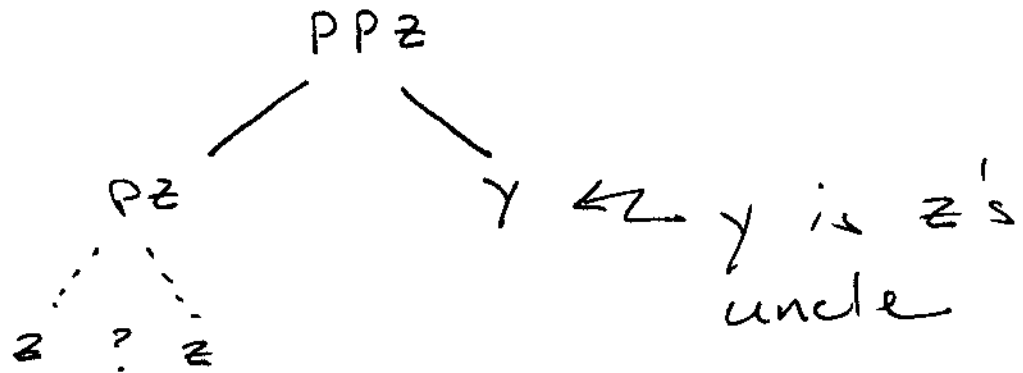
$RB_Insert(T, z)$

$RB_InsertFixUp(T, z)$

How does $RB_InsertFixUp(T, z)$ work?

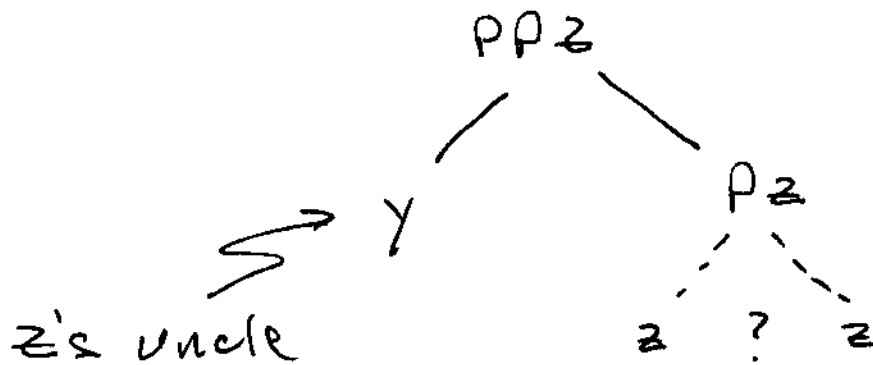
if $color[z]$ is Red, have violation of RBT #4. Two cases

- $P[z]$ is left child of its Parent



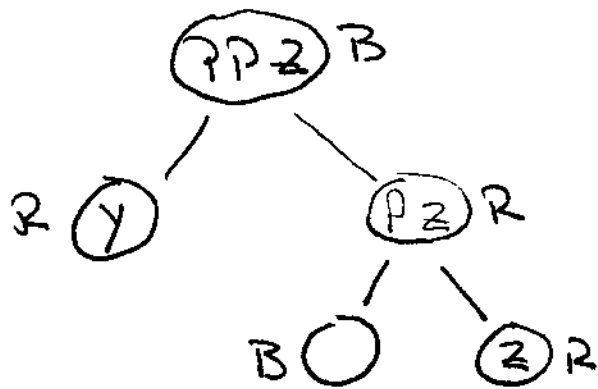
Subcases 1, 2, 3

- $P[z]$ is right child of its Parent

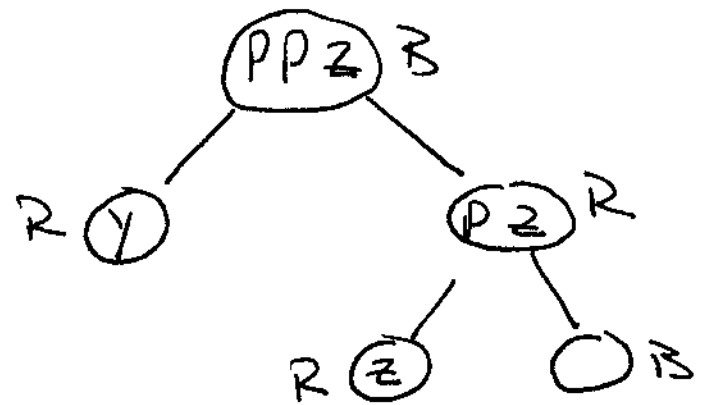


Subcases 3, 4, 5.

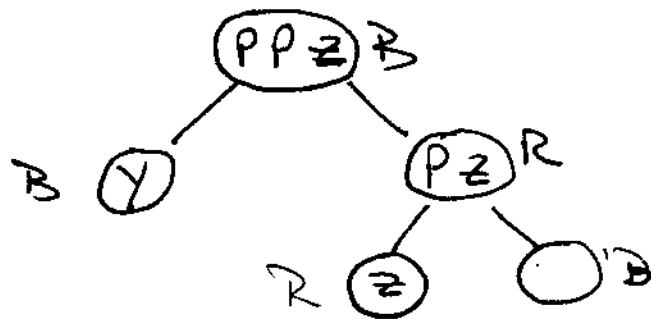
Case 4: y is RED



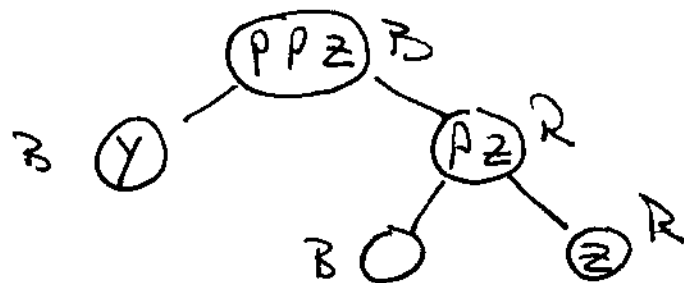
or



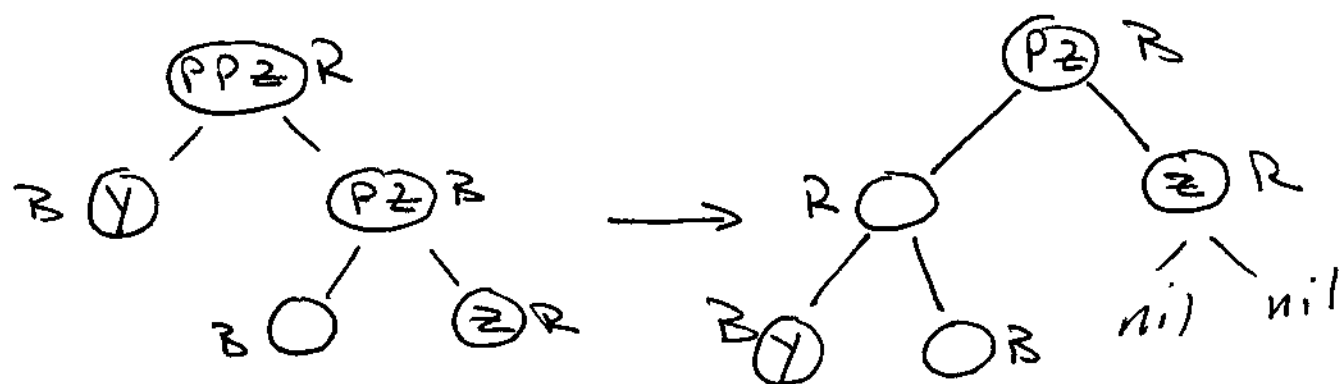
Case 5: y is Black, z is Pz 's left child



Case 6: y is Black, z is Pz 's right child

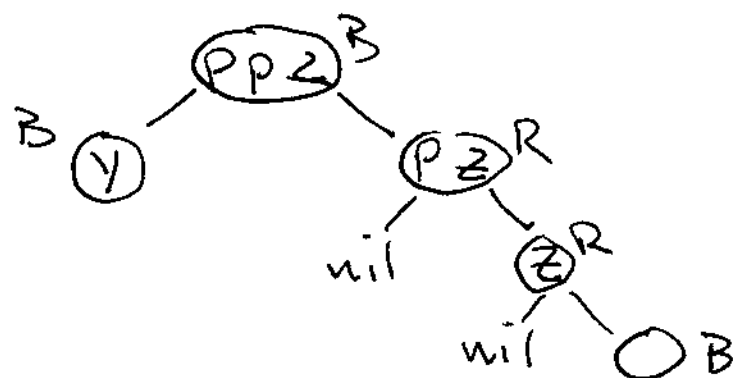


- In case 6 : color P_z BLACK, color PP_z RED, then Left Rotate about PP_z



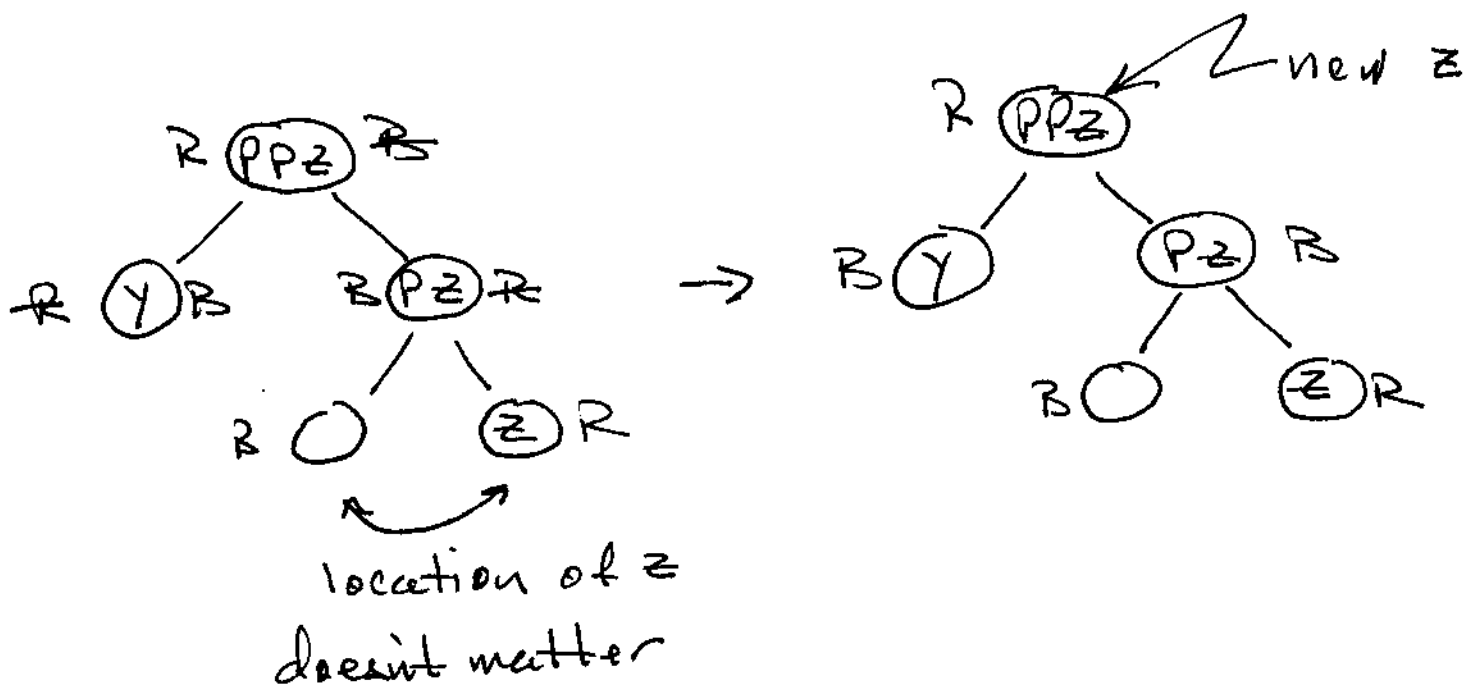
note RBT #4 is satisfied and RBT #5 is also satisfied.

- In case 5 : convert to case 6
let z 'climb up' to P_z , then Right Rotate about z . we get



- In case 4 :

fix color relations among y, p_z, pp_z ,
then let z 'climb up' to pp_z



Possible further iteration of while
loop.