cse 101 4-9-20

containers

ADTs

- Stack
- Queue
- List
- Dictionary
- Priority Queue
- Disjoint Sets

- Graph
- Integer
- Polynomial
- ⋮

mathematical objects

Data Structs

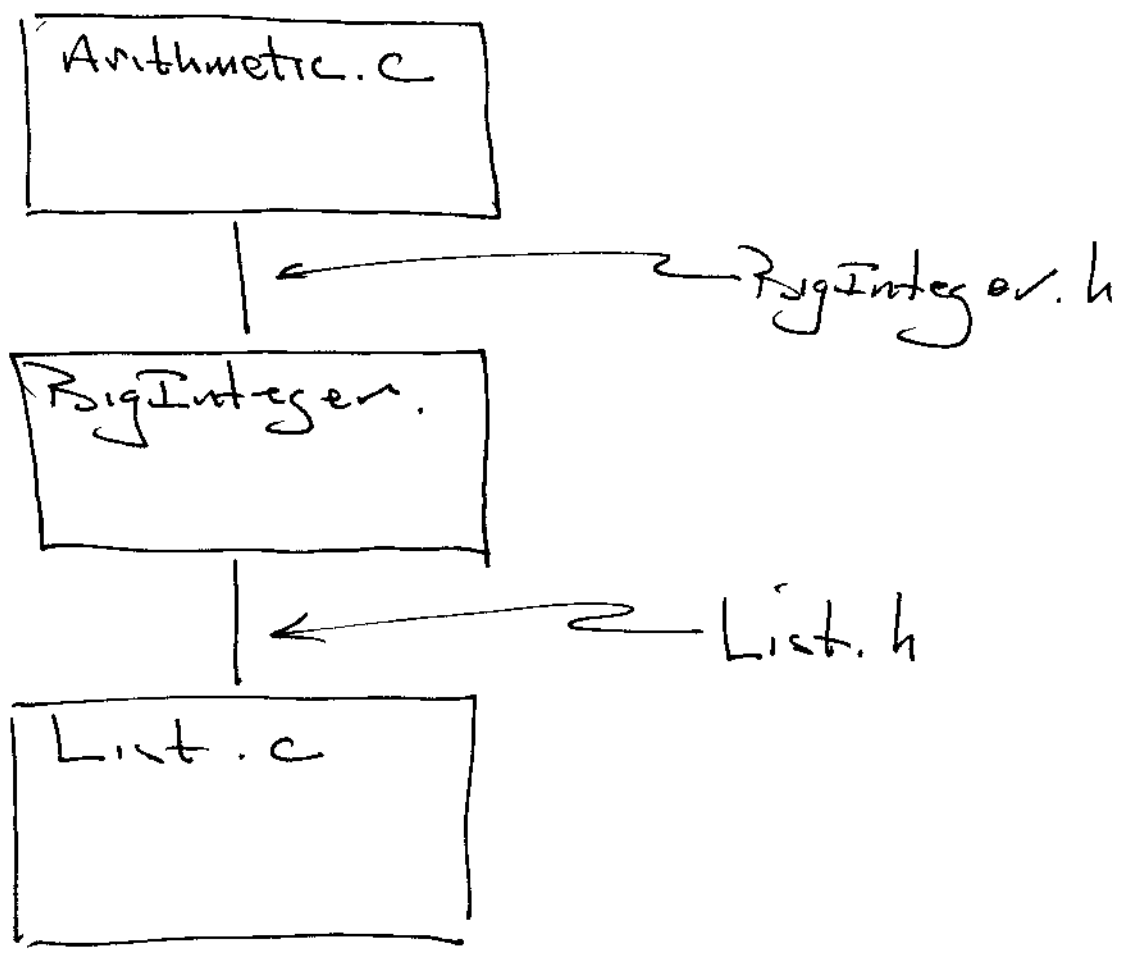
- Array
- linked list

- SkipLists
- BST
- RBT
- HashTable

- Binary Heap
- Fibonacci Heap
- LeftChild-RightSibling tree

- linked list w/ weighted union
- D.S. forest w/ union by rank & path compression

structure chart for Pa2



Graphs

A graph is a pair of sets

$$G = (V, E)$$

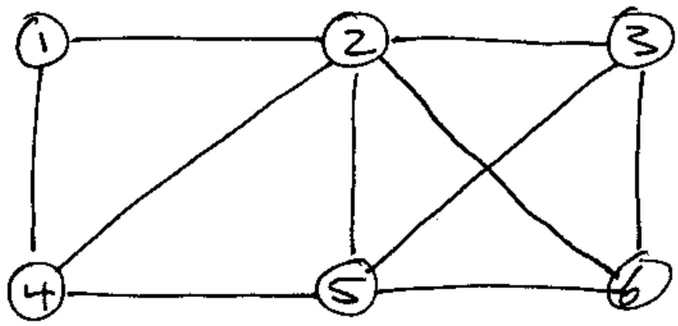
V: vertex set; E: Edge set

where $V \neq \phi$ and $E \subseteq \underbrace{V^{(2)}}_{\text{2-element subsets of } V}$

Ex. $V(G) = \{1, 2, 3, 4, 5, 6\}$

$E(G) = \{12, 14, 23, 24, 25, 26, 35, 36, 45, 56\}$

$\{3,5\}$



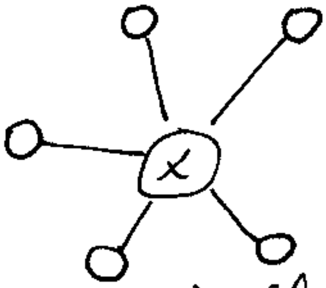
- $\{2,3\}$ *joins* 2 to 3
- 2 is *adjacent* to 3
- 2 & 3 are *ends* of $\{2,3\}$
- 2 & 3 are *incident* with $\{2,3\}$

note:

not allowed since $\{x, x\} = \{x\}$

not allowed since $\{x,y\}$ cannot belong to $E(G)$ twice.

<u>degree</u> of a vertex



$\deg(x) = 5$

is # of edge incidence with x.

<u>degree sequence</u>: degrees arranged in increasing order

Handshake lemma:

$$\sum_{x \in V(G)} \deg(x) = 2|E(G)|$$

Given $x, y \in V(G)$.

- an <u>x-y walk</u> in G is a sequence of vertices

$$x = v_0, v_1, v_2, \ldots\ldots, v_k = y$$

where $\{v_{i-1}, v_i\} \in E(G)$ for $1 \le i \le k$.

call k, <u>length</u> of walk.

<u>Ex.</u>
- $(1, 2, 1, 2, 1, 4, 5, 2, 3)$ is a 1-3 walk of length 8
- (1) is a 1-1 walk of length 0. called a <u>trivial walk</u>.

- we say walk is closed iff $x=y$.

- A <u>trail</u> is a walk in which no edge is repeated

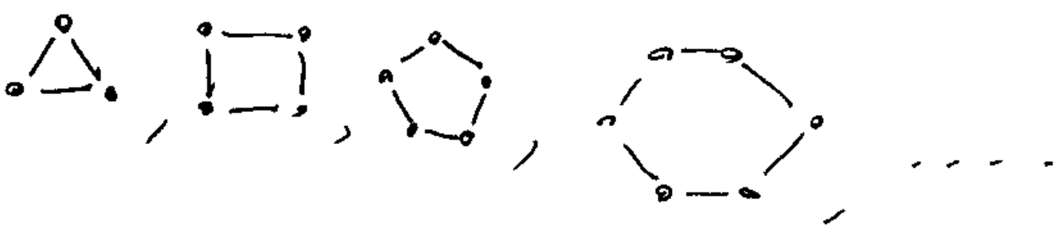
  <u>Ex.</u> $(3,2,4,5,3,6)$ trail of length 5

- A <u>Path</u> is trail in which no vertex is repeated (except possibly when $x=y$)

  <u>Ex</u> $(4,2,6,5,3)$ Path of length 4

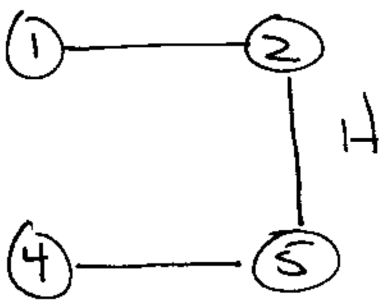
- A <u>cycle</u> is a non-trivial closed Path

<u>Ex.</u>



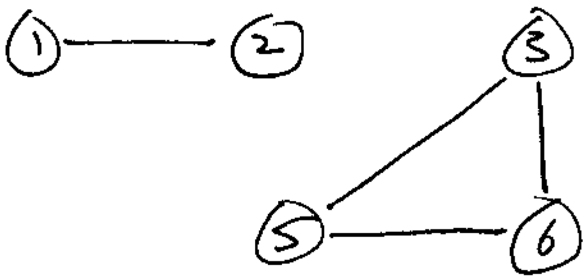
- A <u>subgraph</u> of G is a graph H with $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

<u>Ex.</u>



$V(H) = \{1, 2, 5, 4\}$
$E(H) = \{12, 25, 54\}$

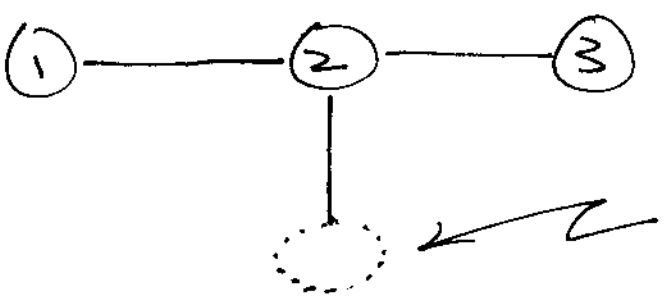
<u>Ex</u>



$V(H) = \{1, 2, 3, 5, 6\}$
$E(H) = \{12, 35, 56, 36\}$

<u>Ex</u> not a subgraph

$(\{1, 2, 3\}, \{12, 25, 23\})$ not a graph



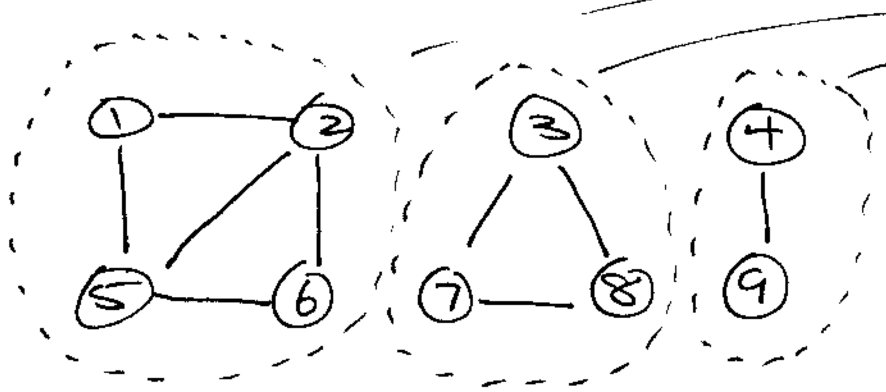
not in vertex set

- if $x, y \in V(G)$ and G contains an x-y path, we say x is <u>reachable</u> from y (or y from x)

- G is <u>connected</u> iff for all $x, y \in V(G)$ x is reachable from y, otherwise G is called disconnected.

<u>Ex</u>

G



connected components of G

- A subgraph H is called a connected component iff
  (1) H is connected, and
  (2) H is maximal w.r.t. property (1)

Problems in Graphs:

- is G connected
- how many connected components does G have.
- what are connected components of G, i.e. what is the reachability relation on G.

- <u>shortest path problem</u>: given $x, y \in V(G)$ determine length of a shortest x-y path (called $d(x,y)$, distance from x to y), i.e.

$$d(x,y) = \begin{cases} \text{len. of min. length } x\text{-}y \text{ path} \\ \infty \text{ if } x \text{ \underline{not} reachable from } y. \end{cases}$$

also find a particular x-y path of length $d(x,y)$, if x reachable from y.

✓ • single source shortest Path (SSSP)

given $s \in V(G)$ (called source), find $d(s,x)$ for all $x \in V(G)$, and for any x reachable from s, find an s–x path of length $d(s,x)$

? • all pairs shortest paths (APSP)

find $d(x,y)$ for <u>all</u> $x, y \in V(G)$.

<u>Next Handout:</u>

Graph Algorithms

BFS

DFS