

CS2 101 4-28-20

1

## Depth First Search (DFS)

vertex attributes:

$color[x]$ : w, g, b

$P[x]$ : Parent (Predecessor)

$d[x]$ : discover time

$f[x]$ : finish time

Two functions

$DFS(G)$

$Visit(x)$   $\leftarrow$  recursive

global variable:  $time \in \{0, 1, 2, \dots, 2n\}$

Predecessor Subgraph

$$G_p = (V_p, E_p)$$

$$V_p = V(G)$$

$$E_p = \{ \underbrace{(P[x], x)}_{\text{ordered pair if digraph}} \mid P[x] \neq \text{nil} \}$$

ordered pair if digraph  
un-ordered pair if graph

Note: Since only white vertices are visited, after which they turn grey then black, there can be no cycles.

$\therefore G_p$  is acyclic

called: DFS forest.

Note :

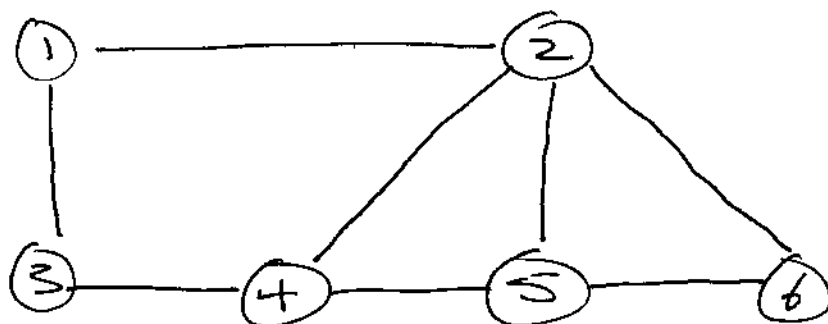
DFS has no source, could supply a source<sup>s</sup> and replace main loop (5-7 of DFS) by single call to visit(s)

could also re-write BFS to not have a source.

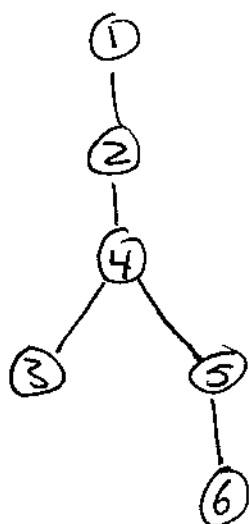
Exercise write the 'other version' of Both algorithms.

Note extremes

BFS ————— DFS  
continuum

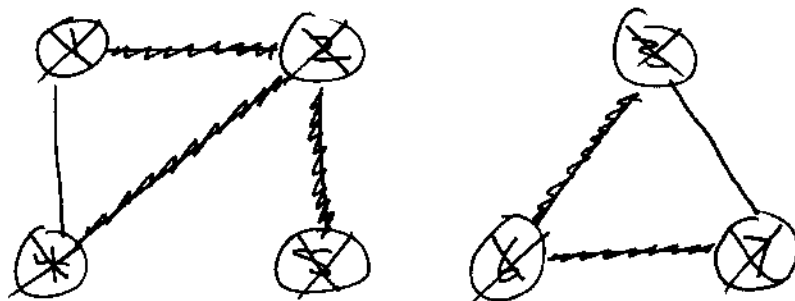
Ex.

|     | adj                                 | color            | d | f  | p              |
|-----|-------------------------------------|------------------|---|----|----------------|
| ✓ 1 | <u>2</u> <u>3</u>                   | <del>w</del> g b | 1 | 12 | n              |
| ✓ 2 | <u>1</u> <u>4</u> <u>5</u> <u>6</u> | <del>w</del> g b | 2 | 11 | <del>n</del> 1 |
| ✓ 3 | <u>1</u> <u>4</u>                   | <del>w</del> g b | 4 | 5  | <del>n</del> 4 |
| ✓ 4 | <u>2</u> <u>3</u> <u>5</u>          | <del>w</del> g b | 3 | 10 | <del>n</del> 2 |
| ✓ 5 | <u>2</u> <u>4</u> <u>6</u>          | <del>w</del> g b | 6 | 9  | <del>n</del> 4 |
| ✓ 6 | <u>2</u> <u>5</u>                   | <del>w</del> g b | 7 | 8  | <del>n</del> 5 |

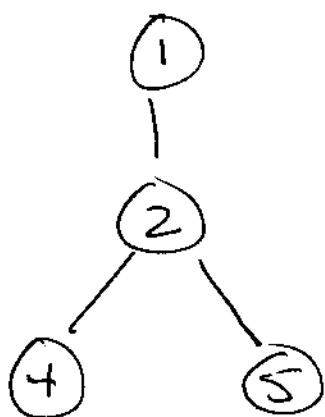
DFS  
foresttime

~~10~~ 6  
~~11~~ 7  
~~12~~ 8  
~~3~~ 4  
~~4~~ 5  
~~5~~ 6  
~~6~~ 7  
~~7~~ 8  
~~8~~ 9  
~~9~~ 10

Ex,



DFS  
Forest



|   | adj   | d  | f  | p |
|---|-------|----|----|---|
| 1 | 2 4   | 1  | 8  | n |
| 2 | 1 4 5 | 2  | 7  | 1 |
| 3 | 6 7   | 9  | 14 | n |
| 4 | 1 2   | 3  | 4  | 2 |
| 5 | 2     | 5  | 6  | 2 |
| 6 | 3 7   | 10 | 13 | 3 |
| 7 | 3 6   | 11 | 12 | 6 |

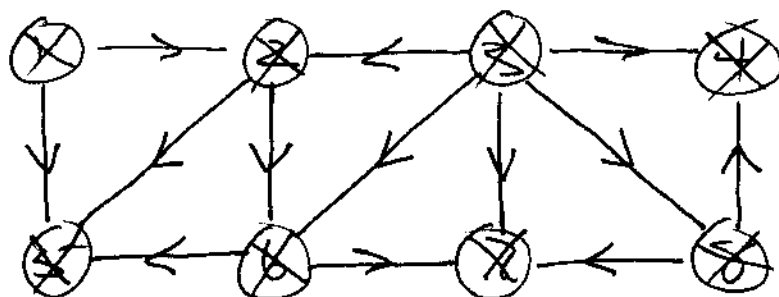
## Exercise

re-write DFS so as to determine (1) # of conn. components in an (undirected) graph, and (2) for any  $x, y \in V$ , determine if they are in same component.

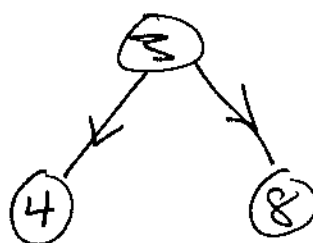
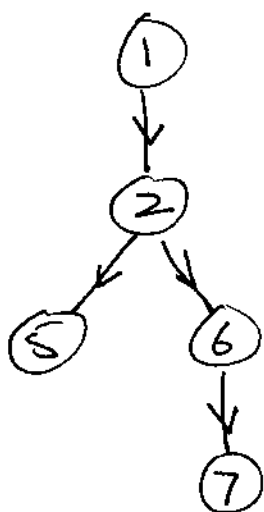
Hint: add another 'global' variable (like time) called  $count$ . increment  $count$  from main loop of DFS, each time  $visit()$  is called

Hint: add a vertex attribute,  $component[x]$ , stamped on  $x$  by  $visit(x)$ ,  $value = count$ .

Ex.

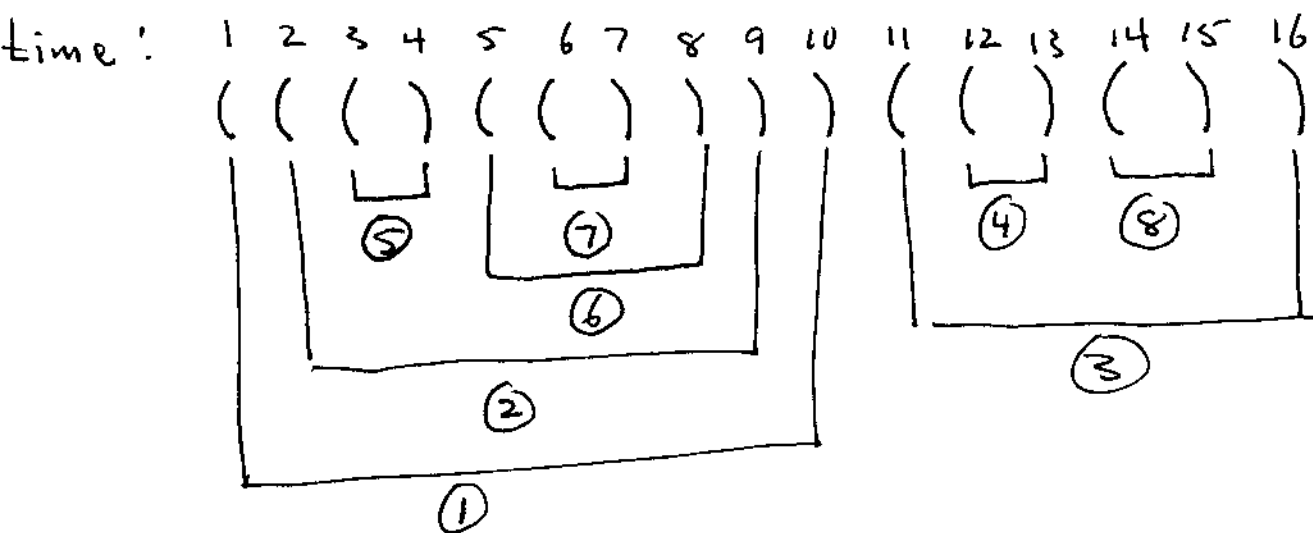


DFS  
FD-est



Paranthesis string

"(" discover event  
")" finish event



## Edge Classification

1. tree edges : belong to  $G_p$
2. back " : join vertex to ancestor
3. forward " : join ancestor to a descendant (other than a child)
4. cross " : all others
  - tree to tree
  - cousin to cousin

### Note

classification depends on order in which we execute

- main loop of DFS (5-6)
- loop 3-6 in  $visit()$

Exercise

modify DFS so it classifies  
edges as it goes

Hint: see p. 546-548 &  
Problem 22.3-4 in CLRS

Sec 23.4 Topological Sort