CSE 101 4-23-20

PA3 : Questions

Asymptotic growth of functions:

let $f(n)$, $g(n)$ be positive functions

Defn $f(n) = O(g(n))$ iff

$0 < \frac{f(n)}{g(n)} \leq B \leftarrow$ some upper bound

(for all suff. large n)

i.e. $f(n) \leq B \cdot g(n)$

we say f is 'big Oh' of g.

<u>Defn</u> $f(n) = \Omega(g(n))$ iff

$$\frac{f(n)}{g(n)} \geq b > 0$$

some lower bound.

(for suff. large n)

i.e. $f(n) \geq b \cdot g(n)$

we say f is 'big Ω' of g.

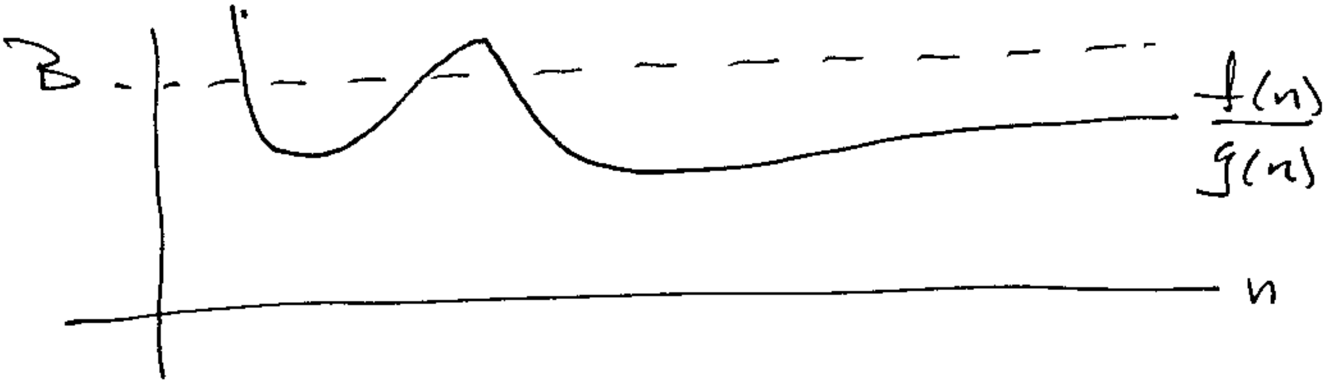
<u>Defn</u> $f(n) = \Theta(g(n))$ iff

$$0 < b \leq \frac{f(n)}{g(n)} \leq B$$

(for suff. large n)

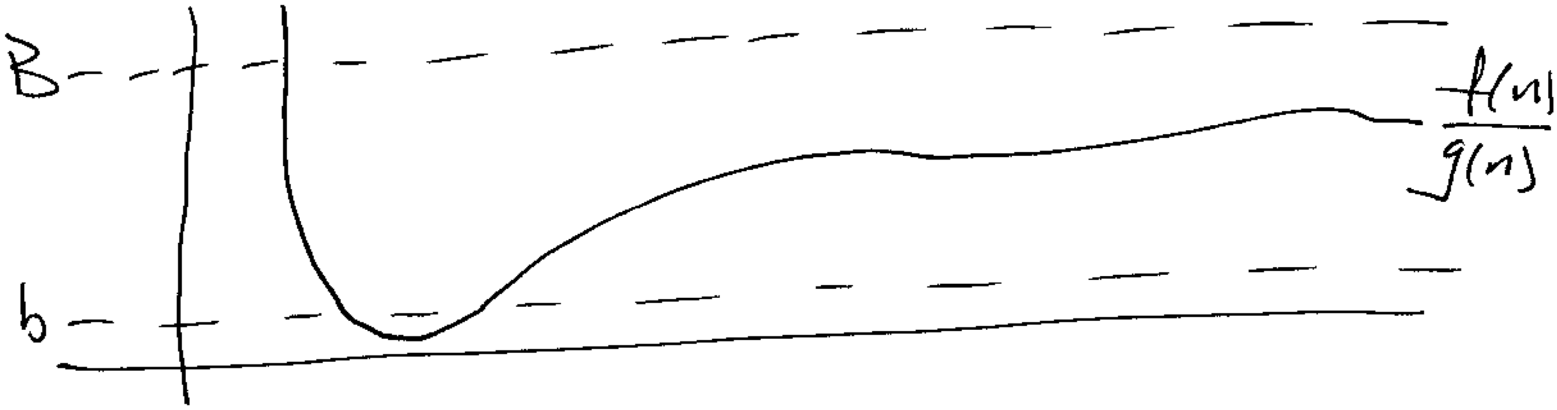
i.e. $b \cdot g(n) \leq f(n) \leq B \cdot g(n)$

$f(n) = O(g(n))$



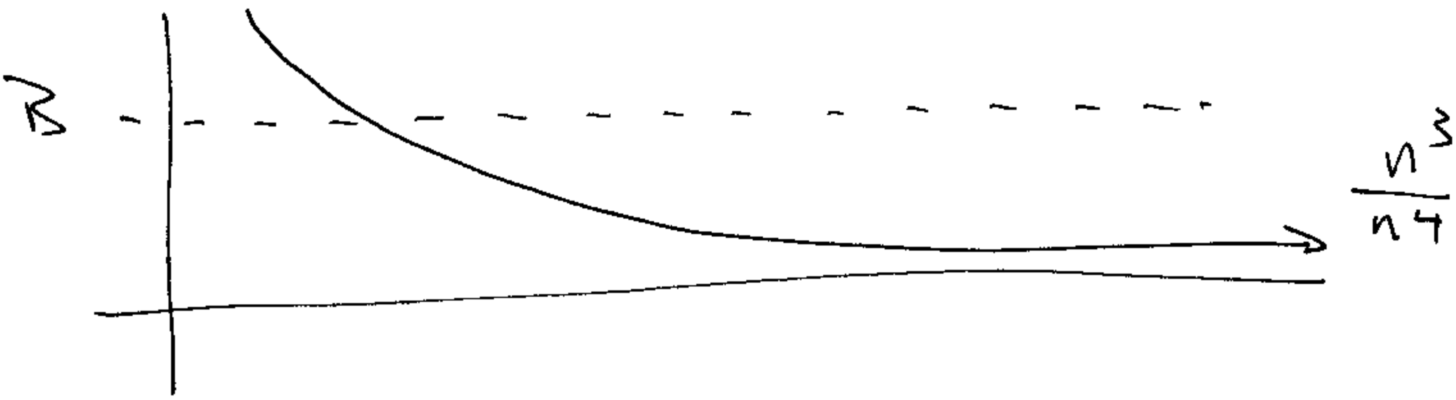
$f(n) = \Omega(g(n))$

$f(n) = \Theta(g(n))$



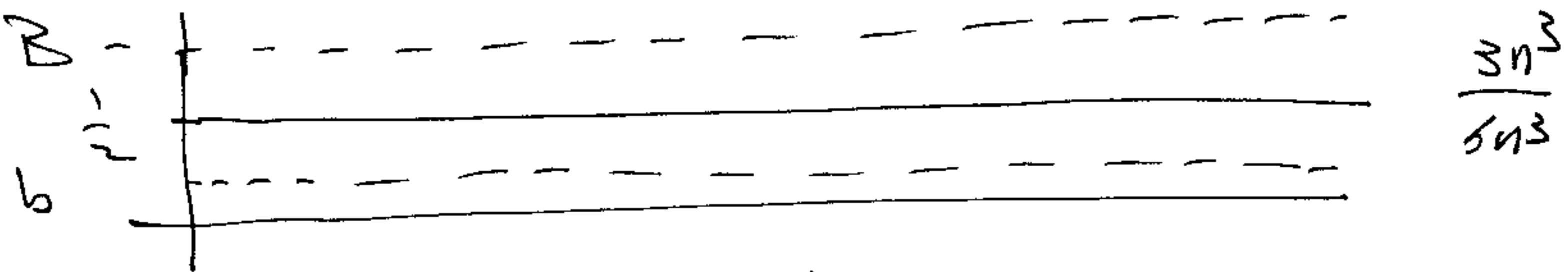
<u>Ex.</u> $n^3 = O(n^4)$

$$\frac{n^3}{n^4} = \frac{1}{n} \longrightarrow 0 \text{ as } n \to \infty$$



<u>Ex.</u> $3n^3 = O(6n^3)$

$$\frac{3n^3}{6n^3} = \frac{1}{2}$$

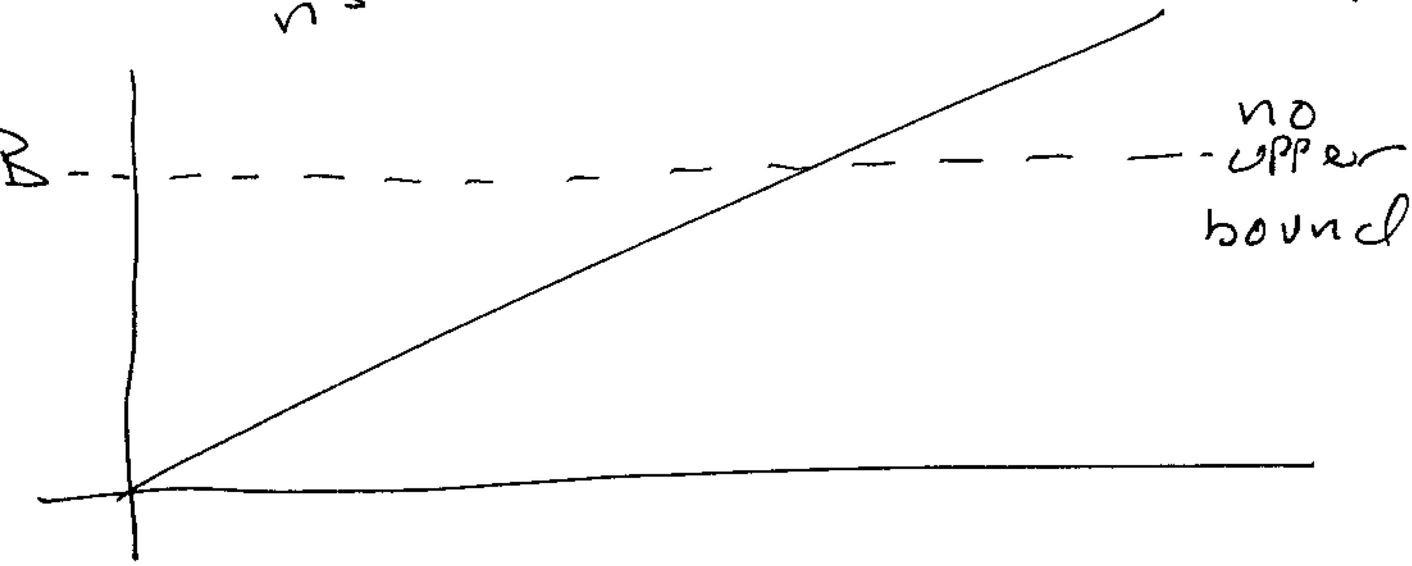


$\therefore 3n^3 = \Omega(6n^3)$

$\therefore 3n^3 = \Theta(6n^3)$

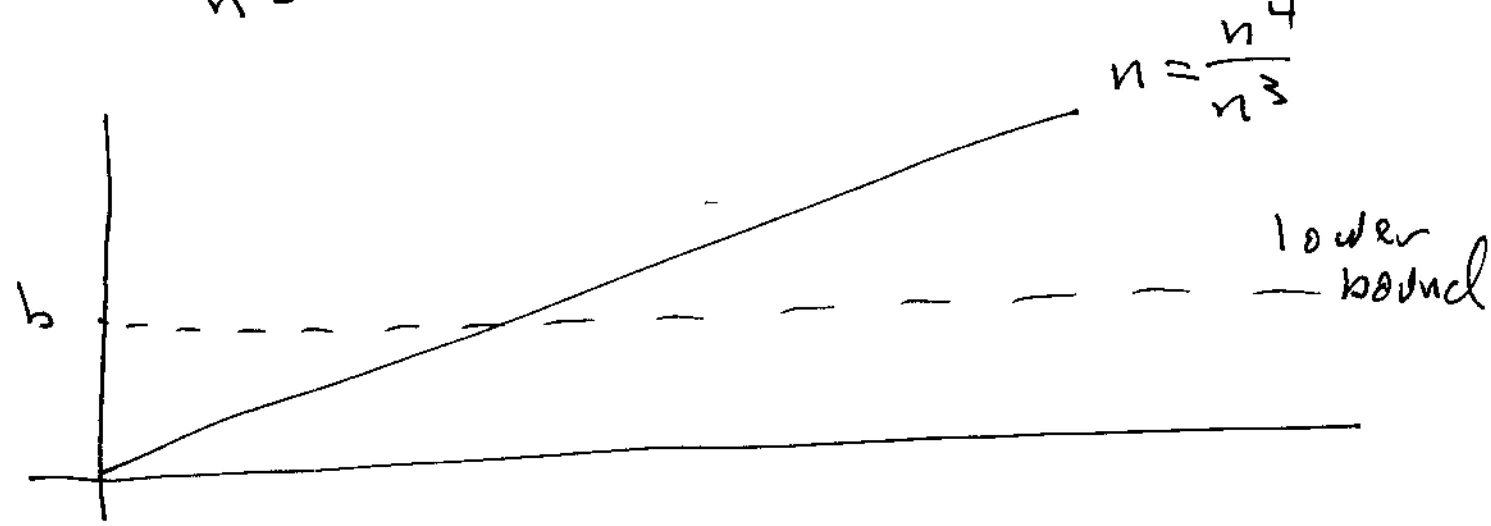
Ex. $n^4 \neq O(n^3)$

$$\frac{n^4}{n^3} = n$$



Ex. $n^4 = \Omega(n^3)$

$$\frac{n^4}{n^3} = n \to \infty \quad \text{as } n \to \infty$$



<u>Fact:</u> $f(n) = O(g(n))$ iff $g(n) = \Omega(f(n))$

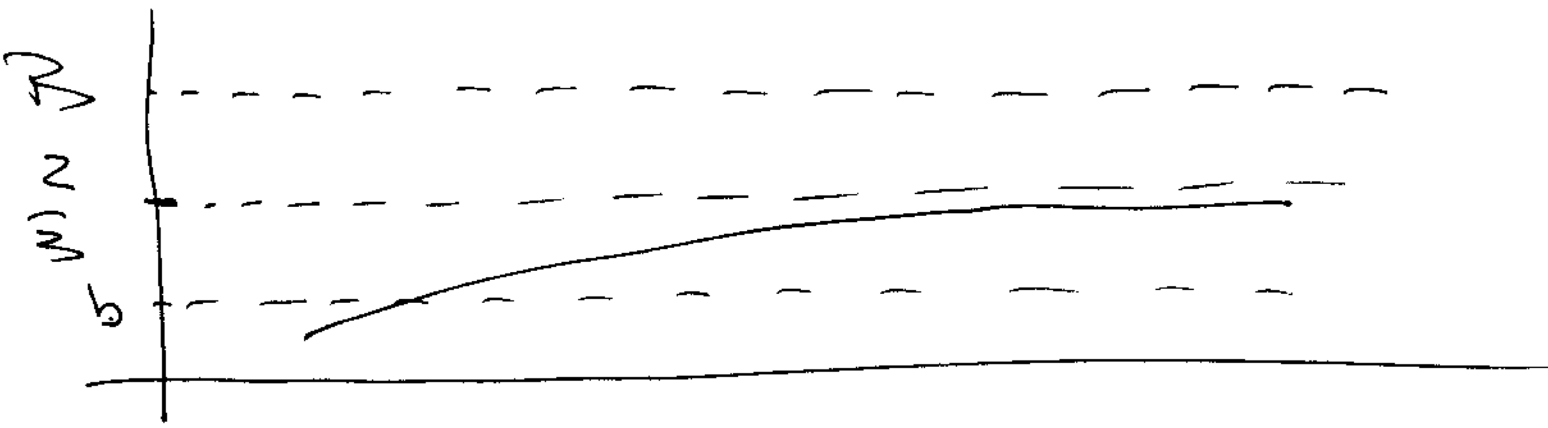
<u>Fact</u> $f(n) = \Theta(g(n))$ iff both

$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

i.e. $\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$.

<u>Ex.</u> $2n^2 + 5n - 1 = \Theta(3n^2 - 6n + 4)$ ✓

$$\frac{2n^2 + 5n - 1}{3n^2 - 6n + 4} = \frac{2 + \cancel{\frac{5}{n}}^{\,0} - \cancel{\frac{1}{n^2}}^{\,0}}{3 - \cancel{\frac{6}{n}}^{\,0} + \cancel{\frac{4}{n^2}}^{\,0}} \to \frac{2}{3} \text{ as } n \to \infty$$



Analogy: analogous to

$f(n) = O(g(n)) \longleftrightarrow x \le y$

$f(n) = \Omega(g(n)) \longleftrightarrow x \ge y$

$f(n) = \Theta(g(n)) \longleftrightarrow x = y$

$f(n) = o(g(n)) \longleftrightarrow x < y$

$f(n) = \omega(g(n)) \longleftrightarrow x > y$

Defn

$f(n) = o(g(n))$ iff

$$\frac{f(n)}{g(n)} \longrightarrow 0 \quad \text{as} \quad n \to \infty.$$

Defn

$f(n) = \omega(g(n))$ iff

$$\frac{f(n)}{g(n)} \longrightarrow \infty \quad \text{as} \quad n \to \infty$$

Fact: $f(n) = o(g(n))$ iff $g(n) = \omega(f(n))$

Ex. $\ln(n) = o(n)$

$$\lim_{n\to\infty} \left(\frac{\ln(n)}{n} \right) = \lim_{n\to\infty} \left(\frac{1/n}{1} \right) = 0$$

Ex. $(\ln(n))^k = o(n^\alpha)$ for any k, α

Ex. $n^k = o(b^n)$ for any k, b

Exercise --

Ex. $n^k = o(e^n)$ for any k

$$\lim_{n\to\infty}\left(\frac{n^k}{e^n}\right) = \lim_{n\to\infty}\left(\frac{k\cdot n^{k-1}}{e^n}\right) = \cdots = 0$$

Ex. $a^n = \begin{cases} o(b^n) & \text{iff} \quad a<b \\ \Theta(b^n) & \text{iff} \quad a=b \\ \omega(b^n) & \text{iff} \quad a>b \end{cases}$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \longrightarrow \begin{cases} 0 & \text{iff} \quad a<b \\ 1 & \text{iff} \quad a=b \\ \infty & \text{iff} \quad a>b \end{cases}$$

Heirarchy of growth rates

$\vdots$

4^n

3^n

exponential fcns $\quad e^n$ $\quad$ intractable algorithms

2^n

$(3/2)^n$

$\vdots$

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n^4

n^3

Polynomial fcns $\quad n^2$ $\quad$ Practical algorithms

n

$\sqrt{n} = n^{1/2}$

$\vdots$

$(\log(n))^3$

log fcns $\quad (\log(n))^2$

$\log(n)$

$\vdots$

What about BFS ?

let $n = |V(G)|$ and $m = |E(G)|$

cost of 1–7 : $3n = \Theta(n)$

" " 8 : const.

" " 9 : "

cost of 10–18 :

Queue ops : $\Theta(n)$

loop 12–17 : cost at most total length of all adj lists

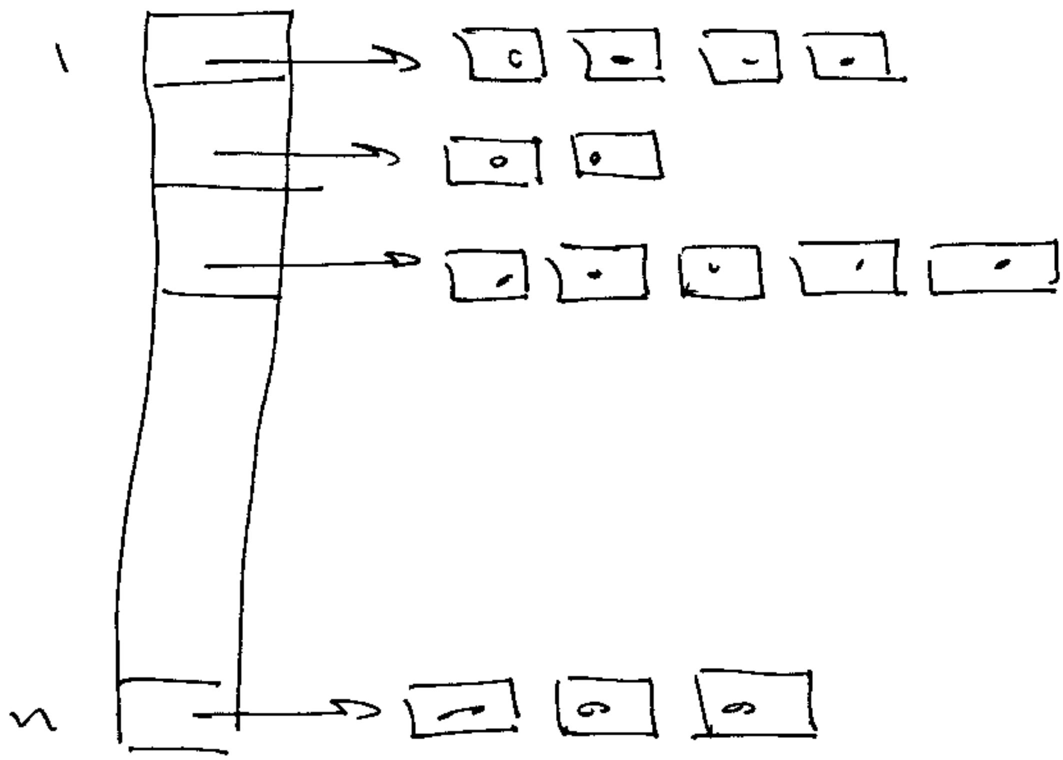
$$= \sum_{x \in V(G)} \deg(x) = \begin{cases} 2m & \text{undir. case} \\ m & \text{dir. case} \end{cases}$$

$$= \Theta(m)$$

$$\boxed{\text{Total cost} = \Theta(n+m)}$$

linear
Polynomial in # bytes to store adj list rep.

memory



$$\# \text{bytes} = \text{const}_1 \cdot n + \text{const}_2 \cdot m$$