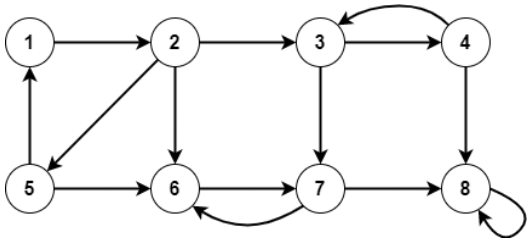
CSE 101
Introduction to Data Structures and Algorithms
Programming Assignment 3

In this assignment you will build a Graph module in C that implements the Depth First Search (DFS) algorithm, then use it to find the strongly connected components of a directed graph. Read the handout on graph algorithms, and sections 20.3-20.5 of the text. Also see the pseudo-code posted on the class webpage at Examples/Pseudo-Code/GraphAlgorithms.

A digraph $G = (V, E)$ is said to be *strongly connected* iff for every pair of vertices $u, v \in V$, vertex u is reachable from v, and vertex v is reachable from u. Most directed graphs are not strongly connected. In general, we say a subset $X \subseteq V$ is *strongly connected* iff every vertex in X is reachable from every other vertex in X. A strongly connected subset that is maximal with respect to this property is called a *strongly connected component* of G. In other words, $X \subseteq V$ is a strongly connected component of G, if and only if (i) X is strongly connected, and (ii) the addition of one more vertex to X would create a subset that is not strongly connected.

Example

G



We can see that this digraph contains 4 strongly connected components: $C_1 = \{1, 2, 5\}$, $C_2 = \{3, 4\}$, $C_3 = \{6, 7\}$, and $C_4 = \{8\}$.

To find the strong components of a digraph G, call DFS(G). As vertices are finished, place them onto a stack. When DFS is complete the stack will store the vertices ordered by decreasing finish times. Next, compute the transpose G^T of G. (The digraph G^T is obtained by reversing directions on all directed edges of G.) Finally run DFS(G^T), but in the main loop (lines 5-7) of DFS, process vertices in order of decreasing finish times from the first call to DFS. This is accomplished by popping vertices off the stack. When the whole process is complete, the trees in the resulting DFS forest span the strong components of G. Note the strongly connected components of G are identical to the strong components of G^T. See the algorithm Strongly-Connected-Components in section 20.5 (p.577) of the text.

Your graph module will again use the adjacency list representation. It will, among other things, provide the capability of running DFS, and computing the transpose of a directed graph. DFS requires that vertices possess attributes for color (white, black, grey), discover time, finish time, and parent. Below is a catalog of required functions and prototypes, constituting the majority of Graph.h.

```
// constructors-destructors ---------------------------------------------------

// newGraph()
// Returns a Graph (directed or undirected) having n vertices and no edges.
Graph newGraph(int n);

// freeGraph()
// Frees heap memory associated with *pG, and sets *pG to NULL.
void freeGraph(Graph* pG);
```

```
// access functions -----------------------------------------------------------

// getOrder()
// Returns the number of vertices in G.
int getOrder(Graph G);

// getNumEdges()
// Returns number of (undirected) edges in G.
int getNumEdges(Graph G);

// getNumArcs()
// Returns number of directed edges in G.
int getNumArcs(Graph G);

// getParent()
// Returns the parent of vertex u, or NIL if DFS() not yet called.
int getParent(Graph G, int u);

// getDiscover()
// Returns the discover time of u, or UNDEF if DFS() not yet called.
int getDiscover(Graph G, int u);

// getFinish()
// Returns the finish time of u, or UNDEF if DFS() not yet called.
int getFinish(Graph G, int u);


// manipulation procedures ----------------------------------------------------

// makeNull()
// Resets G to its original (no edge) state.
void makeNull(Graph G);

// addEdge()
// Creates an undirected edge joining vertex u to vertex v.
void addEdge(Graph G, int u, int v);

// addArc()
// Creates a directed edge from vertex u to vertex v
void addArc(Graph G, int u, int v);

// DFS()
// Runs the Depth First Search algorithm on G.  Input List S contains the vertex
// labels 1, .., n, where n=getOrder(G), and determines the order in which vertices
// are processed in the main loop of DFS(). When complete, output List S contains
// the same vertices sorted by decreasing finish times.
// Pre: getOrder(G)==getLength(S)
void DFS(Graph G, List S);


// other operations -----------------------------------------------------------

// copyGraph()
// Returns a copy of G.
Graph copyGraph(Graph G);

// transpose()
// Returns the transpose of Graph G.
Graph transpose(Graph G);

// printGraph()
// Prints the adjacency list representation of G to FILE* out.
void printGraph(FILE* out, Graph G);
```

Function newGraph() will return a reference to a new graph object containing n vertices and no edges. freeGraph() frees all heap memory associated with a graph and sets its Graph argument to NULL. Function getOrder() returns the number of vertices in G, while functions getParent(), getDiscover(), and getFinish()

return the appropriate field values for the given vertex. Note that the parent of a vertex may be NIL. The discover and finish times of vertices will be undefined before DFS is called. You must #define constant macros for NIL and UNDEF representing those values and place the definitions in the Graph.h. The descriptions of functions addEdge() and addArc() are exactly as they were in pa2. Note that, as in pa2, it is required that adjacency lists are always processed in increasing order by vertex label. It is the responsibility of functions addEdge() and addArc() to maintain adjacency lists in sorted order.

Function DFS() will perform the depth first search algorithm on G. The List argument S has two purposes in this function. First it defines the order in which vertices are to be processed in the main loop (5-7) of DFS. Second, when DFS is complete, it will store the vertices by decreasing finish times (hence S is considered to be a stack). Thus S can be classified as both an input and output parameter to function DFS(). You should utilize the List module from pa1 to implement S and the adjacency lists representing G. DFS() has two preconditions: (i) $\text{length}(S) == n$, and (ii) S contains some permutation of the integers $\{1, 2, \ldots, n\}$ where $n = \text{getOrder}(G)$. You are required to check the first precondition but not the second.

Recall DFS() calls the recursive algorithm Visit() (referred to as DFS-Visit() in the text), and uses a variable called `time` that is static over all recursive calls to Visit(). This function is not mentioned in the above catalog (Graph.h) and therefore is to be considered a private helper function in Graph.c. There are at least three approaches to implementing Visit(). You can define Visit() as a top-level function in Graph.c and let `time` be a global variable whose scope is the entire file. This option has the drawback that other functions in Graph.c have access to `time` and would be able to alter its value. For this reason, global variables are generally considered to be a poor programming practice. The second approach is to let `time` be a local variable in DFS(), then pass the address of `time` to Visit(), making it an input-output variable to Visit(). This is perhaps the simplest option, and is recommended. The third approach is to again let `time` be a local variable in DFS(), then nest the definition of Visit() *within* the definition of DFS(). Since `time` is local to DFS(), its scope includes the defining block for Visit() and is therefore static throughout all recursive calls to Visit(). This may be tricky if you're not used to nesting function definitions since there are issues of scope to deal with. If you pick this option, first experiment with a few examples to make sure you know how it works. Note that although nesting function definitions is not a feature of standard C, and is not supported by many compilers, it *is* supported by the GNU gcc compiler. See

https://gcc.gnu.org/onlinedocs/gcc/Nested-Functions.html

There's actually a fourth option for implementing `time` that may be the easiest of all. Just give Visit() its own local copy of time, then let it take the current value of time as input, and return the new value of time when it's done. No global variable, no passing by reference and no nested function definitions. These design decisions are left to you, and are the type of thing that should be stated in your README.md file.

Function transpose() returns a reference to a new graph object representing the transpose of G, and copyGraph() returns a reference to a new graph that is a copy of G. Both transpose() and copyGraph() may be considered constructors since they create new graph objects. Function printGraph() prints the adjacency list representation of G to the file pointed to by out. Obviously, there is much in common between the Graph module in this project and the one in pa2. If you wish, you may simply add functionality necessary for this project to the previous one, although it is not required that you do so. You should make note of choices such as this in your README.md file.

The client of your Graph module will be called FindComponents. It will take two command line arguments giving the names of the input and output files respectively:

```
$ FindComponents infile outfile
```

FindComponents.c will do the following:

- Read the input file.
- Assemble a graph object G using newGraph() and addArc().
- Print the adjacency list representation of G to the output file.
- Run DFS on G and G^T, processing the vertices in the second call by decreasing finish times from the first call.
- Determine the strong components of G.
- Print the strong components of G to the output file in topologically sorted order.

After the second call to DFS(), the List parameter *S* can be used to determine the strong components of *G*, and to determine a topological sort of those components. You should trace the algorithm Strongly-Connected-Components on several small examples, keeping track of the List *S* to see how this can be done. Input and output file formats are illustrated in the following example, which corresponds to the directed graph on the first page of this handout:

Input:	**Output:**
8	Adjacency list representation of G:
1 2	1: (2)
2 3	2: (3, 5, 6)
2 5	3: (4, 7)
2 6	4: (3, 8)
3 4	5: (1, 6)
3 7	6: (7)
4 3	7: (6, 8)
4 8	8: (8)
5 1	
5 6	G contains 4 strongly connected components:
6 7	Component 1: (1, 5, 2)
7 6	Component 2: (3, 4)
7 8	Component 3: (7, 6)
8 8	Component 4: (8)
0 0	

Observe that the input file format is very similar to that of pa2. The first line gives the number of vertices in the graph, subsequent lines specify directed edges, and input is terminated by the 'dummy' line 0 0. You are required to submit the following eight files:

README.md
Makefile
List.h
List.c
Graph.h
Graph.c
GraphTest.c
FindComponents.c

As usual README.md contains a catalog of submitted files and any special notes to the grader. Makefile should be capable of making the executables GraphTest and FindComponents and should contain a clean

utility that removes all binary files. Graph.c and Graph.h are the implementation and interface files respectively for your Graph module. GraphTest.c will contain your own tests of your Graph module. FindComponents.c implements the top-level client and main program for this project. To get full credit, your project must implement all required files and functions, compile without errors or warnings, produce correct output on our unit tests, and produce no memory leaks under valgrind. By now everyone knows that points are deducted both for neglecting to include required files, for misspelling any filenames and for submitting additional unwanted files but let me say it anyway: do not submit binary files of any kind.

Note that FindComponents.c needs to pass a List to the function DFS(), so FindComponents.c is also a client of the List module. In fact, any client of Graph is also a client of List just by the presence of the List parameter to DFS(). Therefore Graph.h should itself #include the file List.h. (See the handout entitled *C Header File Guidelines* for more on this topic.)

A Makefile for this project will be posted on the course webpage which you may alter as you see fit. As usual start Early and ask questions if anything is not completely clear.