CSE 101 10-7-25

Continue Graph Theory handout..

- The length of walk is the # of edge traversals

$$x = v_0, v_1, \ldots, v_{k-1}, v_k = y \quad \text{length} = k$$

- A trail is a walk where no edge is traversed twice.

- A Path is a trail in which no vertex is visited twice, except possibly when $x = y$.

- The trivial path is a closed path of length 0.

- A non-trivial closed path is called a cycle.

- Let $x, y \in V(G)$. The distance $\delta(x,y)$ is

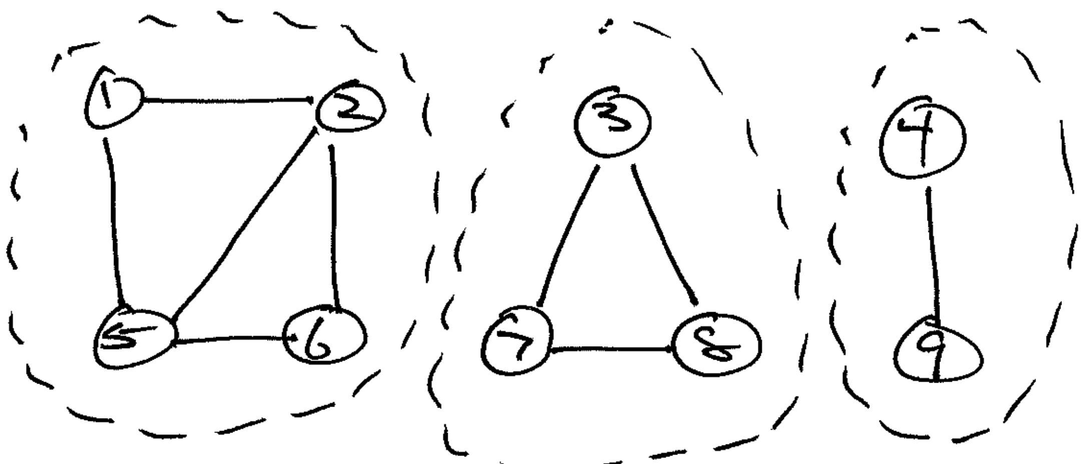
$$\delta(x,y) = \begin{cases} \text{length of a min-length } x\text{-}y \text{ path, if such a path exists} \\ \infty, \text{ otherwise} \end{cases}$$

- We say y is reachable from x iff there exists an x-y path.

- G is called <u>connected</u> iff for all $x, y \in V(G)$, y is reachable from x and x from y (otherwise <u>disconnected</u>).

<u>Ex</u>

G



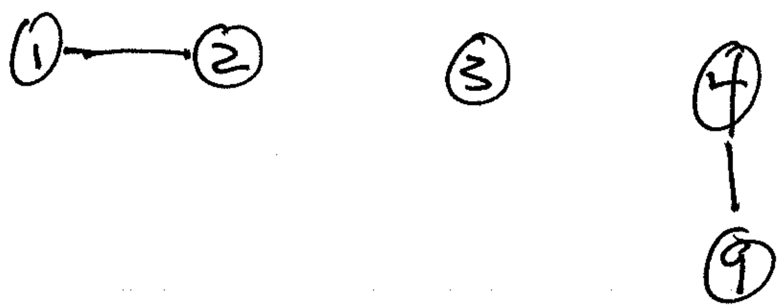
- A <u>subgraph</u> of G is a graph H such that

$$V(H) \subseteq V(G)$$

$$E(H) \subseteq E(G)$$

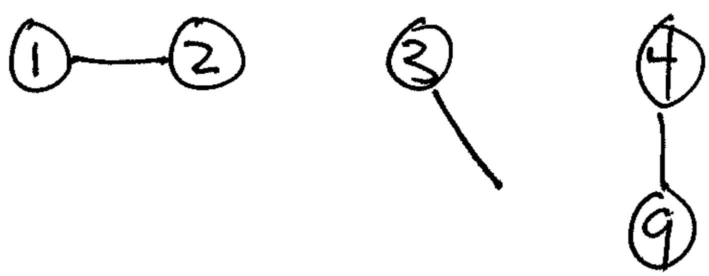
Ex.

- $(\{1, 2, 3, 4, 9\}, \{12, 49\})$



subgraph

- $(\{1, 2, 3, 4, 9\}, \{12, 38, 49\})$



not a graph
so not
a subgraph

Defn

A subgraph H of G is a connected component of G iff

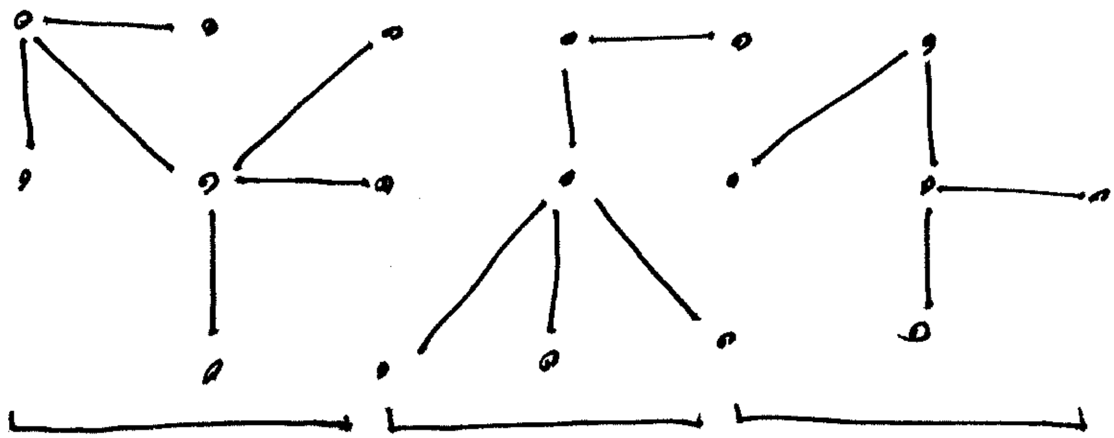
(1) H is connected, and

(2) H is maximal w.r.t. (1)

Defn

A graph is called acyclic iff it contains no cycles. A Tree is a graph that is both acyclic and connected.

Ex forest (acyclic graph)



#vert	7	6	5
#edges	6	5	4

Theorem

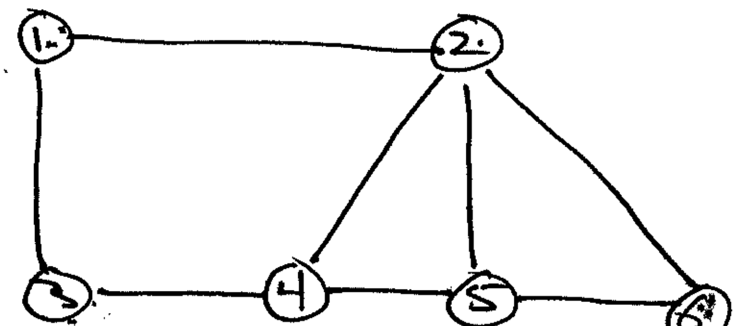
If T is a tree on n vertices, then T has $n-1$ edges.

BFS examples

Ex 1

s=3

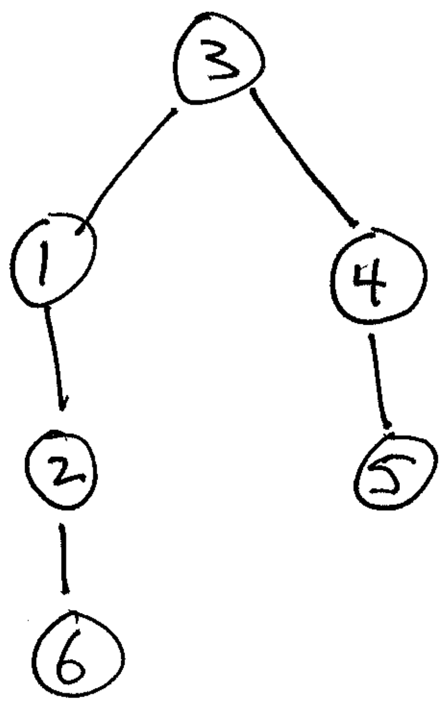
G



	adj	color	dist	Parent
1	(2, 3)	~~w~~ ~~g~~ b	~~∞~~ 1	~~n~~ 3
2	(1, 4, 5, 6)	~~w~~ ~~g~~ b	~~∞~~ 2	~~n~~ 1
3	(1, 4)	~~w~~ ~~g~~ b	0	n
4	(2, 3, 5)	~~w~~ ~~g~~ b	~~∞~~ 1	~~n~~ 3
5	(2, 4, 6)	~~w~~ ~~g~~ b	~~∞~~ 2	n 4
6	(2, 5)	~~w~~ ~~g~~ b	~~∞~~ 3	~~n~~ 2

Q: ~~3~~, ~~1~~, ~~4~~, ~~2~~, ~~5~~, ~~6~~

BFS Tree / Predecessor Subgraph



distance

0

1

2

3