CSE 101 10-30-25

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exercise (4) in handout:

let $a > 1$, $b > 1$. Then

$$a^n = \begin{cases} o(b^n) & \text{if } a < b \\ \Theta(b^n) & a = b \\ \omega(b^n) & a > b \end{cases}$$

Proof

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \longrightarrow \begin{cases} 0 & \text{if } a < b \\ 1 & a = b \\ \infty & a > b \end{cases}$$

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#(7) Compare $2^{3^n}$ to $3^{2^n}$

$$\frac{3^{2^n}}{2^{3^n}} \longrightarrow 0 \quad \text{as } n \to \infty$$

$$\ln\left(\frac{3^{2^n}}{2^{3^n}}\right) = \ln\left(3^{2^n}\right) - \ln\left(2^{3^n}\right)$$

$$= 2^n \cdot \ln(3) - 3^n \cdot \ln(2)$$

$$= 3^n \left( \left(\frac{2}{3}\right)^n \ln 3 - \ln(2) \right) \to -\infty$$

$$\left(\frac{2}{3}\right)^n \to 0$$

$$\therefore \quad 3^{2^n} = o\left(2^{3^n}\right)$$

## SelectionSort(A)

```
1.  n = length[A]
2.  for i = 1 to n-1
3.      imin = i
4.      for j = i+1 to n
5.          if A[j] < A[imin]
6.              imin = j
7.      A[i] ↔ A[imin]   // swap
```

$A_1 \dots A_{i-1}$ | $A_i \; A_{i+1} \dots A_n$

sorted | unsorted.

i-1 smallest elements

Basic operation: array comparison (line 5)

$$\#\text{comp} = (n-1)+(n-2)+(n-3)+\dots+3+2+1$$

$$= \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n = \Theta(n^2).$$

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- BFS $\Theta(n+m) = \Theta(\#\text{bytes to store adj lists})$
- DFS $\Theta(n+m) = \Theta($ " $)$

$n = \#\text{vertices}$, $m = \#\text{edges}$

# Part 4: how to add 2 (sparse) lists

Ex.

P: (10, 1.0) (30, 2.0) (50, 3.0) (60, 4.0)

Q: (20, 5.0) (40, 6.0) (50, 7.0) ----

L: (10, 1.0) (20, 5.0) (30, 2.0) (40, 6.0) (50, 10.0) (60, 4.0)

Product(A, B)

$T$ = transpose

for $i = 1$ to $n$ ← size

if $A_i == 0$ row ! continue

for $j = 1$ to $n$

$$C_{ij} = dot(A_{i,}, T_j)$$