CSE 101 10-23-25

Exercise

- show $c \cdot f(n) = \Theta(f(n))$

must find B_1, B_2, n_0 positive s.t.

$$B_1 \le \frac{c \cdot \cancel{f(n)}}{\cancel{f(n)}} \le B_2$$

for all $n \ge n_0$.

can take $B_1 = B_2 = c$, $n_0 = 1$.

exercise

(1) (a)

assume $L = \lim_{n \to \infty} \frac{f(n)}{g(n)}$ exists.

show that

$$0 \leq L < \infty \Longrightarrow f(n) = O(g(n))$$

Proof

if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L$, then

for all $\varepsilon > 0$, there exists $n_0 > 0$ such that

$$n > n_0 \Longrightarrow \left| \frac{f(n)}{g(n)} - L \right| < \varepsilon$$

Since this is true for all $\varepsilon > 0$. Pick $\varepsilon = 1$. Then there exists $n_0 > 0$ s.t.

$$n > n_0 \Rightarrow \left| \frac{f(n)}{g(n)} - L \right| < 1$$

$$\Rightarrow -1 < \frac{f(n)}{g(n)} - L < 1$$

$$\Rightarrow \frac{f(n)}{g(n)} < L + 1$$

Thus

$$0 \le \frac{f(n)}{g(n)} \le \underbrace{L+1}_{B}$$

for all $n \ge n_0$.

$\therefore$ $f(n) = O(g(n))$. ∎

more exercises

(3) let $\alpha, \beta \in \mathbb{R}$. show

$$n^{\alpha} = \begin{cases} o(n^{\beta}) & \alpha < \beta \quad \checkmark \\ \Theta(n^{\beta}) & \alpha = \beta \quad \checkmark \\ \omega(n^{\beta}) & \alpha > \beta \quad \checkmark \end{cases}$$

Proof

$$\frac{n^{\alpha}}{n^{\beta}} = n^{\alpha-\beta} \longrightarrow \begin{cases} 0 & \text{if } \alpha < \beta \quad \checkmark \\ 1 & \alpha = \beta \quad \checkmark \\ \infty & \alpha > \beta \quad \checkmark \end{cases}$$

$\blacksquare$