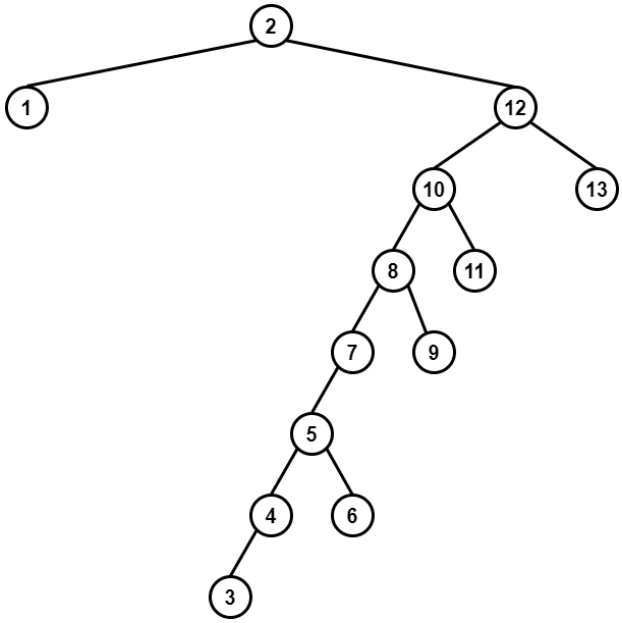
CSE 101
Fall 2020
Quiz 5 **Solutions**

1. (25 Points) Insert the keys: 5 8 3 4 6 1 9 2 7 (in order) into an initially empty Binary Search Tree. Determine all ways in which the colors Red and Black can be assigned to the nodes in this tree so that the RBT properties are satisfied. See https://classes.soe.ucsc.edu/cse101/Fall20/Notes/11-24-20.pdf for a statement of the RBT properties.

   a. (10 Points) State the number of such color assignments.
   Solution:
   There are 2 distinct ways to assign colors to satisfy the RBT properties.

   b. (15 Points) Specify each assignment by stating exactly which nodes are to be colored Red.
   Solution:
   The two ways are:
   (1) assign 2 and 7 to be Red, all others Black
   (2) assign 2, 3, 7, 8 to be Red, all others Black

2. (25 Points) Consider the Binary Search Tree pictured below.



   Use the Theorem on page 8 of https://classes.soe.ucsc.edu/cse101/Fall20/Notes/11-24-20.pdf to explain why it is **impossible** to assign colors Red and Black to the nodes in this tree so as to satisfy the RBT properties. (Note: be sure to include the nil leaves when calculating the height of this tree.)

   Solution:
   According to the stated theorem, we must have $h \leq 2\lg(n+1)$ for any RBT with n internal nodes and (absolute) height h. Observe that the tree above has $n = 13$ and $h = 8$ (counting the nil children of node 3 as the deepest leaves.) But then

$$2\lg(n+1) = 2\lg(14) < 2\lg(16) = 2 \cdot 4 = 8 = h$$

   so the inequality in the theorem is false. Therefore the above tree cannot satisfy the RBT properties.