

Paz: ext 2 days

Adjacency matrix: $A(G)$

requires $V = \{x_1, x_2, \dots, x_n\}$

for $1 \leq i \leq n, 1 \leq j \leq n$

undir.

$$A_{ij} = \begin{cases} 1 & x_i \text{ adjacent to } x_j \\ 0 & \text{otherwise} \end{cases}$$

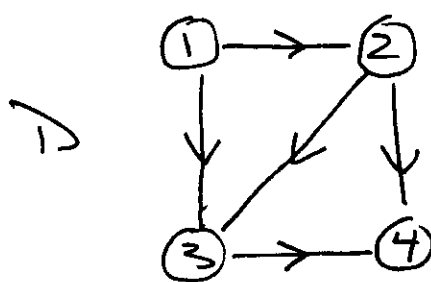
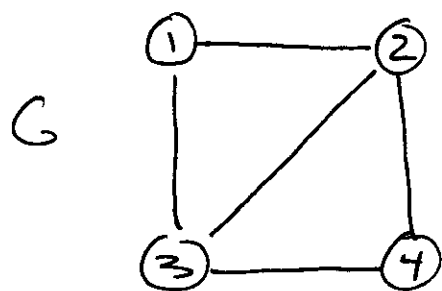


directed

$$A_{ij} = \begin{cases} 1 & \text{if } (x_i, x_j) \in E(G) \\ 0 & \text{otherwise} \end{cases}$$



Ex same



$$A(G) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A(D) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thm

Let $A = A(G)$, and $d \geq 0$. Then

$$(A^d)_{ij} = \# \text{ of } i\text{-}j \text{ walks in } G \text{ of length } d.$$

Adjacency List: $adj(G)$

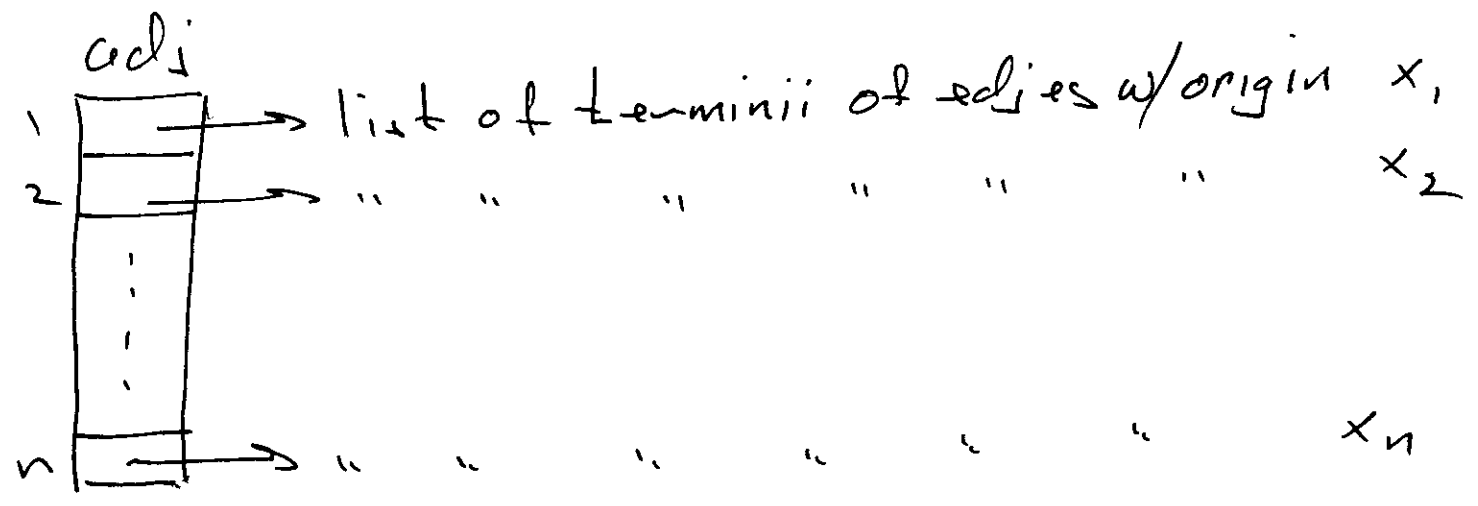
requires $V = \{x_1, x_2, \dots, x_n\}$

$adj(G)$ is a 1-dim of Lists.

undirected

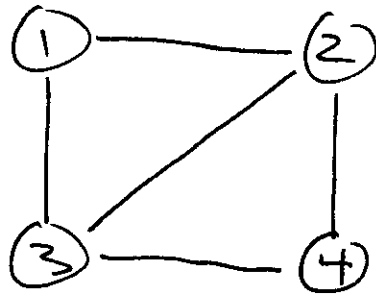
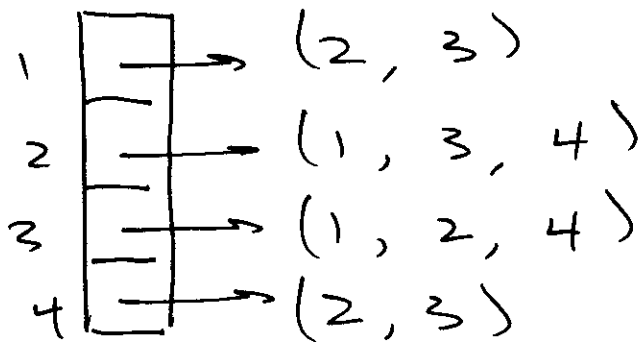


directed

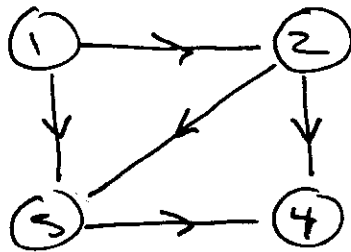
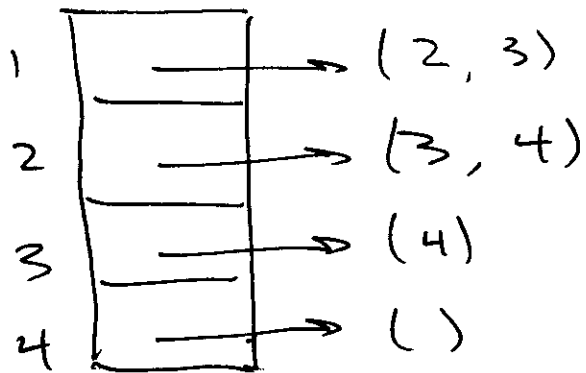


Ex.

G :

 $\text{adj}(G)$ 

D :

 $\text{adj}(D)$ 

G-raph Algorithms

Defn

let G be a g -aph (or dig-aph),

let $x, y \in V(G)$.

$$\delta(x, y) = \begin{cases} (\text{min length of an } x-y \text{ Path}) & \text{if } y \text{ reach. fr. } x \\ \infty & \text{otherwise} \end{cases}$$

Single-source shortest-Paths Problem (SSSP):

Given G (dir. or undir), and fixed $s \in V(G)$ called the source, determine

(1) $\delta(s, x)$ for all $x \in V(G)$

(2) if $\delta(s, x) < \infty$, find an $s-x$ Path of length $\delta(s, x)$.

Breadth First Search (BFS)

input: $adj(G)$ and $s \in V(G)$

maintain, for each $x \in V(G)$

- $color[x]$: white, gray, black
 ↑ ↑ ↑
 undiscovered discovered, but not all neigh. finished, disc. x & all neigh.

- $distance[x]$: $\delta(s, x)$, by the time BFS is complete

- $Parent[x]$: stores predecessor of x along a shortest $s-x$ path.

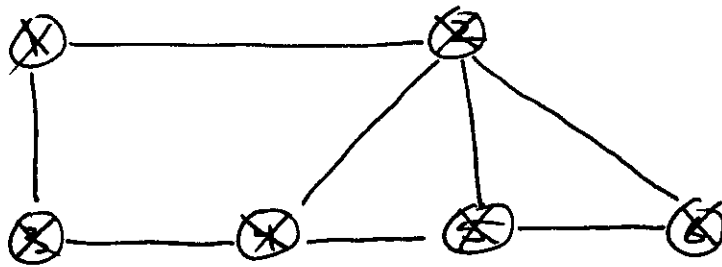
Also maintain a FIFO queue Q .

note

- lines 1-7 init. vertex attributes.
- lines 8-9 init. FIFO Queue Q
- while loop discovers all vertices reachable from src.

Ex.

G



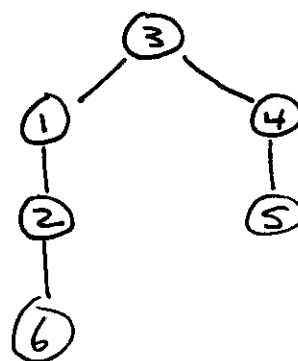
source = 3

	adj	color	d	p
<u>1</u>	<u>2, 3</u>	w g b	∞ 1	* 3
<u>2</u>	<u>1, 4, 5, 6</u>	w g b	∞ 2	* 1
<u>3</u>	<u>1, 4</u>	g b	0	n
<u>4</u>	<u>2, 3, 5</u>	w g b	∞ 1	* 3
<u>5</u>	<u>2, 4, 6</u>	w g b	∞ 2	* 4
<u>6</u>	<u>2, 5</u>	w g b	∞ 3	* 2

Q: ~~3~~ ~~1~~ ~~4~~ ~~2~~ ~~5~~ ~~6~~

BFS Tree

dist



0

1

2

3