CSE 101 10-20-20

PA2 : next week

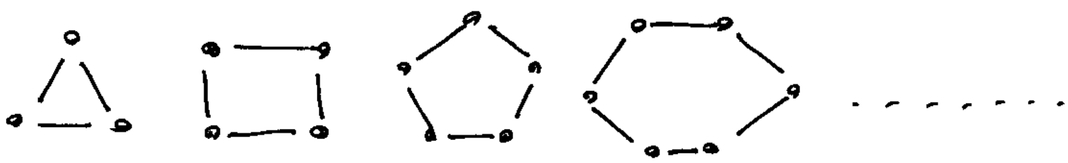
quiz2 : next week

more on graphs.

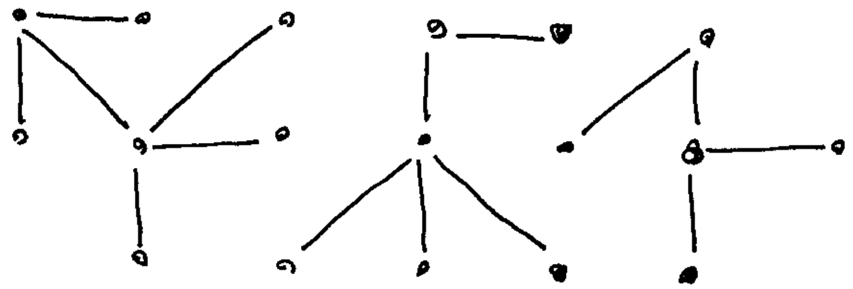
Defn

A graph is called acyclic iff it contains no cycles. also called forest.

Cycles



Ex. acyclic graph



3 conn. components

n:	7	6	5	$n = \|V\|$
m:	6	5	4	$m = \|E\|$

Defn
A tree is a graph that is both connected and acyclic.

Thm
If T is a tree with n vertices, then T has n-1 edges.

Theorem (Treeness)

Let $G=(V,E)$, $n=|V|$, $m=|E|$. Then the following are equivalent.

(a) G is a tree (i.e. conn. & acyclic)

(b) G contains a unique x-y path for all $x, y \in V$.

(c) G is connected, but if any edge e is removed, $G-e$ is disconnected.

(d) G is conn. and $m=n-1$.

(e) G is acyclic and $m=n-1$.

(f) G is acyclic, but if any edge e is added (joining two non-adjacent vertices) then $G+e$ contains a unique cycle.

3 Properties:

(i) connected

(ii) acyclic

(iii) $m = n-1$

Parts (a), (d), (e) together say any 2 of these properties implies the third.

<u>Exercise</u>: find examples

(i)	(ii)	(iii)	
F	F	F	← ex 1
F	F	T	← ex 2
F	T	F	← ex 3
F	T	T	← not possible
T	F	F	← ex 4
T	F	T	← " "
T	T	F	← " "
T	T	T	← any tree.

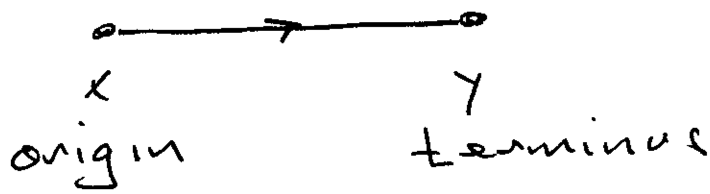
Directed Graphs

Defn A directed graph or digraph is a pair $G = (V, E)$ where

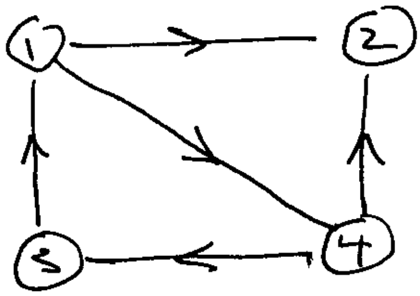
$V \neq \emptyset$

$E \subseteq V \times V$, i.e. ordered pairs of vertices

(x, y)



Ex. $V = \{1, 2, 3, 4\}$
$E = \{(1,2), (1,4), (3,1), (4,2), (4,3)\}$



in degree: $id(1) = 1$, $id(4) = 1$
out degree: $od(1) = 2$, $od(4) = 2$

Thm

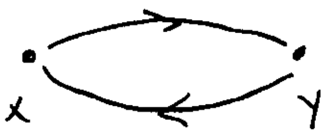
$$\sum_{x \in V} id(x) = \sum_{x \in V} od(x) = |E|.$$

Definitions: (exercises)

- isomorphism of digraphs.
- $x - y$ (directed) walk
- $x - y$ (dir.) trail
- $x - y$ (dir) Path
- trivial (dir.) walk
- closed walk
- directed cycle.

note: 

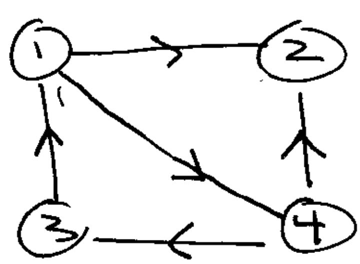
allowed since (x,x)

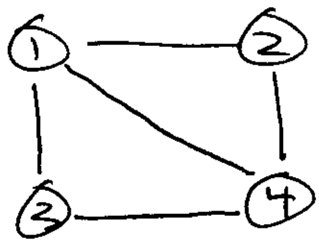


allowed $(x,y) \neq (y,x)$
(if $x \neq y$).

The underlying multi-graph of a digraph G is obtained replacing all ordered pairs by un-ordered pairs

Ex. Previous



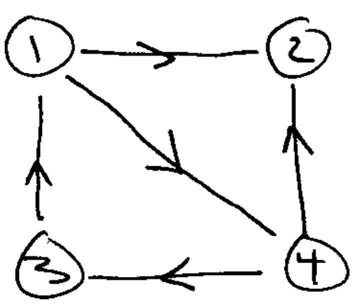


Defn
A digraph is weakly connected iff its underlying multi-graph is connected.

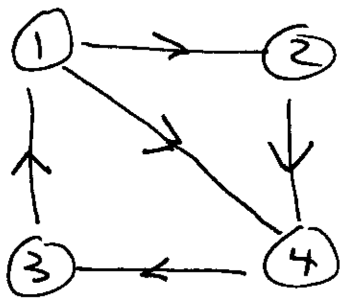
Defn

A digraph G is called <u>strongly connected</u> iff for all $x, y \in V(G)$ y is reachable from x and x is reachable from y.

Ex.



not strongly conn.



is strongly conn.

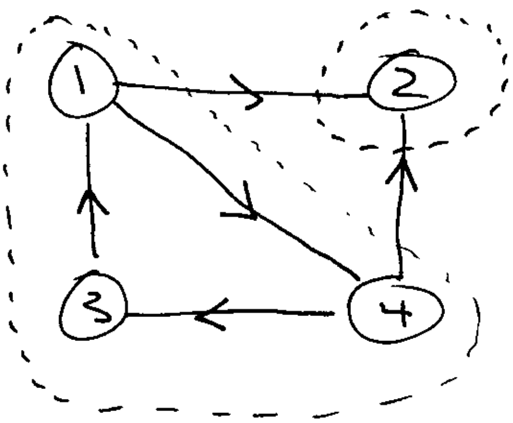
<u>Defn</u>

a subset $S \subseteq V(G)$ is said to be a <u>strongly connected component</u> of G iff

(i) S is strongly connected

(ii) S is maximal w.r.t property (i)

<u>Ex</u>



Representations of Graphs and digraphs.

- Incidence Matrix : 2-dim array
- Adjacency matrix : 2-dim (sq) array
- Adjacency list : 1-dim array of lists.

<u>Incidence matrix</u> $I(G)$

requires: $V = \{x_1, x_2, \dots, x_n\}$ $1 \le i \le n$

labels $E = \{e_1, \dots\dots\dots, e_m\}$ $1 \le j \le m$

undirected:

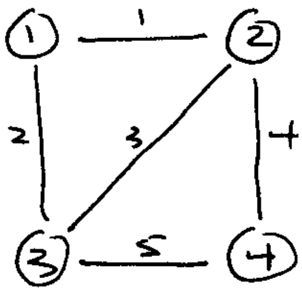
$$I_{ij} = \begin{cases} 1 & \text{if } x_i \text{ incident with } e_j \\ 0 & \text{otherwise} \end{cases}$$

directed

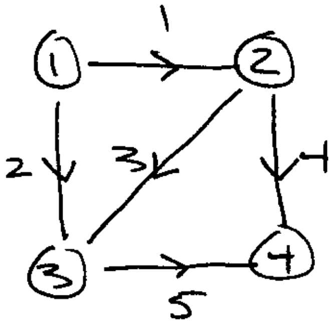
$$I_{ij} = \begin{cases} 1 & \text{if } x_i \text{ is terminus of } e_j \\ -1 & \text{" " " origin " "} \\ 0 & \text{otherwise} \end{cases}$$

Ex.

G



D



$$I(G) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad I(D) = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

<u>Ex.</u> BigInteger

$b = 1000, \quad P = 3$

$$315 \,|\, \underbrace{006}_{\text{long}} \,|\, 201 \,|\, 057$$

long
must print
as
↓
006 not 6

```
#define BASE 1000
#define POWER 3
  :

fprintf(..., ..."%0*d"..., POWER, . , . );
```