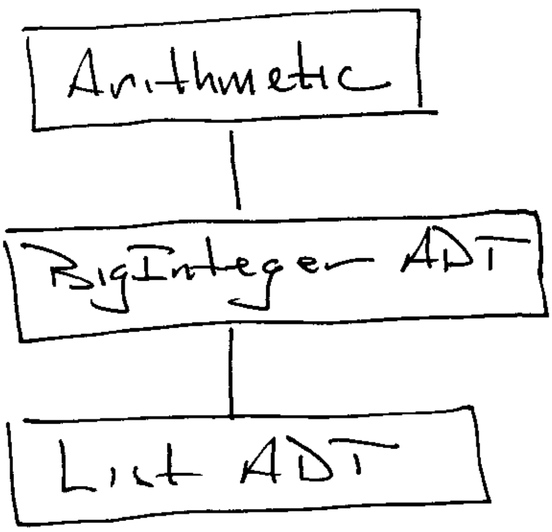
cse 101 10-15-20

- email: Please place cse 101 in subject line.
- Quiz Technical Problems: email Hyejin Han.
- ext Pa1 one last day
- Pa2 is Posted: BigInteger structure chart!



arithmetic ops: add, sub, mult

Ex. base = 100

digits = {00, 01, 02, ..., 98, 99}

$$
\begin{array}{lccccl}
 & \overset{1}{} & & \overset{1}{} & & \\
(& 27 & & 88 & 75) & \leftrightarrow 278875 \\
(& 39 & & 40 & 37) & \leftrightarrow 394037 \\
\hline
(& 67 & & 29 & 12) & \Leftrightarrow 672912
\end{array}
$$

alt way: vector addition

$$
\begin{array}{ccc}
27 & 88 & 75 \\
39 & 40 & 37 \\
\hline
\overset{1}{66} & \overset{1}{128} & 112 \\
67 & 129 & \\
 & -100 & -100 \\
\hline
(67 & 29 & 12)
\end{array}
$$

normalize

note

- not allowed  $\{x,x\} = \{x\}$
- not allowed  $\{x,y\} = \{x,y\}$

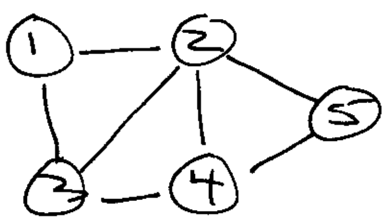
Handshake lemma

$$\sum_{x \in V(G)} \deg(x) = 2|E(G)|$$

degree sequence

seq. of degree in increasing order.

Ex.



deg seq.

$(2, 2, 3, 3, 4)$

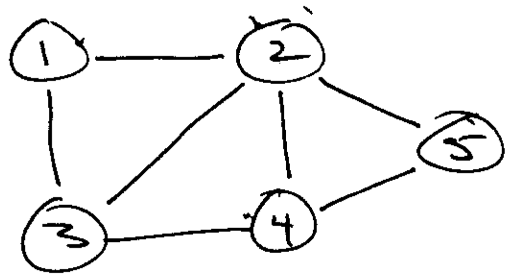
<u>Fact</u> the number of vertices of odd degree must be even.

<u>Exercise</u>: why?

Given $x, y \in V(G)$, an <u>x-y walk</u> in G is a seq. of vertices

$$x = v_0, v_1, v_2, \ldots, v_k = y$$

such that $\{v_{i-1}, v_i\} \in E(G)$ for $1 \le i \le k$. call x <u>origin</u> and y <u>terminus</u> of walk. If $x = y$, walk is called <u>closed</u>. The <u>length</u> of the walk is k, the <u>edge</u> traversals.



length

<u>Walk</u>: (3, 2, 4, 3, 1, 2, 5, 2, 4) 8

we say <u>y is reachable from x</u> iff G contains an x-y walk

<u>trivial walk</u>: $x = v_0 = y$, i.e. no edge traversals.

<u>trail</u>: a walk in which no edge is traversed more than once.

<u>trail</u>: (3, 2, 4, 5, 2, 1) length = 5

Path: trail in which no vertex is visited more than once

Path: (1, 2, 3, 4, 5) length = 4

a cycle is a non-trivial closed Path.

cycle: (1, 2, 4, 3, 1) length = 4

trivial Path: (3) length = 0
not a cycle

note:
could have defined reachability using Paths (or trails) instead of walks.
[G contains an x-y walk] iff
[G contains an x-y Path].

Defn

Let $x, y \in V(G)$. The <u>distance</u> from x to y is

$$\delta(x,y) = \begin{cases} \text{length of a min-length } x\text{-}y \text{ path} \\ \text{if } y \text{ is reachable from } x, \\ \qquad \text{or} \\ \infty \text{ if } y \text{ is not reachable from } x \end{cases}$$

Defn

The <u>diameter</u> of G is

$$\max\{\delta(x,y) \mid x, y \in V(G)\}$$

$\operatorname{diam}(G)$.

Defn

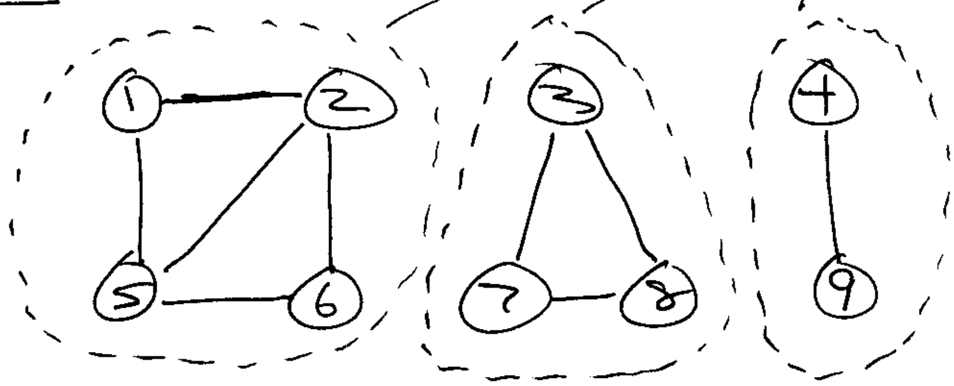
G is called **connected** ~~iff~~

for all $x, y \in V(G)$: y is reachable

from x.

Ex.

connected components



not connected

(**disconnected**)

Defn

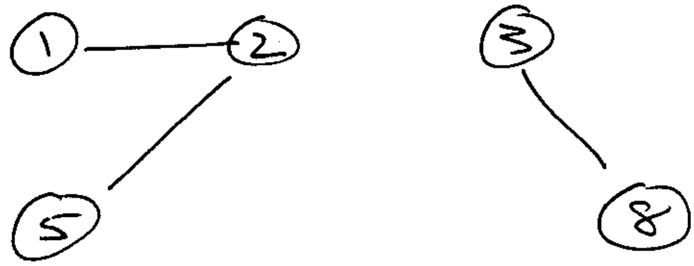
A **subgraph** of a graph G, is

a graph H such that

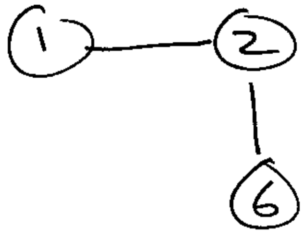
$$V(H) \subseteq V(G)$$

$$E(H) \subseteq E(G)$$

subgraphs

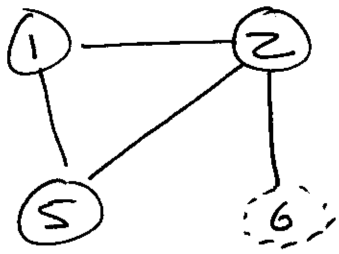


disconnected



connected

not a subgraph



$V = \{1, 2, 5\}$

$E = \{12, 15, 25, 26\}$

not an edge, since no vertex 6

Defn

A subgraph H of G is called a <u>connected component</u> of G iff

(i) H is connected.

(ii) H is maximal with respect to property (i), nothing can be added ~~to~~ H.

continues . . . .