External Shocks under Different Exchange Rate Regimes in Low-Income Economies
reserves and precautionary savings

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Abstract
This paper uses the cash-in-advance model to explore the precautionary savings against external shocks under different exchange rate regimes. The low-income countries mainly face external shocks from the current account, which includes productivity, international aid, terms of trade, external demand, and foreign monetary policy. Agents hold assets to smooth the consumption under the floating exchange rate regime. However, the assets not only stabilize the exchange rate but also smooth consumption under the pegged exchange rate regime. Facing uncertainty, households put more weight on precautionary motive, instead of investment motive. The impulse response function reveals that the initial deviations are more significant with the pegged exchange rate regime because the nominal exchange rate can not absorb the macroeconomic shocks. The floating exchange rate regime could reduce the variance of domestic consumption, while the pegged exchange rate regime would be better to stabilize imported consumption. Furthermore, the foreign monetary policy shocks show significant differences between the two regimes.

1 Introduction
Why do we care about international reserves? There are some reasons for holding international reserves. (1) When a negative shock impacts the current account or capital account, international reserves can buffer this shock and reduce the bad influence. Therefore, households or central bank would accumulate international reserves under precautionary purpose. (2) Using international reserves, the central bank can stabilize the exchange rate. For instance, China pegs the value of its currency to a basket of currencies, using its foreign reserves. (3) The central bank could devalue domestic currency to boost international trade by holding an excess amount of international reserves - Mercantilism. (4) Central bank provides confidence for foreign investors and prevents a sudden flight to quality and loss of capital for the country. (5) Central bank makes sure international reserves would meet external obligations of the country. (6) Central bank diversifies its portfolios and increases asset returns while holding international reserves. Therefore, one question of international finance is the appropriate level of international reserves.
Most papers use the cost-benefit method, by which reserves lower the probability of adverse shocks on current account or capital account and increase attendant costs, to quantify the level of international reserves. They find that the reserves level can be sensitive to country fundamentals and exchange rate regimes, instead of just maintaining reserves equivalent to three months of imports. In short, the cost-benefit method focuses on the role of international reserves in preventing and mitigating absorption drops triggered by large external shocks under considering the opportunity cost of holding reserves. However, other papers present small open economy models to analyze the related reserve levels through the precautionary saving motive of households. In other words, households would prefer to save more international reserves, facing the uncertainty, to smooth their consumption. Although the small open economy model with precautionary purpose is intuitive and explicable, it lacks entirely consideration of central bank monetary policy, such as the impact of different exchange rate regimes on international reserves. The objective of this paper is to combine the small open economy model with different exchange rate regimes and try to find consistent results as the cost-benefit method.

This paper seeks to fill this gap by evaluating precautionary saving under different exchange rate regimes with a small open economy model. As the cost-benefit method found, the exchange rate regime would affect the level reserves. In the following model, the central bank would choose between “floating exchange rate and monetary autonomy” and “pegged exchange rate and renunciation of monetary autonomy”. The first strategy is that the central bank conducts an independent policy rule, such as Jeanne and Sandri [2016], to help households to fulfill the precautionary saving. Whereas, the second strategy includes central bank using international reserves to stabilize the exchange rate. The above allows this paper to address the question, under different exchange rate regimes, what should the level of reserves be? How does each sector (household, firm, and central bank) respond differently, when the negative shock impacts on the economy?

More specifically, the paper examines the dynamic implications of reserves in the context of a simple intertemporal three-sector optimizing real business cycle (RBC) model for low-income countries with external shocks. Compared with emerging market countries, low-income countries are much more financially closed economies. Moreover, emerging market countries and low-income countries are faced with different types of shocks. The shocks from the capital account generally impact emerging market countries, while the shocks from the current account primarily influence low-income countries with limited access to the international financial market.

The model includes productivity shocks, aid shocks, terms of trade shocks, external demand shocks, and foreign monetary policy shocks as exogenous stochastic processes from the current account. Also, it is calibrated using macroeconomic data for 70 low-income countries between 1966 and 2015. The benchmark calibration shows the reserves around

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1Kim et al. [2011] identified the exogenous current account shocks of low-income countries.

2In this paper, low-income countries refer to all countries shown on the IMF’s list of countries eligible for the Poverty Reduction and Growth Trust (PRGT) in October 2015.
the target level, which is close to three months of imports. The prospectus is organized as follows: Section 2 reviews existing literature on cost-benefit and small open economy methods; Section 3 presents the model, highlights the key ingredients, and explains assumptions; Section 4 shows some results from this model with empirical data; Section 5 illustrates the improved model and conclusion.

2 Literature Review

2.1 Cost-Benefit Method Approach to Determining Reserves

First of all, the cost-benefit method found that country fundamentals and exchange rate regimes are essential factors when we try to quantify the level of reserves. Heller [1966] introduced the first formal cost-benefit study of international reserve demand. He found that optimal reserves should satisfy the following condition: reserves should be accumulated until the marginal benefit equals the marginal cost. The marginal benefit of holding reserves is the reduced economic cost of adjusting to a deficit not covered by reserves. The marginal cost of holding reserves is the rate of return foregone by not transforming reserves into real physical capital.

Kim et al. [2011] developed Heller’s methodology and presented a simple cost-benefit framework of precautionary reserve holdings warranted by country characteristics and fundamentals for low-income countries. The basic idea is that international reserves could reduce the likelihood and magnitude of abrupt drops in consumption arising from large external shocks. Also, the cost of holding reserves is the difference between the return in risky but high-yielding domestic capital and safe but low-yielding international reserves. The low-income country seeks to maximize the following function, which involves the trade-off between cost and benefit of holding international reserves.

$$\max_{R} NBR = -qP(R, Z)C(R, Z) - rR$$

Countries would maximize the net benefit of holding reserves (NBR), where $P$ is the conditional probability of a crisis given a large shock, and $C$ is the utility cost (drop in consumption) of a crisis, both depending on reserves $R$ and control variable $Z$. The parameter $q$ is the unconditional probability of a large shock, and the parameter $r$ is the unit cost of holding reserves. The authors empirically estimated the conditional probability of a crisis $P$ and the utility cost of a crisis $C$ in the event of large external shocks. The external shocks include (i) external demand; (ii) terms-of-trade; (iii) FDI; (iv) aid; (v) remittances; (vi) climatic shocks (large natural disasters) for low-income countries. Control variables include the ratio of government balance to GDP, the World Bank’s CPIA index as a proxy for policy and institutional quality, a dummy for flexible exchange rate regime, and a dummy for Fund-supported programs. Finally, the authors found that the traditional rule of thumb of 3 months of imports is more appropriate for countries with flexible exchange rate regimes, but the traditional rule of thumb is likely to be inadequate for countries with fixed exchange
rate regimes.

However, Calvo et al. [2012] used a similar cost-benefit method to explore the reserves of emerging economies. During the recent financial crises\(^3\), largely unexpected cut in international capital flows that happened in emerging economies, which gave them strong incentives to self-insure by accumulating international reserves. The optimal international reserves would balance the probability of sudden stop and carry cost. Their control variables include foreign direct investment, portfolio integration, terms of trade growth, government balance, the exchange rate regime, the ratio M2-to-reserves, and foreign debt as a share of GDP. From their Probit models, unlike the findings of Kim et al. [2011] for low-income countries, the fixed exchange rate regime would insignificantly increase the likelihood of a sudden stop, given the same level of reserves.\(^4\) One potential explanation for this is the Probit estimations also include the change in the real exchange rate, which may capture the explanation of the exchange rate regime. Their main results suggests there is not apparent over-accumulation of reserves in emerging economies, because only 10 out of 27 emerging economies hold the excess international reserves. Similar to the results of Calvo et al. [2008] and Kim et al. [2011], they also found that current account deficits are a critical factor taken into account by the central bank in holding the optimal amount\(^5\) of international reserves.

Moore and Glean [2016] also provided a cost-benefit type approach to evaluating reserve adequacy for small island developing countries. They also followed the idea of Kim et al. [2011] to identify the exogenous shocks (external demand, terms-of-trade, FDI, aid, remittances, and climatic shocks) and crises, and took the policy maker’s objective function of Calvo et al. [2012] to minimize the losses from a sudden stop against the cost of holding reserves. Their estimation took into account government policy and the inherent vulnerability of small countries. In the regression with a dummy variable fixed exchange rate regime, the variable was insignificant, and the results did not change appreciably\(^6\), which was similar to the result of Calvo et al. [2012]. Using the cost-benefit methodology, they found the optimal holding of foreign exchange reserves is approximately 13 weeks higher than the international rule of thumb, depending on the economic characteristics of the country. Indeed, they also found that country with a prudent public expenditure (government policy) can hold a smaller stock of reserves without necessarily impacting on the expected growth for the country, and then the optimal holdings of reserves could fall to just 19 weeks.

The above papers with the cost-benefit method give this paper some critical information. First, the low-income countries, as Kim et al. [2011] mentioned, mainly face the current account shocks, such as external demand, terms-of-trade, FDI, aid, and climatic shocks. Second, exchange rate regimes could influence the level of reserves under the precautionary motive. Kim et al. [2011] presented that low-income countries with fixed exchange rate

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\(^3\)Calvo et al. [2012] presented the fragility of financial markets and institutions is increasing in the last three decades.

\(^4\)Kim et al. [2011] found that the probability of a crisis tends to be significantly lower under flexible exchange rate regimes for low-income countries.

\(^5\)The optimal amount is quantified by the cost-benefit method.

\(^6\)The fixed exchange rate would insignificantly increase the likelihood of a crisis.
regimes should hold more than the traditional rule of thumb under flexible exchange rate regimes. Unfortunately, Calvo et al. [2012] and Moore and Glean [2016] argued the fixed exchange rate regime would insignificantly increase the likelihood of crises (Probit model) in emerging economies and small island developing countries\(^7\) respectively. Third, government policy would affect the holdings of reserves. Moore and Glean [2016] revealed that fiscal policy is an essential factor to quantify the optimal reserves. Fourth, the optimal level of international reserves is slightly higher than the rule of thumb - 3 months of imports.

2.2 SOE Method Approach to Determining Reserves

The small open economy model with a precautionary saving motive is another method to quantify the level of reserves. Fogli and Perri [2015] presented a standard open economy model with time varying macroeconomic uncertainty that can quantitatively account for the relationship between uncertainty and net foreign assets. The crucial mechanism is precautionary motive: more uncertainty induces households to save more, and higher savings are in part channeled into foreign assets. Ljungqvist and Sargent [2004] said self-insurance occurs when the household uses savings to insure himself against income fluctuations. On the one hand, the household could withdraw his savings and avoid large temporary drops in consumption in response to low income realizations. On the other hand, the household could partly save high income realizations in anticipation of bad outcomes in the future. Some papers focus on the precautionary demand for assets that arises from the sudden stop risk in emerging economies, such as Durdu et al. [2009], Alfaro and Kanczuk [2009], and Jeanne and Ranciere [2011]. Jeanne and Ranciere [2011] used a model of the optimal level of international reserves to smooth domestic absorption in response to sudden stops in capital flows (capital account) for a small open economy. However, the recent buildup of reserves in emerging market countries seems in excess of what would be implied by an insurance motive against sudden stops. Others focus on uncertainty, for little integration with capital market as low-income countries, stemming from the current account, such as Dhasmana and Drummond [2008], Barnichon [2008], and Valencia [2010]. Dhasmana and Drummond [2008] used a two-good endowment economy model facing terms of trade and aid shocks in the current account to derive the optimal level of reserves by comparing the cost of holding reserves with their benefits as an insurance against a shock. Their simulations suggested that a few sub-Saharan Africa countries (low-income countries) do not currently carry reserves consistent with the expected output costs associated with expected terms-of-trade or aid shocks. So, international reserve accumulation follows a precautionary motive to build buffers and shield domestic demand from balance of payments (current account and capital account) crises.

Valencia [2010] used the precautionary savings model to compute the level of reserves for Bolivia whose current account shocks are the primary balance of payments risk. In the baseline version of the model, households are assumed to consume only tradable goods and make consumption decisions to maximize the expected present discounted value of the utility derived from consumption. The sequence of events is as follows: at the beginning of the

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\(^7\)The data of small island developing countries includes low-income economies and emerging economies.
period, consumers have net foreign assets $X_t$, conditional on which they make consumption decisions $C_t$, in the middle of the period with the remainder $X_t - C_t$ invested in a risk-free security that yields $R$. At the end of the period - after decisions have been made - income is realized, which determines with how many assets the consumer arrives in period $t + 1$. The budget constraint of households is

$$X_{t+1} = R(X_t - C_t) + \tau_{t+1}Y_{t+1} + A_t$$

, where $Y$ reflects the level of permanent income, $\tau$ denotes transitory shocks to income, assumed to be mean-one, i.i.d., and distributed over a non-negative support, and finally $A$ denotes all other non-export net current receipts. The key factor is $\tau$ which includes two i.i.d. components: export volumes and terms of trade shocks. Next, the extended model adds the investment variable. Because now income is endogenous, consumers save not only for precautionary purposes but also to finance the stock of capital. Therefore, the modified budget constraint is

$$x_{t+1} = R(x_t - c_t - k_t) + \tau_{t+1}k_t^\alpha + (1 - \delta)k_t + a$$

, where $k$ is the capital, and $\tau_{t+1}k_t^\alpha$ denotes income from exports. The differences between baseline and modified model are that the consumer now finds it profitable to cut consumption and finance the stock of capital, and savings need to fulfill the precautionary motive and the investment decision simultaneously. Author suggested the optimal level of reserves from both models result in the range 29 and 37 percent of GDP, which is higher than the standard rule of thumb. These different results illustrate it is important to appropriately account for country-specific risks in order to derive adequate measures of reserve buffers.

Valencia [2010] said this is not a critical issue of who holds these reserves (central bank or household), as long as the central bank’s objectives are in line with households with regards to smoothing demand fluctuations. The policy function for reserve management predicted by the precautionary savings model is non-linear and, therefore, difficult to describe as a simple rule of thumb for the central bank. Jeanne and Sandri [2016] found a simple linear rule can capture most of the welfare gains from optimal reserve management when the central bank holds reserves instead of households. They characterized the optimal management of reserves using an open-economy model of precautionary savings with carrying costs in financially closed economies. They found results are consistent with the rule of thumb (reserves are close to three months of imports) under plausible calibrations for low-income countries, driven by exogenous stochastic processes for the value of exports, the output of nontradable goods, and the real rate of interest. Then, they considered a simple linear rule according to which reserves have to converge towards a target while buffering export shocks. Comparing with the reserves under household precautionary savings motive, they found that

\[8\text{The baseline model uses the standard Euler equation for consumption } u'(c_t) = R\beta E_t u'(c_{t+1}) \text{ and the normalized budget constraint } x_{t+1} = R(x_t - c_t + \tau_{t+1} + a) \text{ to determine the optimal level of reserves } x^* \text{ with the transitory shocks to income } \tau. \text{ The modified model uses the standard Euler equation for consumption } u'(c_t) = R\beta E_t u'(c_{t+1}), \text{ the standard Euler equation for capital } u'(c_t) = \beta E_t u'(c_{t+1})(\alpha \tau_{t+1}k_t^{\alpha - 1} + (1 - \delta)) \text{ and the normalized budget constraint } x_{t+1} = R(x_t - c_t - k_t) + \tau_{t+1}k_t^\alpha + (1 - \delta)k_t + a \text{ to determine the optimal level of reserves } x^* \text{ with the transitory shocks to income } \tau. \]
the linear rule approximates fairly well. In short, they pointed out that it is more important to properly adjust reserves in response to shocks than choosing a particular reserve target because the welfare gains from reserves management come from using the reserves rather than keeping them close to the target.

These papers, using the small open economy model for low-income countries, show the main idea that reserves play a straightforward and primary role of precautionary savings against current account shocks. In the closed economy, households with a precautionary savings motive, facing uncertainty of economy, can use assets to smooth the consumption. In the open economy, if the international asset market is accessible, households could also hold foreign assets against shocks. Also, Jeanne and Sandri [2016] provided a central bank policy rule which could capture the optimal reserves under precautionary savings motive. Except for the first reason of holding international reserves, other papers illustrate rest reasons. Korinek and Serven [2016] analyzed how reserve accumulation can serve to undervalue a country’s real exchange rate and increase economic growth - reason (3) mercantilism. They argued the accumulation of international reserves as a second best policy in economies with learning-by-investing externalities. In the other direction, Benigno and Fornaro [2012] also found that the welfare gains when reserve policy is large by introducing credit shocks in the model. During periods of financial stress, the government uses foreign exchange reserves to provide liquidity to the tradable sector, which involves financial transactions with foreign investors. Espinoza and Winant [2014] still explored reason (4) with considering reason (6). In periods of sudden stops, governments optimal investment levels, as well as the financing and central bank reserves decisions, are affected by the risk. His model solves a portfolio decision involving external debt, central bank reserves, and physical capital; and illustrates different functions of these assets.

2.3 Contributions to Literature

In general, the main objective of Dhasmana and Drummond [2008], Barnichon [2008], Valencia [2010], and Jeanne and Sandri [2016] is to explain the holding reserves under precautionary saving motive against current account shocks, similar to capital account shocks in the researches of Durdu et al. [2009], Alfaro and Kanczuk [2009], and Jeanne and Ranciere [2011]. However, the main idea of Benigno and Fornaro [2012] is that international reserves provide confidence for foreign investors, prevent a sudden flight to quality and loss of capital for the country. The shortcoming of these papers, which use the small open economy model, is that they quantify the optimal reserves only under each reason individually. In other words, they do not analyze the interactional effects of each reason on international reserves. The contribution of this paper will combine the first and the second reasons for holding international reserves to explore the amount under different exchange rate regimes. What is the level of reserves after considering a precautionary saving motive and stable exchange rate motive simultaneously? Are the results consistent with the cost-benefit method under low-income countries, as Kim et al. [2011]? Also, the foreign assets as reserves involve two purposes - precaution and investment in the standard models. This paper separates two purposes: liquidity (money) related reserve account for precautionary motive and foreign
bonds for investment motive.

Last but not least, the model’s structure in this paper is motivated by Einarsson et al. [2002]. He develops a simple two good small open economy model with domestic resource shock, which is the main driving force of the economy’s business cycle. Households rent capital and provide labor to firms, and have access to an international bond market. Additionally, the central bank has different monetary policies under two alternative exchange rate regimes (floating exchange rate regime and monetary union). With a floating exchange rate, the central bank can implement an independent monetary policy. Under this regime, the exchange rate plays a vital role in absorbing macroeconomic shocks. However, under monetary union, the central bank could not take an independent monetary policy, since the money stock passively adjusts to any changes in the balance of payments. Under this regime, the domestic price plays a vital role in absorbing macroeconomic shocks. Klein and Shambaugh [2015] showed that a central bank could only achieve simultaneously two out of capital mobility, monetary autonomy, and pegged exchange rate objectives. If there is no constraint on capital mobility, the central bank could choose monetary autonomy or pegged exchange rate. That is the reason why the model of Einarsson et al. [2002] could choose an independent monetary policy under the floating exchange rate regime.

This paper combines Einarsson et al. [2002] - model’s structure, Kim et al. [2011] - identified current account shocks, Valencia [2010] - precautionary savings, and Jeanne and Sandri [2016] - central bank policy rule. The main objective of this paper is to compare the reserves with a precautionary saving motive under the floating exchange rate regime with those under the pegged exchange rate regime for low-income countries. First, Einarsson et al. [2002] provided a suitable model’s structure, including household, firm, central bank with two goods small open economy. The most important part of Einarsson et al. [2002] is that the central bank could implement different monetary policies under two alternative exchange rate regimes, so I can compare results between floating and pegged exchange rate regimes using this model’s structure. Second, this paper will focus on low-income countries, which are mostly financially closed economies. Therefore, the current account shocks that were identified by Kim et al. [2011] mainly impact on these countries. Third, Valencia [2010] illustrated the mechanism of the precautionary saving - households would prefer to save more international reserves to smooth consumption facing the uncertainty. Furthermore, Jeanne and Sandri [2016] found a linear policy rule could capture the optimal reserves under the precautionary savings motive of households. Now, this paper can obtain a central bank policy rule, which is in line with households with regards to smoothing demand fluctuations. This paper uses it for independent monetary policy under floating exchange rate regime, by which the model could get the baseline result for precautionary saving motive only as Dhasmana and Drummond [2008], Barnichon [2008], Valencia [2010], and Jeanne and Sandri [2016]. Whereas the central bank renunciates the monetary autonomy under a pegged exchange rate regime. Then, the model would get the level of reserves by considering a stable exchange rate. Finally, we can compare the reserves under different exchange rate regimes.

Facing current account shocks (including aid, terms-of-trade, external demand, foreign monetary policy, and productivity shocks), the infinitely lived representative household (with
labor and capital income, and international aid) consumes imported and domestic goods by using cash (cash-in-advance). In addition to those uncertain income, the household directly holds money (reserves) for a precautionary saving motive under the central bank system. Because of the limited access to international financial markets for low-income countries, the current account shocks are the driving force of the economy’s business cycle. Floating exchange rate and monetary autonomy: the central bank could pursue the independent rule which follows, as Jeanne and Sandri [2016] suggested, reserves on behalf of households under the floating exchange rate. Therefore, households could smooth the consumption in response to these shocks by “holding money (reserves)” The appropriate amount of international reserves is the household’s optimal level of precautionary savings (money) in response to these shocks in the model. However, the pegged exchange rate and renunciation of monetary autonomy: the central bank should use the international reserves to stabilize the exchange rate (change money supply) and keep the balance of payments equal to zero under the pegged exchange rate. Then, this paper can analyze the behavior of households to quantify the level of money (reserves) under different exchange rate regimes.

3 The Theoretical Model

3.1 Sectors in Model

The model is a neoclassical dynamic general equilibrium model of a small economy that includes a representative household, a single aggregative firm, and a central bank. The economy receives international aid every period from the rest of the world. Additionally, the low-income countries treat aid, terms of trade, external demand, and foreign monetary policy as exogenous variables.

Firstly, the household consumes imported goods and domestic goods, also can buy foreign bonds and hold money for a precautionary savings motive, and does domestic capital investment. The amount of international reserves - assets of the central bank should be equal to money - the liabilities of the central bank. Therefore, the household’s income comes from working, investment, international aid, and money holdings.

Furthermore, as Arellano et al. [2009] said, the firm in the tradable sector is labor-intensive industry, such as agriculture, mining, and manufacturing. However, the firm in the non-tradable sector is capital-intensive industry, such as water, electricity, and telecommunications. Also, Brock and Turnovsky [1994] and Goldstein and Lardy [2005] had a similar argument. Therefore, low-income countries typically specialize in low-skill labor-intensive industries and export these goods; and most non-tradable firms focus on highly capital-intensive infrastructure projects. To keep matters as simple as possible, the model assumes that a single aggregative firm would produce goods for domestic and foreign households.

Last, the central bank provides money to the household. Also, a central bank balance sheet is always balanced. The central bank can choose open economy policy between pegged
exchange rate and monetary autonomy. Under the regime of a floating exchange rate, the central bank has the option of conducting an independent policy rule. The central bank is benevolent and manages money to help households fulfill the precautionary savings. However, under the regime of a pegged exchange rate, the stock of nominal balances evolves according to the balance of payments. For instance, if the current account is a deficit, the central bank would use international reserves corresponds to stabilize the foreign exchange rate.

3.2 Household Sector

Household maximizes expected lifetime utility, has preferences over a composite bundle of imported and domestic goods and includes disutility of labor, where \( \gamma \) is the coefficient of relative risk aversion, \( \chi \) specifies the preference weight of hours in utility, and the Frisch elasticity for labor supply is \( \frac{1}{\psi} \).

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\gamma}}{1-\gamma} - \chi \frac{l_t^{1+\psi}}{1+\psi} \right]
\]

The consumption - composite goods follow the constant elasticity of substitution (CES) form, where \( \theta \) is the elasticity of substitution of consumption between imported and domestic goods, and \( \omega \) is the weight household places on imported consumption.

\[
c_t = \left[ \omega \frac{1}{\theta} (c_t^M)^{\frac{\theta-1}{\theta}} + (1-\omega) \frac{1}{\theta} (c_t^N)^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta}}
\]

The consumption price index \( P_t \) is CES form also, where \( P_t^M \) is the imported goods price, \( P_t^N \) is the domestic goods price, \( p_t \equiv \frac{P_t}{P_t^*} \) and \( p_t^M \equiv \frac{p_t^M}{P_t^*} \).

\[
P_t = \left[ \omega (P_t^M)^{1-\theta} + (1-\omega)(P_t^N)^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

\[
p_t = \left[ \omega (p_t^M)^{1-\theta} + (1-\omega) \right]^{\frac{1}{1-\theta}}
\]

The real exchange rate is defined as the nominal exchange rate multiplied by the ratio of the foreign CPI to the domestic CPI, where \( S_t \) denotes the nominal exchange rate (the price of one unit of foreign currency in terms of domestic currency), \( P_t^* \) denotes the nominal price of consumption in the foreign country in units of foreign currency, and \( P_t \) denotes the nominal price of consumption in the domestic country in the units of domestic currency.

\[
e_t \equiv \frac{S_t P_t^*}{P_t}
\]

Household has access to an international bond market, holds money for consumption and capital investment, receives labor income, and international aid, where aid \( D_t^* \) is exogenous.
stochastic processes. The household’s budget constraint:

\[ P_t c_t + S_t B_t^* + I_t + M_t = P_t^N (w_t l_t + r_t^K k_{t-1}) + S_t i_t^* B_{t-1}^* + S_t D_t^* + M_{t-1} \]

Household cash-in-advance constraint\(^9\):

\[ P_t c_t + I_t \leq M_{t-1} \]

If \( i_t^* \geq 1 \), money is always dominated by foreign bond assets, and the cash-in-advance constraint would always hold with equality. The precautionary demand for liquidity leads to increased holding of liquid assets in response to income risk.

The law of motion for domestic capital:

\[ k_t = (1 - \delta) k_{t-1} + \frac{I_t}{P_t^N} \]

In the real term:

\[ p_t c_t + p_t e_t b_t^* + k_t - (1 - \delta) k_{t-1} + p_t m_t \]

\[ = w_t l_t + r_t^K k_{t-1} + p_t e_t b_{t-1}^* i_{t-1}^* \frac{i_t^*}{\pi_t} + p_t e_t d_t^* + \frac{p_t m_{t-1}}{\pi_t} \]

\[ = p_t c_t + k_t - (1 - \delta) k_{t-1} = p_t m_{t-1} \frac{\pi_t}{\pi_{t-1}} \]

where \( b_t^* \equiv \frac{B_t^*}{P_t^*} \), \( m_t \equiv \frac{M_t}{P_t} \), \( \pi_t^* = \frac{P_t^*}{P_{t-1}^*} \), \( d_t^* \equiv \frac{D_t^*}{P_t^*} \), and \( \pi_t = \frac{P_t}{P_{t-1}} \).

### 3.3 Firm Sector

The aggregative firm produces output with a Cobb-Douglas and constant returns to scale form, where \( A_t \) is an exogenous productivity shock. Also, the firm is competitive, choose labor and capital to maximize profit.

To keep matters as simple as possible, the model assumes that a single aggregative firm would produce goods for domestic and foreign households. Firm’s production function:

\[ y_t = A_t (l_t^d)^\alpha (k_t^d)^{1-\alpha} \]

\(^9\)In particular the CIA constraint may not be bound under the timing assumption of Svensson [1985]. Similar to the household’s borrowing constraint, the borrowing constraint is not a necessary condition to generate a precautionary motive. Carroll [2004] and Sandri [2014] argued that, only given a big negative shock, the borrowing constraint would be bound. The model assumes that the CIA constraint is bound.
The profit of the firm in a perfectly competitive industry:

\[ \Pi_t = y_t - w_t l_t^d - r_t^K k_t^d \]

The productivity shock of the firm:

\[ \ln A_t - \ln \bar{A} = \rho_A (\ln A_{t-1} - \ln \bar{A}) + \epsilon_t^A \]

### 3.4 Central Bank

In international macroeconomics: the central bank can only simultaneously achieve two out of the three objectives, which include capital mobility, monetary autonomy, and pegged exchange rate. Hence, the central bank can choose “floating exchange rate and monetary autonomy” or “pegged exchange rate and renunciation of monetary autonomy”.

Under the floating exchange rate regime, the central bank could conduct an independent policy rule. Einarsson et al. [2002] presented the monetary authority has the option of conducting an independent policy rule, which may or may not be state dependent. The policy rule could be influenced by asset holdings, resources, productivity, terms of trade \( \frac{P_X}{P_M} \), and external demand \( x_t^* \), such as \( M_t^s = F(A_t, d_t^*, \frac{P_X}{P_M}, x_t^*, M_{t-1}^s) \). The model assumes a simple ad hoc state dependent policy rule, as Jeanne and Sandri [2016], in which money growth partly accommodates output growth. This independent policy rule implies more price stability than a constant money growth. In this model, the liabilities of the central bank are money and assets are reserves. Therefore, the net change in money shows the net change in reserves in domestic currency \( V_t = M_t^s - M_{t-1}^s \).

\[
M_t^s = \bar{\mu} \left( \frac{y_t}{y_{t-1}} \right)^\zeta M_{t-1}^s \\
M_t^s = M_{t-1}^s + V_t
\]

where \( \bar{\mu} \) is the mean gross growth rate, and \( \zeta \) measures the degree of money growth in response to output growth.

In the real term:

\[
m_t^s = \bar{\mu} \left( \frac{y_t}{y_{t-1}} \right)^\zeta \frac{M_{t-1}^s}{\pi_t} \\
m_t^s = \frac{m_{t-1}^s}{\pi_t} + v_t
\]

Under the pegged exchange rate regime, the stock of money evolves according to the balance of payments, where \( V_t \) is the balance of payments residual (changed reserves). The balance of payments should be equal to the sum of the current account and capital account.
The current account involves net export, net income from abroad, and net current transfers. The capital account shows the net change in foreign bond investments and reserves.

\[ V_t = P_t^X x_t^* - P_t^M c_t^M + S_t D_t^* + S_t i_{t-1}^* B_{t-1}^* - S_t B_t^* \]
\[ M_t^s = M_{t-1}^s + V_t \]

The pegged exchange rate regime needs

\[ S_t = S_{t-1} \]

In the real term:

\[ p_t v_t = p_t^X x_t^* - p_t^M c_t^M + p_t e_t d_t^* + p_t e_t \frac{i_{t-1}^*}{\pi_t^*} b_{t-1}^* - p_t e_t b_t^* \]
\[ m_t^s = \frac{m_{t-1}^s}{\pi_t^*} + v_t \]
\[ \frac{\pi_t^*}{\pi_t} = \frac{e_t}{e_{t-1}} \]

where \( v_t \equiv \frac{V_t}{P_t} \).

### 3.5 Equilibrium

A competitive equilibrium of the model includes the following conditions.

Labor holds the factor market clearing.

\[ l_t = l_t^d \]

Capital also holds the factor market clearing.

\[ k_{t-1} = k_t^d \]

Output market equilibrium:

\[ P_t^N y_t = P_t^X x_t^* + P_t^N c_t^N + I_t \]

In the real term:

\[ y_t = p_t^X x_t^* + c_t^N + k_t - (1 - \delta)k_{t-1} \]
Import price and terms of trade:

\[ P_t^M = S_t P_t^* \]
\[ tot_t = \frac{P_{X,t}}{P_t^M} \]

In the real term:

\[ p_t^M = p_t e_t \]
\[ tot_t = \frac{p_{X,t}^t}{p_t^M} = \frac{p_{X,t}^t}{p_t^M} \]

Money market equilibrium:

\[ M_t^* = M_t \]
\[ M_t = M_{t-1} + V_t \]
\[ M_t = \bar{\mu} \left( \frac{y_{t-1}}{y_t} \right)^\zeta M_{t-1} \quad \text{ (floating)} \]
\[ S_t = S_{t-1} \quad \text{ (pegged)} \]

Balance of payments equilibrium:

\[ V_t = M_t - M_{t-1} = P_{t,X} x_t^* - P_{t,M} c_{t,M} + S_t D_t^* + S_t i_{t-1}^* B_{t-1}^* - S_t B_t^* \]

### 3.6 Model in Full

For the small open economy, the low-income countries face external shocks (stationary and Markov) each period, including \( d_t^* \) measures international aid shocks; \( tot_t \) measures terms of trade shocks; \( x_t^* \) measures external demand shocks; and \( i_t^* \) measures foreign monetary policy shocks. The formulas follow the same forms as productivity shocks.

After setting up the problem for low-income countries, the competitive equilibrium of this model should satisfy the following conditions, such as (i) household maximizes utility subject to the budget constraint and cash-in-advance constraint given prices, (ii) aggregative firm maximizes profit given prices, (iii) the central bank budget constraint is satisfied, and (iv) markets are clear.

The model in full: By normalizing the foreign price level \( P_t^* \) to unity, the model does not distinguish between real and nominal terms in the foreign sector. Endogenous state variables include \( k_t, b_t^*, m_t \). Endogenous control variables include \( c_t, c_t^N, c_t^M, l_t, r_t^K, w_t, p_t, p_{t,X}^t, \pi_t, e_t \). Exogenous variables include \( A_t, d_t^*, tot_t, x_t^*, i_t^* \).

14
\[ p_t \frac{\chi^\psi_t}{w_t} = \beta E_t \left( \frac{1}{\pi_t} c^\gamma_{t+1} \right) \]  

(1)

\[ p_t \frac{\chi^\psi_t}{w_t} e_t = \beta E_t (p_{t+1} \frac{\chi^\psi_{t+1}}{w_{t+1}} e_{t+1}^*) \]  

(2)

\[ \frac{c^\gamma_t}{p_t} = \beta E_t [r^K \frac{\chi^\psi_{t+1}}{w_{t+1}} + (1 - \delta) \frac{c^\gamma_{t+1}}{p_{t+1}}] \]  

(3)

\[ (p_t e_t)^\theta = \frac{\omega}{1 - \omega} \frac{c^N_t}{c^M_t} \]  

(4)

\[ p_t c_t + k_t - (1 - \delta) k_{t-1} = p_t \frac{m_{t-1}}{\pi_t} \]  

(5)

\[ c_t = [\omega^\frac{1}{\pi} (c^M_t)^{\frac{1}{\pi} - 1} + (1 - \omega) \frac{1}{\pi} (c^N_t)^{\frac{1}{\pi} - 1}] \frac{1}{\pi - 1} \]  

(6)

\[ p_t = [\omega (e_t p_t)^{1 - \theta} + (1 - \omega)] \frac{1}{1 - \theta} \]  

(7)

\[ \alpha A_t (l_t)^\alpha (k_{t-1})^{1 - \alpha} = w_t l_t \]  

(8)

\[ (1 - \alpha) A_t (l_t)^\alpha (k_{t-1})^{1 - \alpha} = r^K_t k_{t-1} \]  

(9)

\[ A_t (l_t)^\alpha (k_{t-1})^{1 - \alpha} = p_t^X x_t^* + c_t^N + k_t - (1 - \delta) k_{t-1} \]  

(10)

\[ p_t^X = p_t e_t \pi_t \]  

(11)

\[ p_t (m_t - \frac{m_{t-1}}{\pi_t}) = p_t^X x_t^* - p_t e_t c_t^M + p_t e_t d_t^* + p_t e_t i_t^* b_t^* - p_t e_t b_t^* \]  

(12)

\[ m_t = \bar{\mu} \left( \frac{y_t}{\pi_{t-1}} \right) \frac{m_{t-1}}{\pi_t} \]  

(floating)

\[ \frac{1}{\pi_t} = \frac{e_t}{e_{t-1}} \]  

(pegged)

15
\ln A_t - \ln \bar{A} = \rho_A (\ln A_{t-1} - \ln \bar{A}) + \epsilon^A_t \tag{14}
\ln d^*_t - \ln \bar{d}^* = \rho_d (\ln d^*_{t-1} - \ln \bar{d}^*) + \epsilon^d_t \tag{15}
\ln \text{tot}_t - \ln \bar{\text{tot}}_t = \rho_T (\ln \text{tot}_{t-1} - \ln \bar{\text{tot}}_t) + \epsilon^T_t \tag{16}
\ln x^*_t - \ln \bar{x}^* = \rho_x (\ln x^*_{t-1} - \ln \bar{x}^*) + \epsilon^x_t \tag{17}
\ln i^*_t - \ln \bar{i}^* = \rho_i (\ln i^*_{t-1} - \ln \bar{i}^*) + \epsilon^i_t \tag{18}

The sequence of decisions for low-income countries in this model is as follows. At time $t$, the household has the capability for consumption and capital investment only the cash carried over from the previous period $t-1$, so cash balances must be chosen before the household knows how much spending they will wish to undertake. Then, the household realizes exogenous shocks: the productivity, aid, terms of trade, external demand, and foreign monetary policy shocks. Domestic capital, foreign bonds, and money are predetermined. The household chooses consumption (given level of composite goods, minimize the cost of imported goods and domestic goods) and capital investment first subject to the cash-in-advance constraint, then makes foreign bonds decision. The firm chooses capital and labor to produce output. Central bank supplies money and also holds reserves ($M_t - M_{t-1} = V_t$). Therefore, money (as reserves) could measure lower bound precautionary saving motive from the household side. Prices and exchange rates are determined at this time.

In short, facing the uncertainty, the household under this model tries to balance the interrelationships of investment, money, and foreign bonds. Because the uncertainty is realized after money balances are chosen, a household may find that the holding money is too low to finance the desired consumption and capital investment level. Alternatively, the household maybe hold more money than needs, thereby giving up the interest income. For explaining these interrelationships, Valencia [2010] introduces two terms “prudence” (the agent saves international reserve in order to minimize the impact of future shocks on consumption) and “impatience” (the agent prefers to spend today rather than tomorrow). The optimal level of international reserves should satisfy the condition - these interrelationships are exactly balanced.

### 3.7 Precautionary Savings

Crucially, the agent’s asset holding - reserves and foreign bonds ($a_t \equiv m_t + e_t b^*_t$) can smooth the consumption against the uncertainty. Comparing with the standard precautionary saving problem, we can simplify the above model to understand this motive clearly. However, the standard model involves two purposes - precaution and investment for the saving assets.

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \]
subject to\textsuperscript{10} 
\begin{align*}
p_t c_t + k_t - (1 - \delta)k_{t-1} + p_t a_t \\
= w_t l_t + r^K_t k_{t-1} + p_t e_t d_t^* + p_t \frac{i_{t-1}}{\pi_t} a_{t-1} - p_t \frac{(i_{t-1} - 1)}{\pi_t} m_{t-1}
\end{align*}

rearrange budget constraint
\begin{align*}
a_t = \left( \frac{i_{t-1}}{\pi_t} a_{t-1} - c_t - \frac{k_t}{p_t} \right) + \frac{y_t}{p_t} + (1 - \delta) \frac{k_{t-1}}{p_t} + e_t d_t^* - \frac{(i_{t-1} - 1)}{\pi_t} m_{t-1}
\end{align*}

The rearranged budget constraint is similar to the extended precautionary savings model of Valencia [2010] $x_{t+1} = R(x_t - c_t - k_t) + \tau_{t+1} k_t + a$, where $x_t$ is net foreign assets, $R$ is a risk-free security yield, $c_t$ is consumption, $k_t$ is the capital, $\tau_{t+1} k_t$ is income from exports, and $a$ is all other non-export net current receipts. However, the last term of rearranged budget constraint is a cost to holding money when the net nominal interest rate $(i_{t-1} - 1)$ is positive. Due to the carry cost of money, the overall asset holding could focus on precautionary purpose.

For the relation between the asset holdings and carry costs, the reduced form model adds the carry cost of reserves, as shown by Jeanne and Sandri [2016] 
\begin{align*}
\xi = \frac{G^\gamma}{\beta} - (1 + r^*)
\end{align*}
, where $G$ is the growth rate of the economy. This is another way to decompose these two purposes. Therefore, $\frac{G^\gamma}{\beta}$ would be the interest rate under financial autarky. $1 + r^*$ is the rate of return of holding reserves. Then, the difference between the two terms is the carry cost. If the carry cost $\xi$ is positive, holding reserves should face an opportunity cost that ensures a finite level of reserves. The higher carry cost $\xi$ would cause the reserves (money) to decrease.

In the low-income small open economy, the social planner is isolated from almost all asset markets and can only hold some amounts of international reserves (risk-free asset) to acquire “self-insurance” against the uncertain income. For instance, the social planner can draw international reserves, in response to low income realizations, and avoid large temporary drops in consumption. In contrast, the social planner can buy international reserves, when the income is high, in anticipation of poor outcomes in the future. Also, the social planner now finds it profitable to cut consumption and finance the stock of capital. In shorts, total savings could fulfill the precautionary motive and finance the capital stock. Plugging UIP into the constraint gets
\begin{align*}
a_t = \left( \frac{e_t}{e_{t-1}} \frac{i^*_t}{\pi_t} a_{t-1} - c_t - \frac{k_t}{p_t} \right) + \frac{y_t}{p_t} + (1 - \delta) \frac{k_{t-1}}{p_t} + e_t d_t^* - \frac{(e_t}{e_{t-1}} \frac{i^*_t}{\pi_t} - \frac{1}{\pi_t}) m_{t-1}
\end{align*}

The precautionary savings could be influenced by the real exchange rate and disposable income fluctuations under external shocks.

\textsuperscript{10}UIP is $i_{t-1} = \frac{s_{t-1}}{s^*_t} i^*_t$, in the real term $\frac{i_{t-1}}{\pi_t} = \frac{e_t}{e_{t-1}} \frac{i^*_t}{\pi_t}$. 

17
Due to the precautionary saving motive, a social planner can choose immediately to use his saving against the negative wealth effect to smooth consumption over time in low-income countries. After understanding the carry cost and precautionary motive, back to our original model, the central bank could help consumers to hold reserves indirectly and to get a similar allocation in the equilibrium to occupy most welfare level as a social planner’s problem. Therefore, the original model under a floating exchange rate regime assumes the independent policy rule should care about the precautionary motive, such as money supply depends on output shocks while buffering income fluctuations. However, the original model under a pegged exchange rate regime assumes the central bank’s objective is pegging the exchange rate. Then, we can explore external shocks under different exchange rate regimes with the precautionary saving motive.

4 Results

4.1 Data

This paper collects the panel data of 70 low-income countries\textsuperscript{11} between 1966 and 2015 from the World Bank’s World Development Indicators (WDI) database, Shambaugh exchange rate classification data set\textsuperscript{12}, and Emergency Events Database (EM-DAT) published by the Center for Research on the Epidemiology of Disasters (CRED). Firstly, Figure 2 displays the reserves-to-imports ratio in months over time for different countries. There is an upward trend around the 3-months-of-imports rule of thumb. Figure 3 shows the distribution of reserves-to-imports ratio in months, which summarizes mean, median, and skewness to know the reserves of these low-income countries.

Then, the parameters in this model should be calibrated to replicate the key characteristics. Some parameters are taken from other papers and researches. For the relative risk aversion coefficient $\gamma$ is set to 2 (Jeanne and Sandri [2016]); the elasticity of substitution between imported and domestic consumption $\theta$ is equal to 0.76 (Ostry and Reinhart [1992]); and the depreciation rate $\delta$ is 0.05, which is taken from Arellano et al. [2009]. Due to the standard values used in the RBC model, the preference weight of labor in utility $\chi$ is 1; Frisch elasticity for labor supply $\frac{1}{\psi}$ is 1/0.6; the world interest rate $\bar{r}^*$ is 1.02; the discount factor $\beta$ is 1/1.02. Others base on the panel data: the labor share of production $\alpha$ is 0.79.11 The countries include: Afghanistan; Bangladesh; Benin; Bhutan; Burkina Faso; Burundi; Cambodia; Cameroon; Cabo Verde; Central African Republic; Chad; Comoros; Congo, Dem. Rep.; Congo, Rep.; Cote d’Ivoire; Djibouti; Dominica; Eritrea; Ethiopia; Gambia, The; Ghana; Grenada; Guinea; Guinea-Bissau; Guyana; Haiti; Honduras; Kenya; Kiribati; Kyrgyz Republic; Lao PDR; Lesotho; Liberia; Madagascar; Malawi; Maldives; Mali; Marshall Islands; Mauritania; Micronesia, Fed. Sts.; Moldova; Mozambique; Myanmar; Nepal; Nicaragua; Niger; Papua New Guinea; Rwanda; Samoa; Sao Tome and Principe; Senegal; Sierra Leone; Solomon Islands; Somalia; South Sudan; St. Lucia; St. Vincent and the Grenadines; Sudan; Tajikistan; Tanzania; Timor-Leste; Togo; Tonga; Tuvalu; Uganda; Uzbekistan; Vanuatu; Yemen, Rep.; Zambia; Zimbabwe. See Figure 1.

12Shambaugh [2004], Klein and Shambaugh [2008], Obstfeld et al. [2010], and Klein and Shambaugh [2012] provided the classification of an exchange rate regime. Here, the paper would use this measure of exchange rate regime based on the classification in Shambaugh [2004].
by the production regression; the mean gross growth rate of money $\bar{\mu}$ is 1; buffering output shocks $\zeta$ is 0.26; weight of imported consumption in the gross consumption $\omega$ is 0.43 and mean levels of some external shocks are in the following table, which uses the similar methods as Arellano et al. [2009] and Jeanne and Sandri [2016]. The stochastic process for all the external shocks can get $\rho$ and $\sigma$ using detrended panel data. Table 1 presents all the parameters used in the calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>1/1.02</td>
</tr>
<tr>
<td>Relative risk aversion coefficient</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
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</tr>
<tr>
<td>Labor share of production</td>
<td>$\alpha$</td>
<td>0.79</td>
</tr>
<tr>
<td>Mean gross growth rate of money</td>
<td>$\bar{\mu}$</td>
<td>1</td>
</tr>
<tr>
<td>Buffering output shocks</td>
<td>$\zeta$</td>
<td>0.26</td>
</tr>
<tr>
<td>Preference weight of labor in utility</td>
<td>$\chi$</td>
<td>1</td>
</tr>
<tr>
<td>Frisch elasticity for labor supply</td>
<td>$1/\psi$</td>
<td>1/0.6</td>
</tr>
<tr>
<td>Elasticity of substitution between imported and domestic consumption</td>
<td>$\theta$</td>
<td>0.76</td>
</tr>
<tr>
<td>Weight of imported consumption in the gross consumption</td>
<td>$\omega$</td>
<td>0.43</td>
</tr>
<tr>
<td>Productivity level</td>
<td>$\bar{A}$</td>
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</tr>
<tr>
<td>Aid level</td>
<td>$\bar{d}^*$</td>
<td>0.202</td>
</tr>
<tr>
<td>Terms of trade level</td>
<td>$\bar{t}^*$</td>
<td>1</td>
</tr>
<tr>
<td>World interest rate</td>
<td>$\bar{i}^*$</td>
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</tr>
<tr>
<td>Autocorrelation coefficient of productivity shocks</td>
<td>$\rho_A$</td>
<td>0.43</td>
</tr>
<tr>
<td>Autocorrelation coefficient of aid shocks</td>
<td>$\rho_d$</td>
<td>0.43</td>
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<tr>
<td>Autocorrelation coefficient of terms of trade shocks</td>
<td>$\rho_T$</td>
<td>0.47</td>
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<tr>
<td>Autocorrelation coefficient of external demand shocks</td>
<td>$\rho_x$</td>
<td>0.47</td>
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<tr>
<td>Autocorrelation coefficient of foreign monetary policy shocks</td>
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<tr>
<td>Standard deviation of productivity shocks</td>
<td>$\sigma_A$</td>
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<tr>
<td>Standard deviation of aid shocks</td>
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<td>Correlation between terms of trade and external demand</td>
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</tr>
<tr>
<td>Correlation between terms of trade and foreign monetary policy</td>
<td>$\text{corr}(\bar{e}_T^<em>, \bar{e}_i^</em>)$</td>
<td>0.0563</td>
</tr>
<tr>
<td>Correlation between external demand and foreign monetary policy</td>
<td>$\text{corr}(\bar{e}_x^<em>, \bar{e}_i^</em>)$</td>
<td>-0.0044</td>
</tr>
</tbody>
</table>

Table 1: Calibration for the benchmark model

### 4.2 Impulse Response Function

Figure 4 to Figure 7 shows the impulse response functions to the external shocks in productivity, aid, terms of trade, external demand, and foreign monetary policy under the floating

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13 This paper uses Hamilton [2018] method to detrend data.
exchange rate regime. The largest shock is terms of trade, and the smallest is foreign monetary policy (federal funds rate - FFR). When negative shock impacts on productivity with one standard deviation, it will decrease output and increase inflation. The substitute effect would cause households to consume more import and reduce investment with the CIA constraint. It decreases a tiny amount of precautionary savings, which spend a long time going to the new steady state. Espinoza and Winant [2014] found that natural disasters (one component of productivity shocks) do not increase the probability of drawing down reserves. The effects of aid shocks are minimal for output and inflation, compared with others. The findings between consumption and investment are consistent with Arellano et al. [2009] who said a permanent flow of aid mainly finances consumption rather than investment, as the historical failure of aid inflows to translate into sustained growth. However, the effect causes more labor supply and less precautionary savings. Importantly, negative export shocks, including terms of trade and external demand, will decrease output and inflation. The decrease in terms of trade means export price relative to import price is cheaper, so domestic consumption rises and import falls. The negative shocks on external demand would cause the current account deficit, so the same results are as decrease in terms of trade. The real exchange rate depreciates, and the nominal exchange rate has a lagged effect. Under both shocks, there is a shift between the precautionary motive and investment motive. Facing uncertainty, the households save more money (reserves) for consumption smoothing and reduce foreign bond holdings. Then, the overall precautionary savings fall because of an increase in carry costs after this shift. Last but not least, the negative impact on foreign monetary policy would cause foreign output and inflation to increase. Therefore, the external demand and import price increases for the trade channel. What’s more, the real and nominal exchange rates appreciate due to a decrease in the foreign interest rate. For the financial channel, households would decrease foreign bond holdings with less return. The domestic output and domestic consumption are a slight decrease after this shock. However, composite consumption and import increase a lot. Because this is a good scenario for the domestic economy, the precautionary savings are accumulated with the current account surplus.

Figure 8 to Figure 11 illustrates the effects of external shocks under the pegged exchange rate regime. The patterns are very similar to the effects under the floating exchange rate regime, except ones under the foreign monetary policy shock. Although the patterns are similar, the initial deviations are more significant with the pegged exchange rate regime. Figure 10 presents the impulse response functions of precautionary savings to the external shocks under the pegged exchange rate regime. The agents will use assets to stabilize the exchange rate and smooth consumption against the external shocks. So, the initial deviation of precautionary savings with foreign monetary policy shock is negative, instead of positive under the floating exchange rate. Under the floating exchange rate regime, agents cut the precautionary savings to smooth domestic absorption. Compared with the floating exchange rate, the external shocks will consume much more reserves initially and similarly cause a permanent effect to lower the international reserves (except negative foreign monetary policy shock) under the pegged exchange rate. Figure 9 hints that households have more volatile composite consumption in order to satisfy the pegged exchange rate purpose.

14 Agents would accumulate the precautionary savings, facing negative foreign monetary policy shock.
In other words, if the country wants to fulfill both pegged exchange rate and smoothing consumption purpose, the central bank will hold more reserves and choose softpegs\textsuperscript{15}.

After analyzing the smoothing consumption and precautionary savings, the model also shows variables with adverse monetary policy shocks would be influenced differently under different exchange rate regimes. The Euler equation implies uncovered interest parity, so there is a strong relationship between the exchange rate and interest rate. Furthermore, higher interest rates increase the value of a country’s currency. The different exchange rate regimes would cause different responses of variables with negative monetary policy shocks. Unlike the floating exchange rate regime, the increase in import price with pegged nominal exchange rate would cause an increase in low-income country CPI inflation. Additionally, the pegged nominal exchange rate triggers less volatile deviations in import and real exchange rate at the beginning. However, labor and output decrease a lot under the pegged exchange rate regime. Although the external demand increases, the decrease in terms of trade has a large negative impact on the domestic economy.

From Table 2, the real output\textsuperscript{16} and real domestic consumption more volatile, but import for consumption is less volatile under pegged exchange rate regime. The pegged exchange rate could reduce the variance of import price in domestic currency, so it not only narrows volatility of import but also decreases the deviation of composite price and CPI inflation. As a result, the real exchange rate has a small variance under the pegged exchange rate regime. The real money (reserves) is less volatile with the floating exchange rate, whereas the real foreign bonds are more volatile. Under the floating exchange rate regime, the nominal exchange rate, including real exchange rate and CPI, can absorb the macroeconomic shocks. However, the inverse inflation is equal to the changed real exchange rate under the pegged exchange rate regime. The agents should use more precautionary savings to maintain the pegged exchange rate and smooth consumption. Also, a significant shift under terms of trade, external demand, and foreign monetary policy shocks from investment motive to precautionary motive for both regimes, when agents face uncertainty. Therefore, the welfare gains from precautionary savings come from using the reserves rather than keeping them close to the target.

5 Conclusion

This work to date uses the theoretical model and simulation to illustrate how precautionary savings change in response to the current account shocks. For low-income countries, the

\textsuperscript{15}Obstfeld et al. [2010] described 4 possibilities of softpeg: (1) maintains exchange rate within 5% up or down bands and has a maximum monthly change of less than 1%, but is not a peg; (2) maintains exchange rate within 5% bands against the base currency but outside of 2% bands and has some month where the change is greater than 1%; (3) has no month in which the exchange rate changes by more than 2% up or down, but violates the 5% band rule; (4) 0% change in 11 out of 12 months.

\textsuperscript{16}The empirical data - real GDP per capita also shows significant variance under the pegged exchange rate regime.
main external shocks are from the current account. The model includes five shocks: productivity, international aid, terms of trade, external demand, and foreign monetary policy. Under the floating exchange rate regime, the results show the assets (money/reserves and foreign bonds) with the precautionary motive to smooth the consumption. However, the assets not only stabilize the exchange rate purpose, but also smooth consumption under the pegged exchange rate regime. Facing uncertainty, households put more weight on precautionary motive, instead of investment motive. The impulse response function from the benchmark model simulation reveals that the initial deviations are more significant with the pegged exchange rate regime. The floating exchange rate regime could reduce the variance of domestic consumption, while the pegged exchange rate regime would be better to stabilize imported consumption. Interestingly, the foreign monetary policy shocks show significant differences between the two regimes. Although the federal funds rate (world interest rate) shocks are the smallest among these five shocks, the effects on some variables are as large as the effects of productivity shocks. However, future research may develop other aspects.

First, this neoclassical dynamic general equilibrium model of a small economy ignores the vital role of sticky prices. The impulse response function shows the dynamic deviations of CPI/PPI gaps with different shocks under two regimes. The price channel is a significant and crucial part of transferring current account shocks of low-income countries. Therefore, the New Keynesian small open economy model is more suitable. Second, this model assumes a simple ad hoc state dependent policy rule, which may not be an excellent monetary policy rule to mimic real world behavior. Jeanne and Sandri [2016] implies that foreign bond holding partly accommodates the prior foreign bond level and exogenous shocks ($\frac{\pi_{PN}}{\pi_{P}}$) is terms of trade,

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regime</th>
<th>Symbol</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>real output</td>
<td>[float]</td>
<td>y</td>
<td>0.008879</td>
</tr>
<tr>
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Table 2: Moments of simulated variables
$x_t$ is the external demand - export shocks). They found reserves have to converge towards a target level and buffer terms of trade and external demand shocks (export shocks). Based on their findings, central bank policy rule is $M_t = M_{t-1} + \mu(M - M_{t-1}) + \tau(P^X_t x_t - P^X x)$. In this policy rule, parameter $\mu$ captures the speed of convergence to the target level, and the parameter $\tau$ is buffering the export shocks. If the central bank ignores the money (reserve) target ($\mu = 0$), the money will be more volatile following the external shocks as the original model. The CIA model could shed some light on the precautionary saving problem with the central bank strategy, but the results would be sensitive to the monetary policy rule equation. However, the New Keynesian model involves the interest rate as a tool under the Taylor rule instead of the money supply, which is a more popular assumption in the theoretical model. Third, this model finds that the economy is a different response to foreign monetary policy shocks under two regimes. The main transmission channels of federal funds rate shocks are the exchange rate, the international trade market, and the international financial market. In this model, the exogenous correlations of terms of trade, external demand, and foreign monetary policy shocks capture the mechanism of the international trade market channel. A negative shock of foreign monetary policy would cause foreign economy output and inflation to increase. Then, the domestic economy is more likely to face an increase in external demand. However, the exchange rate could fall with this negative shock, and import price in foreign currency would rise. So, the total effect on terms of trade depends on the net effect from both domestic and foreign economies. The missing part of this model is the foreign economy (the rest world), which would show the endogenous mechanism of the international trade market channel and reduce model assumptions. Also, this model does not include the government spending or transfer and domestic bonds, which would also influence precautionary savings. Moore and Glean [2016] found that the fiscal policy of government would influence reserves. If countries feel less need to hold precautionary reserves, macro-prudential policies may lead to less international reserves; or if policies help prevent outflows of international reserves, it could lead to more international reserves. Aizenman et al. [2015] found the negative impact of macro-prudential policies for developed countries and the positive one for developing countries.

This model reveals the precautionary savings of low-income countries against current account shocks. Are the findings also consistent with emerging market countries that are financially open economies? Now, we need to consider capital account shocks. Also, some emerging market countries are not small so that the countries would affect the price of tradable goods. China is a good case to quantify how much “mercantilist” and “precautionary” motives have contributed to the enormous reserves. Aizenman and Lee [2007] constructed a minimal model and a new empirical analysis to identify the contributions of precautionary and mercantilist motives to international reserves build-up. They found that trade openness and financial crises are essential in explaining the patterns of hoarding reserves, but the mercantilist motive is not. Also, Schröder [2017] undertook an empirical investigation and found precautionary motives are the dominant reason instead of mercantilist motives under two vastly differing approaches. Another avenue for future research is to test whether the findings from the model are consistent with empirical results. Using the data of low-income or emerging market countries, empirical research can estimate how the external shocks would affect the economy. Moreover, the empirical field could analyze the interaction effect of variables
to solve the endogenous problem. Finally, monetary policy shocks would affect the economy through different channels. The empirical tests could reveal the result of each channel.

Overall, the objective is to develop a better theoretical and empirical understandings of the effects of external shocks. Currently, this paper focuses on the interactional effect between the precautionary motive and investment motive, and between consumption smoothing and exchange rate stabilization. Using a small open economy model for low-income countries, this paper shows that asset holdings (money/reserves and foreign bonds) smooth consumption under different exchange rate regimes and presents how agents shift motivations in response to the current account shocks. In the model, the precautionary savings include money and foreign bonds. The households choose money (reserves) for precautionary purposes and foreign bonds for investment motives. At the same time, the central bank supplies money to fulfill different exchange rate regimes. The next one will plan to extend the model to the New Keynesian model adding missing parts and find detailed empirical results of low-income or emerging market countries to supplement the theoretical model.
References


Tor Einarsson et al. Small Open Economy Model with Domestic Resource Shocks: Monetary Union vs. Floating Exchange Rate. 2002.


Appendices

A Graphs

Figure 1: 70 low-income countries GDP per capita in 2014 (USD)
Figure 2: The reserves-to-imports ratio in months

Figure 3: The distribution of reserves-to-imports ratio in months
Figure 4: Floating regime - IR to external shocks

Figure 5: Floating regime - IR to external shocks
Figure 6: Floating regime - IR to external shocks

Figure 7: Floating regime - IR to external shocks
Figure 8: Pegged regime - IR to external shocks

Figure 9: Pegged regime - IR to external shocks
Figure 10: Pegged regime - IR to external shocks

Figure 11: Pegged regime - IR to external shocks