EART 121 PROBLEM SET #4

Due: Friday, Feb 1.

1. Carbon cycle [7 pts]

Look at the Keeling curve (CO\(_2\) vs time at a site in Hawaii) shown in class. Let’s say that each year, the difference in CO\(_2\) in the atmosphere is about 6 ppm between fall and spring. Assuming the atmosphere of the northern hemisphere is well-mixed, calculate how much biomass carbon per m\(^2\) of the northern hemisphere land mass this corresponds to (answer will be in units of kg of C per m\(^2\) of land area). Assume the northern hemisphere is approximately 40% land and 60% ocean.

2. Aerosol indirect effect [13 pts]

The Earth’s average albedo is currently about 0.30. Of this 0.30, 70% is due to clouds, with the remainder of the sunlight reflected directly by aerosols and the Earth’s surface.

(a) Describe in words what the aerosol indirect effect on climate is.

(b) Currently, what is the albedo of Earth considering only the clouds?

(c) Consider a cloud containing cloud drops with number concentration N (units are number of drops per cm\(^3\) of air). Assume all drops are the same size, radius r. Write expressions for (i) the total surface area A of these drops, and (ii) the total volume V of these drops, both in terms of N and r.

(d) Now we want to consider a situation where N increases due to the addition of anthropogenic particles. We want to determine an expression (don’t substitute values for variables yet!) for dA/dN assuming V is constant (i.e. assuming the total volume of cloud drops remains the same).

(e) Using the expression in (d), estimate the change in surface area, ΔA, given ΔN = 20 cm\(^3\) and if we start with a cloud with N = 100 cm\(^3\) and r = 10 \(\mu\)m.

(f) If cloud albedo is determined by surface area (because light is reflected by these drops in proportion to their surface area), what is the new cloud albedo? What, then, is the new global albedo?
1. CO$_2$ Model [30 pts]

Let’s make a model to predict atmospheric carbon dioxide content starting in
the year 2000 and going to the year 2200. Let’s define the following terms:

- $t =$ time in years, ranging from 0 to 200 years
- $c(t) =$ CO$_2$ amount in atmosphere, in units of GtC, at year $t$
- $c_0 =$ initial carbon dioxide amount in atmosphere = 730 GtC
- $E(t) =$ emission rate of CO$_2$ in any given year, units of GtC/yr
- $S(t) =$ sink rate of CO$_2$ in any given year, units of GtC/yr

(a) Using the above terms, write a differential equation expressing the mass
balance of CO$_2$.

Now we need to set up $E(t)$ and $S(t)$.

$E(t):$ From Slide #69 in the climate change lecture, the emission scenario A2
has $E(t)$ as roughly a straight line. Let’s write this line as $E(t) = D + At$.

$S(t):$ Let’s assume that the more CO$_2$ is in the atmosphere, the faster it will be
removed (a reasonably good assumption). If this removal rate is linear, then
$S(t) = k'c(t)$, where $k$ has units of “per year”. Let’s write this in a more
intuitive form as $S(t) = c/B$, where $B$ has units of years and is a characteristic
removal time scale.

(b) For part (b) only, assume that $E(t) = 0$ (i.e. no human emissions at all) and
derive an equation for $c(t)$. Use this solution to show that $B$ is an “e-folding
time”, i.e. that at time $t = B$, the CO$_2$ concentration has decreased such that
$c = c_0/e$ (where $e$ has the standard meaning of 2.718).

(c) Now integrate the full differential equation for $c(t)$.

(d) Your solution of (c) should contain an unknown constant $c_1$, which we solve
for by using the initial condition $c(0) = c_0$. Derive an equation for $c_1$ in terms of
$A$, $B$, $D$ and $c_0$. Now show that the units of the solution in (c) work out.

(e) We have a predictive model! Let’s assume that $B = 100$ years. Use slide
#69 and Scenario A2 to determine $A$ and $D$. Make sure you have the units
correct. Compute $c_1$.

(f) Now let’s plot! Plot the CO$_2$ concentration for 200 years.

(i) Compare the answer in year 100 with that from Slide #70. How does it agree?
(ii) Using your plot, determine the year this model predicts that CO₂ will double, triple and quadruple the pre-industrial CO₂ concentration (which is 280 ppm [yes, you need to convert this to GtC]). Draw lines on your plot to show these milestones (it can be approximate - a pencil and ruler is fine).

(g) Let’s say that Scenario A2 is too pessimistic. Instead, let’s say our per year emissions grow by 40% less than in A2. What is the new A? Plot the new solution and determine the year this new model predicts that CO₂ will double, triple and quadruple the initial CO₂ concentration (add lines to the plot again). Given that this is a very drastic reduction, what do you conclude about the likelihood of substantial climate change?