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DISCUSSION

A Solution to a Problem for Bayesian Confirmation Theory

ABSTRACT

Charles Chihara has presented a problem he claims Bayesian confirmation theory cannot handle. Chihara gives examples in which he claims the change in belief cannot be construed as conditionalizing on new evidence. These are situations in which the agent suddenly thinks of new possibilities. I propose a solution that incorporates the important ideas of Bayesian theory. In particular, I present a principle which shows that the change of belief in Chihara's example is due to simple conditionalization.

1 INTRODUCTION

In a recent article Charles Chihara (1987) presents a problem that he claims classical Bayesian theory cannot handle. According to Chihara there are instances of rational belief change that cannot be construed as conditionalizing on new evidence. After presenting Chihara's problem, I propose a solution which retains all the important elements of classical Bayesian theory. In particular, I show how in Chihara's example the new beliefs can be seen to follow from simple conditionalization.

2 THE PROBLEM OF NEW POSSIBILITIES

In order to illustrate his problem Chihara presents a delightful story in which an agent is successively given six numbers, and he is to guess what the seventh number will be. After receiving the six numbers he notices no strong pattern that would indicate to him what the seventh number will be. However, suddenly he thinks of a new hypothesis that he had not previously thought of: perhaps the numbers are the Gödel numbers spelling 'crimson'. He checks the first number, 47, which turns out to be the Gödel number of 'c'; he then believes much more strongly that the seventh letter will be 69, the Gödel number of 'n'.

In this story the agent's degree of belief that the seventh number will be 69 increases upon thinking of the new hypothesis. But contrary to Bayesianism, it appears this change takes place without any new evidence. Let *H* be the hypothesis that the numbers are the Gödel numbers spelling 'crimson', let H^* be the hypothesis that the seventh number is 69, and let *E* be the evidence that the first number is 47, the second number is 77, . . ., and the sixth number is 71. According to Chihara the problem for Bayesianism is to account for the change of belief in H^*. The changes that took place were dependent upon the agent thinking of *H*, and not upon any new evidence or reflection upon logical relations among the old evidence. Contrary to Bayesianism, it looks as if this change of belief was not a function of the agent's previous beliefs and some new evidence. According to Chihara this shows that 'just thinking of new possibilities will produce in rational people important redistributions of subjective probability' (p. 559). But this is a problem for Bayesianism, because this change in probabilities does not appear to be via conditionalization.

3 A SOLUTION

In order to show how Bayesianism can account for Chihara's problem we must first become clear as to what Bayesianism requires and what a Bayesian solution would look like. According to traditional Bayesianism, an agent has a degree of belief in some evidence *E*, and as a result of experience the agent changes his degree of belief in *E* to 1. The agent then uses conditionalization to modify his other beliefs in response to the change of belief in *E*. Richard Jeffrey argued that it is unrealistic to suppose that the only way experience affects our beliefs is by causing us to believe some proposition to degree 1; experience may cause us to change our degrees of belief in various propositions, without believing any of them to degree 1. He then developed a theory of probability kinematics to show how our other beliefs can be modified as a result of experience changing our degrees of belief in some propositions, without raising them to degree 1. But both traditional Bayesianism and Jeffrey's probability kinematics share the following traits. Experience directly affects some of our beliefs. What beliefs are affected and our new degree of belief in them are not dependent upon the agent's old probability function; instead it is dependent upon the experience. Once the agent has new degrees of belief in certain propositions as a result of experience, the agent must change his other degrees of belief to account for the changes as a result of experience. So there are two parts to rational belief change for a Bayesian. First, some changes in degrees of belief are a result of experience and are not based upon any rule using prior probabilities; they are independent of the previous probability function. Second, some changes in degrees of belief are not based directly on experience, but are based on how the agent changes his degrees of belief in other propositions; these changes are made according to some rule, such as conditionalization, that uses the agent's prior probabilities and the probabilities changed as a result of experience. To solve Chihara's problem we need to show how the changes in degrees of belief in his story can be seen as instances of those two ways that beliefs can be changed.

One aspect of strict Bayesianism that generates Chihara's problem is the requirement that the agent's probability function be defined over all propositions. In reality, there are many propositions that we don't assign a probability, or that we don't assign a probability conditional on all other propositions. Even if the proposition is in our language we may not assign any probability to it, because we have never considered it. Our probability functions have gaps. Allowing probability functions to have gaps is a more realistic form of Bayesianism that provides a framework to discuss the problem Chihara raises. Let $\mathrm{P_o}$ be the agent's old probability function and let $\mathrm{P_n}$ be the agent's new probability function. In Chihara's problem, $\mathrm{P_o}$ is defined for H^* and E, but there are no probabilities involving H. For example, neither $\mathrm{P_o}(H)$, $\mathrm{P_o}(H^*/H)$, nor $\mathrm{P_o}(H/E)$ are assigned any value, because there is a gap around H in $\mathrm{P_o}$. But both $\mathrm{P_n}(H)$ and $\mathrm{P_n}(H^*/H)$ are defined, because the gap has been 'filled in'. We might say that the langauge of $\mathrm{P_o}$ has been expanded to form the language of $\mathrm{P_n}$.

One way to construe the agent's suddenly thinking of H in Chihara's story is as the agent expanding his probability function and filling in some of the gaps. H is not in the domain of $\mathrm{P_o}$, but as a result of thinking of H it is in the domain of $\mathrm{P_n}$. In filling in gaps the agent can assign probabilities in any way he pleases; he is not constrained by any prior probability of propositions in the gaps. $\mathrm{P_o}$ does not constrain in any way the value of $\mathrm{P_n}(H)$, because $\mathrm{P_n}(H)$ is not based on $\mathrm{P_o}$ in any way; it is based on the experience of thinking of H. We saw earlier that according to Bayesianism one way that beliefs can change is due to experience; as a result of experience some beliefs are simply changed. When an agent thinks of a new possibility this can be seen as the agent having a new experience. As a result of this experience the agent expands his probability function, he fills in gaps in the old probability function. This is in accord with the first way beliefs change according to Bayesianism. Bayesianism does not require that there be a rule for assigning probabilities to propositions that are newly considered, just as it does not give a rule for assigning probabilities to propositions that are directly affected by experience. I propose we look at the new belief in H as follows: as a result of experience the agent now has a degree of belief in H. Thus Bayesianism can account for the change of belief in H in Chihara's story.

After thinking of new possibilities the agent must then change his other degrees of belief by means of conditionalization or probability kinematics to account for these new beliefs. This is the second part of rational belief change.

In Chihara's story we need to explain the change in belief in H^*; the change in belief in H^* must be seen as conditionalizing on some new evidence. In other words, Bayesianism requires there be some evidence such that $P_n(H^*) = P_o(H^*/\text{evidence})$. I wish to propose the obvious: the new evidence is the filling in of a gap in the agent's probability function. The change of belief is due to the agent thinking of a new possibility and assigning a probability to a proposition that previously had no probability assignment.

We now need to formalize the intuition that the change is due to the agent filling in a gap is his probability function. Consider the following principle, which I will call 'GAP':

GAP $P_o[H^*/\text{there exists a } H \text{ in the domain of } P_n \text{ not in the domain of } P_o \text{ such that } P_n(H) = x \text{ and } H \text{ implies } H^*] \geqslant x$.

Although this is formulated in terms of H and H^*, it holds for any propositions and any future probability assignment. GAP justifies the agent's change of belief in H^*, because the change would be due to conditionalizing on the new evidence. In this case the new evidence is how the gap in P_o is filled in: a future probability function assigns a value to a new possibility, and this new possibility implies H^*. This accounts for the agent's change of belief in H^*; since the agent assigns a high probability to H and H implies H^*, the agent also assigns a high probability to H^*. Thus Bayesianism can account for changes of belief in propositions that are not in the gaps by conditionalizing on how the gaps are filled in.

GAP is actually a weaker version of van Fraassen's principle of reflection [1984]. Reflection requires that my current probability of H^*, conditional on the future probability of H^* being x, be equal to x: $P_o(H^*/P_n(H^*) = x) = x$. The information conditionalized on in GAP implies that my future probability of H^* will be greater than or equal to x; thus my current probability of H^* conditional on that information must be greater than or equal to x. Because GAP is based on reflection we can justify GAP in the same way that reflection is justified; violating it will result in a Dutch strategy being made against the agent. Let # stand for what is conditionalized on in GAP, namely there exists a H in the domain of P_n not in the domain of P_o such that $P_n(H) = x$ and H implies H^*. Suppose the agent violates GAP and assigns a probability of y to the probability in GAP, where $y < x$. The following bets would then be considered fair by the agent, where time o is the time of P_o and time n is the time of P_n:

Bet 1. A bet at time o against H^* conditional on #. Pay $(1-y)\$$ to get 1 if H^* is false, and 0 otherwise. If # is not true, the bet is off.

Bet 2. An unconditional bet for H^* made at time n if # is true. Since $P_n(H) = x$ and H implies H^*, coherence requires that $P_n(H^*) \geqslant x$. Let $P_n(H^*) = z$. Pay z to get 1 if H^* is true, 0 otherwise.

Bet 3. A bet at time o that # is true. If $P_o(\#) = r$, pay $r(x-y)$ to get $(x-y)$ if # is true, 0 otherwise.

If # is true, the agent loses $r(x-y)+(z-x)$ and if # is false the agent loses $r(x-y)$; either way the agent is guaranteed to lose at least $r(x-y)$. Thus the agent has had a Dutch strategy made against him. In order to avoid having a Dutch strategy made against him the agent must accept GAP. But then his change of belief in H^* is seen to be by simple conditionalization on #, which is how gaps in his probability function were filled in.

One limitation of GAP is that it requires that H implies H^*; fortunately GAP is easily generalized to remove that requirement. First consider the case in which the agent comes to fully believe H:

GAP* $P_o[H^*/\text{there exists a } H \text{ in the domain of } P_n \text{ not in the domain of } P_o \text{ such that } P_n(H^*/H)=y \text{ and } P_n(H)=1]=y$.

If the agent fills in the gaps in such a way that $P_n(H^*/H)=y$ and $P_n(H)=1$ coherence will require that $P_n(H^*)=y$; GAP* can then be seen to be a simple case of reflection. However, it may be unusual for the gaps to be filled in such a way that $P(H)=1$. To account for this likely possibility, consider the following:

GAP** $P_o[H^*/\text{there exists a } H \text{ in the domain of } P_n \text{ not in the domain of } P_o \text{ such that } P_n(H^*/H)=y,\ P_n(H^*/\neg H)=z, \text{ and } P_n(H)=x] = xy+(1-x)z$.

Principle GAP** incorporates the ideas of Jeffrey's probability kinematics into GAP*. This allows for cases in which H does not imply H^* and in which the agent is not certain of H. A more general version would be one in which it is not required that the only new possibility thought of is H; perhaps the agent thinks of several new possibilities at the same time:

GAP*** $P_o[H^*/\text{there exists a } H_1, H_2, \ldots, H_m \text{ in the domain of } P_n \text{ not in the domain of } P_o \text{ such that } \sum_{i=1}^{m} P_n(H_i) = 1,\ P_n(H^*/H_i) = y_i, \text{ and } P_n(H_i) = x_i] = \sum_{i=1}^{m} y_i x_i$.

This principle accounts for the case in which many gaps are filled in. Since Jeffrey's probability kinematics was designed to account for cases in which our experience affects many of our beliefs directly, GAP*** uses it to account for cases in which we may fill in many gaps in our probability function. All of the above versions of GAP are special cases of reflection and would be justified in the same manner that reflection is justified.

We have seen that Chihara's problem of new possibilities can be accounted for by a modified Bayesianism. If we allow agents to have and fill in gaps in their probability functions, we can view the filling of gaps as being the result of new insight, which is a type of experience. These changes of belief in response to experience is consistent with Bayesianism. It is also natural to think that the filling of these gaps will have effects on other beliefs. The change in beliefs not in the gaps can be accounted for by some version of conditionalization. We presented several principles that describe what an agent's conditional probabilities must be, conditional on how the gaps might be filled in. Thus we see that Bayesian principles can explain situations where an agent thinks of new possibilities.

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