CHAPTER 4
Appendix

ADDITIONAL TOPICS IN DEMAND THEORY

The Constant Elasticity Demand Curve

The demand curves discussed so far have been linear demand curves, which, as noted, have the property that price elasticity declines as we move down the demand curve. Not all demand curves have this property, however; on the contrary, there are demand curves for which price elasticity can remain constant or even rise with movements down the demand curve. The *constant elasticity demand curve is* the name given to a demand curve for which elasticity does not vary with price and quantity. Whereas the linear demand curve has the general form $P = a - bQ$, the constant elasticity demand curve is instead written

$$P = \frac{k}{Q^{1/h}}, \qquad \text{(A.4.1)}$$

where k and h are positive numbers, specific values of which determine the exact shape and position of the curve.[1] An example with $k = 2$ and $h = 1$ is pictured in Figure A.4-1.

[Figure A.4-1]

Let us examine some points on the curve pictured in Figure A.4-1 and verify that they do indeed have the same price elasticity. Consider first the point $P = 2$, $Q = 1$, and calculate price elasticity as the product of the ratio P/Q and the reciprocal of the slope of the demand curve. To calculate the slope of the demand curve, we need to calculate the ΔQ that occurs in response to a very small ΔP near the point (1, 2). Suppose, for example, we use a price change of +0.001. If $P = 2.001$, we can solve from the demand curve (that is, from the equation $P = 2/Q$) to get the corresponding $Q = 2/2.001 = 0.9995$. Thus $\Delta Q = 0.9995 - 1 = -0.0005$, and the slope of the demand curve at (1, 2) may be calculated as $\Delta P/\Delta Q$, or $0.001/(-0.0005) = -2$. The reciprocal of the slope is $-{}^1/_2$, and so the price elasticity is $2(-{}^1/_2) = -1$

Consider now the point (2, 1). Again using a ΔP of 0.001, we get a new Q of $2/1.001 = 1.998$, or a ΔQ of -0.002. Thus the slope of the demand curve at (2, 1) is $0.001/(-0.002) = -{}^1/_2$, and its reciprocal is -2. The price elasticity at (2, 1) is therefore $({}^1/_2)(-2)$, or again -1.

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[1]Using the formal definition of elasticity, it is easy to show that the elasticity at any price-quantity pair along this demand curve is - η:

$$\frac{P}{Q}\frac{dQ(P)}{dP} - \frac{k/Q^{1/h}}{Q}\frac{1}{(-1/h)kQ^{-1/h-1}} = -h$$

• EXERCISE A.4-1

Try several other points along the demand curve in Figure A.4-1 and verify that the price elasticity in every instance is equal to -1. [The answer at the end of the chapter uses the points (0.5, 4) and (4, 0.5.]

The demand curve given by $P = k/Q$ *is* a special case of the constant elasticity demand curve called the *constant expenditure demand curve. At* every point along such a demand curve, total expenditure is given by the product $PQ = k$, where k *is* again a positive constant. Thus, unlike the case of the straight-line demand curve, here people spend exactly the same amount when price is high as they do when price is low. Someone who spends her entire allowance on compact discs each month, for example, would have a constant expenditure demand curve for compact discs. The constant k would be equal to the amount of her allowance.

As we move downward along any constant elasticity demand curve ($P = k/Q^{l/\eta}$), the fall in the ratio P/Q is exactly counterbalanced by the rise in the reciprocal of the slope. A constant elasticity demand curve with $\eta > 1$ has the property that a price cut will always increase total expenditures. For one with $\eta < 1$, by contrast, a price cut will always reduce total expenditures.

• EXERCISE A.4-2

What happens to total expenditure when price falls from 4 to 3 along the demand curve given by $P = 4/Q^{1/2}$?

SEGMENT-RATIO METHOD

The price elasticity at a given point along a straight-line demand curve may be given one other useful geometric interpretation. Suppose we divide the demand curve into two segments *AC* and *CE*, as shown in Figure 4-2. The price elasticity of demand (in absolute value) at point *C*, denoted $|\eta_c|$, will then be equal to the ration of the two segments.[2]

[2]To see why this is so, we can make use of some simple high school geometry. First, note that the reciprocal of the slope of the demand curve in Figure 4-2 is the ration *GE/GC* and that the ratio of price to quality at point C is *GC/FC*. Multiplying these two, we get $|\eta_c| = (GE/GC)\,(GC/FC) = GE/FC$. Now note that the triangles *AFC* and *CGE* are similar, which means that the ratios of their corresponding sides must be the same. In particular, it means that the ratio *GE/FC*, which we just saw is equal to the price elasticity of demand at point *C*, must also be equal to the ration *GE/AC*. And this, of course, is just the result we set out to establish.

$$|\eta_c| = \frac{CE}{AC} \qquad \text{(A.4.2)}$$

[Figure A.4-2]

Equation A.4.4 is called the *segment-ratio* for calculating price elasticity of demand.

Knowing that the price elasticity of demand at any point along a straight-line demand curve is the ratio of two line segments greatly simplifies the task of making quantitative statements about it. Consider the demand curve shown in the top panel of Figure A.4-3. At the midpoint of that demand curve (point *M*), for example, we can see at a glance that the value of price elasticity is -1. One-fourth of the way down the demand curve (point *K* in Figure A.4-3), the elasticity is -3; three-fourths of the way down (point *L*), $-^{1}/_{3}$; and so on. The bottom panel of Figure A.4-3 summarizes the relation between position on a striaght-line demand curve and the price elasticity of demand.

[Figure A.4-3]

The Concept of Arc Elasticity

Suppose we start on a hypothetical straight-line demand curve at a point with $P_0 = 10$ and $Q_0 = 100$. Now, let price rise by 10 so that *P*1 = 20, and suppose the resulting quantity demanded is *Q1* = 50. What is the price elasticity of demand for this good? Suppose we try to answer this question using the formula $\eta = (\Delta Q/Q)/(\Delta P/P)$. It is clear that $\Delta P = 10$ and $\Delta Q = -50$. But what values do we use for *P* and *Q?* If we use the initial values, P_0 and Q_0, we get an elasticity of (-50/100)/(10/10) = $-^{1}/_{2}$. But if we use the new values, P_1 and Q_1, we get an elasticity of (-50/50)/(10/20) = -2.

Thus, if we reckon price and quantity changes as proportions of their initial values, we get one answer, but if we compute them as proportions of their new values we get another answer. Neither of these answers is incorrect. The fact that they differ is merely a reflection of the fact that the elasticity of demand differs at every point along a straight-line demand curve.

Strictly speaking, the original question ("What is the price elasticity of demand for this good?") was not well posed. To have elicited a uniquely correct answer, it should have been, "What is the price elasticity of demand at the point (100, 10)?" or, "What is the price elasticity of demand at the point (50, 20)?" Economists have nonetheless developed a convention for answering ambiguous questions like the one originally posed. It is to use the formula for the so-called *arc elasticity of demand,* which is given by

$$h = \frac{\Delta Q/[(Q_0 + Q_1)/2]}{\Delta P/[(P_0 + P_1)/2]}. \qquad \text{(A.4.3)}$$

The arc elasticity approach thus sidesteps the question of which price-quantity pair to use by using averages of the new and old values. The formula reduces to

$$h = \frac{\Delta Q/(Q_0 + Q_1)}{\Delta P/(P_0 + P_1)}, \qquad \text{(A.4.4)}$$

and this is the last time you will see it in this text. (The only reason it appears here at all is that questions on standardized tests in economics sometimes refer to it.) Hereafter, all questions having to do with elasticity will be taken up using the measure discussed earlier, which is called *point elasticity*.

The Income-Compensated Demand Curve

The individual demand curves we saw in this chapter take into account both the substitution and income effects of price changes. For many applications, such demand curves will be the relevant tool for predicting people's response to a change in price. Suppose, for example, that gasoline prices rise because of a new OPEC agreement. Such a price increase will have both income and substitution effects, and the individual demand curve described earlier will be the appropriate device for predicting a person's response.

In other situations, however, this demand curve will not be the right tool. During the Carter administration, for example, there was a proposal to tax foreign oil, then cushion the burden of the tax by simultaneously reducing the tax on wage earnings. A tax on oil, taken by itself, would increase the price of oil and produce the corresponding income and substitution effects. But the effect of the simultaneous earnings tax reduction, roughly speaking, would have been to eliminate the income effect of the price increase. The tax comes out of one pocket, but is put right back into the other.

To analyze the effect of such a policy, we must use the *income-compensated demand curve*, which tells the amounts consumers would buy if they were fully compensated for the income effects of changes in price. To generate this curve for an individual, we simply eliminate the income effect from the total effect of price changes. The top panel of Figure A.4-4 shows the income and substitution effects of an increase in the price of shelter from $6/sq yd to $12/ sq yd for a consumer whose weekly income is $120. The ordinary demand curve for shelter for the individual pictured here would associate $6 with 10 sq yd/wk and $12 with 6 sq yd/wk. The income-compensated demand curve is always constructed relative to a fixed reference point, the current price. Thus like the ordinary demand curve, it too associates 10 sq yd/wk with the price $6. But with the price $12 it associates not 6 sq yd/wk but 7 sq yd/wk, which is the amount of shelter the consumer would have bought at $12/sq yd if he had been given enough income to remain on the original indifference curve, I_0.

The individual whose responses are described in Figure A.4-4 happens to regard shelter as a normal good, one for which the quantity demanded increases as income rises. For normal goods, the income-compensated demand curve will necessarily be steeper than the ordinary demand curve. In the case of an inferior good, however, the ordinary demand curve will always be the steeper of the two. The relationship between the two demand curves for an inferior good is as pictured in Figure A.4-5.

[Figure A.4-4]

[Figure A.4-5]

In applications, the distinction between ordinary and income-compensated demand curves turns out to be particularly important for questions of tax policy. In the case of Jimmy Carter's gasoline tax proposal, there was an explicit provision for the proceeds of the tax to be returned to the people who paid it. But even without such a provision, the practical impact of a new tax would be roughly the same. After all, when the government raises more revenue from one source it needs to raise less from others. The end result is that the relevant demand curve for studying the effects of a tax on a good is the income-compensated demand curve.

As a practical matter, the distinction between the two types of demand curves is relevant only for goods for which income effects are large in relation to the corresponding substitution effects. In order for the income effect of a price change for a particular good to be large, it is necessary (but not sufficient) that the good account for a significant share of total expenditures. Many of the individual goods and services we buy, however, account for only a tiny fraction of our total expenditures. Accordingly, for such goods the distinction between the two types of demand curve will be unimportant. Even for a good that accounts for a large budget share, the income effect of a price change will sometimes be small. (The good might lie on the border between a normal and an inferior good.) For such goods, too, the distinction between ordinary and income-compensated demand curves will be of little practical significance.

GIFFEN GOODS

A *Giffen good* (a good whose demand curve is upward sloping) is one for which the total effect of a price increase is to increase, not reduce, the quantity purchased. Since the substitution effect of a price increase is always to reduce the quantity purchased, the Giffen good must be one whose income effect not only works against but also overpowers the corresponding substitution effect. That is, the Giffen good is a particular kind of inferior good—one so strongly inferior that the income effect is actually larger than the substitution effect.

A much cited example of a Giffen good was the potato during the Irish potato famine of the nineteenth century. The idea was that potatoes were such a large part of poor people's diets to begin with that an increase in their price had to severe adverse effect on the real value of purchasing power. Having less real income, many families responded by cutting back on meat and other more expensive foods and buying even more potatoes (see Figure A.4-6). Or so the story goes.

Modern historians dispute whether the potato ever was really a Giffen good. Whatever the resolution of this dispute, the potato story does illustrate the characteristics a Giffen good would logically have to possess. First, it would not only have to be inferior, but would also have to occupy a large share of the consumer's budget. Otherwise an increase in its price would not create a significant reduction in real purchasing power. (Doubling the price of keyrings, for example, does not make anyone appreciably poorer.) The second characteristic required of a Giffen good is that it have a relatively small substitution effect, one small enough to be overwhelmed by the income effect.

In practice, it is extremely unlikely that a good will satisfy both properties required of a Giffen good. Most goods, after all, account for only a tiny share of the consumer's total expenditures. Moreover, as noted, the more broadly a good is defined, the less likely it is to be inferior. Finally, inferior goods by their very nature tend to be ones for which there are close substitutes. The consumer's tendency to substitute ground sirloin for hamburger, for example, is precisely what makes hamburger tend to be an inferior good.

The Giffen good is an intriguing anomaly, chiefly useful for testing students' understanding of the subtleties of income and substitution effects. Unless otherwise stated, all demand curves used in the remainder of this text will be assumed to have the conventional downward slope.

Question for Review:

1. *True or false*: All Giffen goods are inferior. Explain

Answers to Exercises

A.4-1. First consider the point (0.5, 4). If we again let ΔP be 0.001 so that the new P is 4.001, the resulting Q is 2/4.001 = 0.499875, which means that ΔQ is -0.000125. Price elasticity is therefore equal to (4/0.5)(-0.000125/0.001) = -1. Now consider the point (4, 0.5). If we again let ΔP be 0.001, so that the new P is 0.501, the resulting Q is 2/0.501 = 3.992, which means that ΔQ is -0.008. Price elasticity is therefore equal to (0.5/4)(-0.008/0.001) = -1.

A.4-2. For P = 4, we have $4 = 4/\sqrt{Q}$, which yields Q = 1, so total expenditure is 4(1) = 4. For P = 3, we have $3 = 4/\sqrt{Q}$, which yields $Q = {}^{16}/_{9}$, so total expenditure is $(3)({}^{16}/_{9}) = {}^{16}/_{3}$. So with $\eta = 2$, total expenditure rises with a decrease in price.