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Hot Spot Policing: A Study of Place-Based Strategies for Crime Prevention

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Hot spot policing is a popular policing strategy that addresses crime by assigning limited police resources to areas where crimes are more highly concentrated. We analyze this strategy using a game theoretic approach. The main argument against focusing police resources on hot spots is that it would simply displace criminal activity from one area to another. We provide new insights on the nature of the displacement effect with useful implications for the empirical analysis of crime-reduction effects of police reallocation. We also propose alternative place-based policies that display attractive properties in terms of geographic spillovers of crime reduction via optimal police reallocation.

JEL Classification: D7, K4, R1

1. Introduction

Crime mapping is a powerful tool used by analysts in law enforcement agencies to visualize and study crime patterns. Such maps indicate that crimes are often not evenly distributed across geographic locations. Instead, clusters of crimes occur in specific areas, or hot spots. Hot spot policing is a place-based strategy that attempts to reduce crime by assigning limited police resources to places where crimes are more highly concentrated. This approach to crime prevention is relatively new and many crime experts argue it is one of the main reasons why New York City has achieved a dramatic decrease in crimes during the past two decades.[1] We analyze this policy via a game theoretic approach and propose alternative strategies that display interesting features in terms of geographic spillovers of crime reduction. We pay special attention to the so-called displacement effect and discuss the implications of our results for the empirical research on police effectiveness regarding crime reduction. While our results support the doctrine on hot spot policing, they also raise some concerns regarding the extreme implementation of this policing strategy.

The model we develop to study the effectiveness of hot spot policing incorporates various crime theories that capture different aspects of crime decisions. These theories have been so far studied in isolation and, by combining them in a single model, we are able to make predictions that are more consistent with observed patterns of crimes. Specifically, our approach is based on the rational choice model and uses game theory to incorporate strategic interactions among potential offenders into the analysis. We also borrow from the theory of environmental criminology,

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[1] See, for example, Zimring (2012).

which highlights the role of spatial factors in the choice of crime location.[2] More formally, we propose a two-stage game. We first divide the region under study into a finite number of areas that differ in terms of attractiveness for potential offenders. For instance, if the overall region represents NYC, then an area may correspond to one of its neighborhoods. We capture crime attractiveness via two attributes, namely, risk of apprehension and potential productivity. The riskiness of a place for a potential offender can be thought of as an index function that captures structural factors affecting the successful apprehension of offenders in that location, such as the presence of illumination or video cameras. The second attribute, potential productivity, relates to the expected gains from committing a crime in that place, such as the presence of a shopping mall or a bank. In the first period of the two-stage game, the enforcement agency decides how to allocate the limited police resources across alternative areas. In the second period, upon observing police allocation, people decide whether to commit a crime and, in case of doing so, where to perform the criminal act.

Using the standard backward induction principle, we solve the game by first modeling people's choices for a given police assignment. This first part of our work sheds light on one of the most controversial issues associated with hot spot policing, namely, the displacement effect. That is, the possibility that increasing police resources in hot spots would simply displace criminal activity from one area to the others.[3] Empirical research has found some evidence against this argument (see, e.g., Di Tella and Schargrodsky 2004; Braga 2008, Cook and MacDonald 2011). Our model features this characteristic when the value of the outside option (not to commit a crime) does not depend on the number of people who opt not to become a criminal and there are no complementarities in the criminal activity. Under these circumstances, the value of the outside option regulates people's utilities, and increasing resources in an area simply discourages people in that area from committing a crime. This lack of displacement effect also relies on the assumption that, in order to increase the police resources in a specific area, the enforcing agency does not need to diminish them from an alternative one. When at least one of these conditions fails, the previous result does no longer hold. Under these alternative circumstances, increasing police resources in areas with high criminality pushes criminals from these locations to the others. This result raises a simultaneity issue in the empirical studies that aim to measure the effect of increases in police on crime reduction using panel data. More precisely, any such study should take into account that the crime rate in each area depends not only on the police resources allocated to that specific area but on the whole vector of police allocation. We provide specific conditions under which researchers should use more sophisticated econometric approaches.

After we characterize the decisions of potential offenders, we go back to the first stage of the game and describe the crime-minimizing police allocation. We find that, at the optimal police allocation, areas that are a priori more attractive to offenders (i.e., display a larger productivity-to-risk ratio) receive indeed more police attention. However, in our model, the extra efforts in these areas do not fully offset the impact of their initial structural differences. That is, some hot spots still remain under the crime-minimizing allocation strategy. We then contrast the optimal police allocation with the behavior of an enforcement agency that aims to eliminate crime differences across areas. This egalitarian policy can be thought of as taking the implementation of hot spot policing to the limit. We find that the latter implies a higher overall crime rate and its opportunity cost

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[2] See, for example, Brantingham and Brantingham (1991).

[3] Reppeto (1976) offers an early discussion on the different types of displacement effects in criminal activity. Ellen, Lacoe, and Sharygin (2013) study the displacement of crimes due to foreclosures.

increases with the variability of the productivity-to-risk ratio across areas. In summary, we find that the optimal crime reduction strategy does involve differential targeting of potentially high crime locations, as argued by the doctrine on hot spot policing. Nevertheless, our results also suggest that extreme implementations of this strategy should be carefully evaluated in terms of the ultimate objectives, as it might have the unintentional effect of increasing the overall crime rate. These results are robust to all the extensions we consider for our initial model. In particular, they remain valid independently of the displacement effects.

We then study an alternative place-based strategy. Specifically, we analyze the implications of introducing structural changes that aim to reduce the productivity-to-risk ratio of a certain area (e.g., installing video cameras on the streets). This policy has been suggested by a number of crime theorists, including Braga and Weisburd (2010), who state that:

> The attributes of a place are viewed as key factors in explaining clusters of criminal events... To reduce and better manage problems at crime hot spots, the police need to change the underlying conditions, situations, and dynamics that make them attractive to criminals and disorderly persons.

We find that this policy reduces crime not only in the target area but also in all other locations. Positive (or negative) external effects of structural changes have been proposed earlier (see, e.g., Ehrlich 1973; Glaeser and Gottlieb 2008). An interesting aspect of our result is the mechanism that produces this outcome. The direct effect of the policy is to make the target area less attractive for potential offenders, thereby reducing its criminal activity. The indirect effect is due to subsequent police reallocation from the improved area to the other ones, where the criminal activity diminishes as well. In other words, structural improvements in an area generate geographic spillovers of crime reduction in all other locations via optimal police reallocation.

We finally consider various extensions to our initial model. The outside option people face (not to commit a crime) could be interpreted as the possibility to get a legal job. We then introduce alternative specifications of the outside option and study the effects of improvements in the job market on the proportion of people who choose not to commit a crime, for example, the labor supply of the economy. We find that, when the outside option displays congestion effects, increasing police resources in a given area pushes some criminals from this area to the others—even if we do not reduce the police resources from these other areas. Congestion effects in the outside option might occur if, for instance, an increment in the number of people searching for a legal job pushes salaries down or increases unemployment, thereby making this outside option less attractive. As we mentioned earlier, this result raises a simultaneity issue in the empirical studies that address the effect of police levels on crime reduction using panel data. Finally, we investigate the consequences of reversing the interaction effects among potential offenders. We show that, when interactions are positive—as in Freeman, Grogger, and Sonstelie (1996) and Sah (1991)—interventions become a delicate matter. (By positive interactions, we mean that the payoff of committing a crime in a specific area increases when more people decide to do so.) The reason is that these models often display multiple equilibria and policy interventions can easily affect equilibrium selection (see Blume 2006). In this context, the displacement effect can also take place and be quite extreme. We explore this possibility with a simple example.

Literature Review

Our research contributes to work in both criminal studies and economics. In an early study, Becker (1968) examines individual decisions to commit crimes from an economic perspective.[4] His cost-benefit analysis is consistent with the rational choice approach used by Cornish and Clarke (1986), which we follow as well.[5] Our study also relates to subsequent work on the importance of social interactions in motivating criminal behavior (see, e.g., Sah 1991; Freeman, Grogger, and Sonstelie 1996, Glaeser, Sacerdote, and Scheinkman 1996; Ballester, Calvó-Armengol, and Zenou 2006, 2010; Chen and Shapiro 2007). Bayer and Timmins (2005) develop the equilibrium properties of an estimable model of location choice that incorporates social interactions. Following the latter, we assume that each individual decision depends on other people's criminal choices. To incorporate spatial factors into the analysis, we model people's expected payoffs as in Hugie and Dill (1994), who study habitat selection by modeling the behavior of predators and prey (see also Helsley and Zenou 2014).

As we mentioned above, several studies find empirical evidence of no displacement effect in the criminal activity. Among them, Di Tella and Schargrodsky (2004) analyze the aftermath of a terrorist attack in Argentina in 1994, which led to massive police presence in certain sensitive locations. They find large effects on motor vehicle theft with very limited evidence of displacement. Cook and MacDonald (2011) evaluate private security activities by business improvement districts. They also document no evidence that crime is displaced to neighboring areas. See also Braga (2008) and Braga and Weisburd (2010) for a large review of empirical evidence on this phenomenon. As noted earlier, we derive specific conditions under which the displacement effect does and does not take place.

Our work is also related to Espejo, L'Huillier, and Weber (2011), who provide an evaluation of hot spot policing using a leader and follower model, as we do in this investigation. Our article differs from theirs in a few dimensions. First, the model of Espejo, L'Huillier, and Weber (2011) does not include an outside option for the potential criminals. As a consequence, the overall level of criminality does not depend on the police allocation and they are not able to characterize the displacement effect—which is one of our main focuses. Second, their model does not provide insights regarding the determinants of payoffs and, therefore, they cannot evaluate the alternative place-based policies we study here. Finally, Espejo, L'Huillier, and Weber (2011) do not discuss the possibility of positive interactions.

In this study, we assume that people observe police allocations and make subsequent choices regarding crime decisions and the location of the criminal acts. Lazear (2006) shows that it may be optimal, under certain circumstances, to keep police allocations secret. Although our model is quite different from his—for instance, we introduce social interactions—it is important to remark that our results remain unchanged if we allow for randomized police allocations. The reason is that the effect of police enters people's utilities in a linear fashion. It would be interesting to study the advantages of secret police allocations in a model similar to the one of Lazear (2006) but with the additional feature of social interactions. We leave this analysis for future research.

Braga and Weisburd (2010) offer a deep analysis of place-based policies to crime fighting.[6] In addition to new interesting insights about hot spots and crime prevention, they provide a thorough

[4] See also Ehrlich (1973).

[5] Durlauf, Navarro, and Rivers (2010) provide a general description of criminal choices at the individual level to understand the implicit assumptions in aggregate crime regressions. They highlight the relevance of modeling the microfoundations of the empirical analysis of crimes.

[6] See also Braga (2008), Eck et al. (2005), Felson and Clarke (1998), Sherman (1995), and Weisburd and Green (1995).

and updated overview of the theoretical and empirical research regarding this topic. Our theoretical modeling assumptions are inspired by all their discussions and the literature therein. On the applied side of the literature, Fu and Wolpin (2013) perform a structural estimation of the effects of police reallocation on crime reduction. They assume that increasing the police resources in a given area does not affect the criminal activity in other areas. We show that, in our model, this happens if the interactions are negative and the outside option (e.g., working in a legal activity) does not display crowding out effects. This result provides some key lower-level restrictions under which the assumption in Fu and Wolpin (2013) holds.

The rest of the article is organized as follows. Section 2 describes our model. Section 3 presents our main findings. Section 4 evaluates the decision of an enforcement agency that aims to reduce crime by changing the attributes of a certain area. Section 5 discusses three extensions of our model. Section 6 concludes. We collect all proofs in the Appendix.

2. The Model

Main Variables

This subsection describes the main variables of our model, making a clear distinction between the features that we assume are exogenous to the incumbents (i.e., people and the enforcement agency) and those that are under their control. Sections 4 and 5 examine some extensions to this initial model structure.

Exogenous Variables

We let N and M represent the size of the mass of people and police, respectively. There are K alternative areas where criminal activity can take place. With only a slight abuse of notation, K represents the set as well as the number of locations. These areas differ with respect to three attributes, namely, size of the area, risk of apprehension, and productivity of the criminal activity, which we describe next.

S_k refers to the geographic size of Area k (e.g., in square feet). Riskiness R_k is a probability measure of the successful apprehension of offenders in Area k. Differences in riskiness across areas capture different structural characteristics of the areas that affect the ability of police to capture offenders. For example, better street lighting may increase the risk of apprehension as offenders are more likely to be seen by someone who might call the police. Conversely, the presence of nearby highways may reduce this risk, as it becomes easier for criminals to escape. We use f to indicate the fee an offender must pay if apprehended. The fee f could capture, for instance, the opportunity cost of time spent in prison.

Productivity A_k captures the richness of Area k in terms of expected benefits to criminals. For example, a rich area may be a neighborhood that is populated by high-income people whose houses contain high-value items. It may also be a location with stores or banks available as potential targets.

Endogenous Variables

The incumbents in the model are the people and the law enforcement agency. Specifically, people decide whether to commit a crime and, if they do so, where to perform the criminal act.

In our model, p_k represents the fraction of total people N who decide to commit a crime in location k. We indicate the density of offenders in that location by $d_k = p_k N / S_k$; that is, the number of criminals in the area $p_k N$ divided by the size S_k of that area.

The enforcement agency decides how to assign police resources to the different areas. We let q_k denote the proportion of total police resources M that is assigned to location k; consequently, $e_k = q_k M / S_k$ is the corresponding police density (i.e., the number of police officers in the area q_k M divided by the size S_k of that area.). We assume $M/S_k \leq 1$, for all $k \in K$, so that the per capita apprehension rate (defined below) lies between 0 and 1.

Payoffs of People

We model encounters between police and offenders as a random process, such that overall apprehension in location k is given by

$$\mathcal{A}(k, p_k, q_k) = d_k e_k R_k S_k.$$

That is, overall apprehension $\mathcal{A}(k, p_k, q_k)$ is the product of the density of criminals in the location d_k, the density of police officers in the area e_k, the riskiness of the location R_k, and the geographic size of the area S_k. It represents the expected number of criminals to be captured in Area k.

Furthermore, the per capita apprehension rate of an offender in location k is denoted by

$$\mathcal{P}(k, p_k, q_k) = \mathcal{A}(k, p_k, q_k) / d_k S_k = R_k e_k.$$

In other words, the per capita apprehension rate of an offender $\mathcal{P}(k, p_k, q_k)$ in location k is just the riskiness of the area R_k times the density of police resources e_k in that area, and represents the probability that a criminal is captured by the police in Area k.

It follows from the last two expressions that the expected penalty for a person who commits a crime is

$$\mathcal{P}(k, p_k, e_k) f$$

Thus, the cost-side of each individual's analysis depends on both his perceived probability of being apprehended $\mathcal{P}(k, p_k, e_k)$ and the penalty f he would have to pay in that case.[7] (In section 5, we allow for congestion effects in the cost-side of the crime decision. These congestion effects can be motivated by the fact that a police officer cannot be in two different places at the same time; thus, as the criminal activity in a certain area increases, the probability of being apprehended in that area goes down.)

Conversely, the offender's expected benefit of committing a crime in Area k is

$$\mathcal{Y}(k, p_k, q_k) = A_k / d_k.$$

It follows that the expected payoff of the criminal act increases with the productivity of the area A_k; by contrast, it decreases with the density of offenders in the area d_k, as the total potential productivity has to be shared among more people.

[7] Durlauf and Nagin (2011) suggest that increasing the perceived risk of apprehension seems to have considerable deterrent effects on crimes.

Thus, the overall expected utility of an offender in location k is given by

$$\mathcal{U}(k, p_k, q_k) = \mathcal{Y}(k, p_k, q_k) - \mathcal{P}(k, p_k, q_k) f = A_k / d_k - R_k e_k f.$$

Recall that our model allows people not to commit a crime. This outside option can be thought of as the possibility to work in a legal activity. Under this interpretation, the number of people who opt not to commit a crime comprises the labor supply in the economy. To simplify the exposition, we initially assume the expected payoff of this outside option is 0. We relax this restriction in section 5 to evaluate the impact on crimes of public policies that affect the labor market in the economy. We refer to the outside option as $k = 0$, so that $\mathcal{U}(0) \equiv 0$ and the choice set of each person is $K_0 \equiv 0 \cup K$. The outcome of their decisions is a probability vector $\mathbf{p} \equiv (p_k)_{k \in K} \in \Delta^{K_0}$ where

$$\Delta^{K_0} \equiv \left\{ \mathbf{p} : p_k \geq 0 \quad \text{and} \quad \sum\nolimits_{k=0}^{K} p_k = 1 \right\}.$$

Thus, p_0 represents the proportion of total people N who decide not to commit a crime, and expression $p_0 N$ represents the number of people who choose so (i.e., the labor supply in the economy).

Payoffs of Police Allocation Strategies

The public authority decides how to assign the police to different locations. Specifically, it chooses $\mathbf{q} \equiv (q_k)_{k \in K} \in \Delta^K$, where q_k denotes the fraction of total police M allocated to Area k and

$$\Delta^{K} \equiv \left\{ \mathbf{q} : q_k \geq 0 \quad \text{and} \quad \sum\nolimits_{k=1}^{K} q_k = 1 \right\}$$

is the set of all possible police allocations. If the purpose of the enforcement agency is to minimize the overall level of criminal activity, then its payoff function is represented by

$$\mathcal{V}(\mathbf{p}) = p_0 \geq 0$$

and its objective is to maximize p_0 (i.e., the fraction of people who decide not to commit a crime). In our subsequent analysis, we contrast the behavior of a public authority interested in minimizing the overall crime rate to that of a public authority aiming to minimize criminality while it keeps an even distribution of crimes across areas.

Structure of the Game

We model interactions between incumbents using a leader and follower game, with the public authority as the leader and potential offenders as the followers.

In this game, the public authority first decides how to assign police to different locations with the objective of minimizing the overall crime rate. This problem can be specified as follows

$$\max\nolimits_{\mathbf{q}} \left\{ \mathcal{V}(\mathbf{p}) : \mathbf{q} \in \Delta^K \right\}.$$

Upon observing the distribution of police $\mathbf{q}$, each person, taking as given the decisions of the others, decides whether to commit a crime and, in that case, where to perform the criminal act. Thus, the problem faced by each person is

$$\max_k\{\mathcal{U}(k,p_k,q_k) : k \in K_0\}.$$

In the next section, we solve the game using the standard backward induction principle.

3. Equilibrium Analysis

People's Choices

The second stage of the game is itself a game among potential offenders. We use Nash equilibrium as our solution concept. Given a strategy profile $\mathbf{p}$, we let $\mathrm{b}(\mathbf{p})$ indicate the best-response correspondence of an arbitrary person, that is,

$$\mathrm{b}(\mathbf{p})=\{k' \in K_0 : k' \in \arg\max_k\ \mathcal{U}(k,p_k,q_k)\}.$$

It follows that $\mathbf{p}(\mathbf{q}) \in \Delta^{K_0}$ is a Nash equilibrium if, for each $k' \in K_0$, we obtain

$$p_{k'}(\mathbf{q}) > 0 \quad \text{if } k' \in \mathrm{b}(\mathbf{p}) \quad \text{and} \quad p_{k'}(\mathbf{q})=0 \quad \text{otherwise.}$$

Given an initial police assignment and some beliefs regarding crime location, all people face the same choice set and expected payoffs. Thus, any action that is selected with a strictly positive probability will be among the options with the highest expected value. As people are indifferent across these possibilities, we can interpret $\mathbf{p}(\mathbf{q})$ as either an asymmetric equilibrium in pure strategies or a symmetric mixed strategy equilibrium (see Hugie and Dill 1994).

To simplify notation, we define $\theta_k \equiv (S_k)^2 A_k/R_k MNf$, for all $k \in K$. We can now describe, for each police assignment $\mathbf{q}$, the distribution of criminal activity across areas.

Proposition 1. Fix some $\mathbf{q} \in \Delta^K$. The proportion of the population that decides to commit a crime in location k, for each $k \in K$, is given by

$$p_k(\mathbf{q})=S_k A_k/N[u(\mathbf{q})+R_k q_k Mf/S_k],$$

with $u(\mathbf{q}) \geq 0$. Moreover, $p_0(\mathbf{q}) > 0$ if and only if $\sum_{k\in K}\theta_k/q_k < 1$, in which case $u(\mathbf{q})=0$. The equilibrium is unique.

Remark. $u(\mathbf{q})$ is the utility level obtained by each person at the second-stage equilibrium when the police assignment is $\mathbf{q}$. In addition, the proportion of people who opt not to commit a crime is given by $p_0(\mathbf{q})=1-\sum_{k\in K}p_k(\mathbf{q})$. According to Proposition 1, $p_0(\mathbf{q}) > 0$ if and only if $\sum_{k\in K}p_k(\mathbf{q})=\sum_{k\in K}\theta_k/q_k < 1$. The latter condition can be re-expressed as $M > \sum_{k\in K}(S_k)^2 A_k/R_k Nfq_k$. This means that, in our model, some people will opt not to commit a crime if and only if the mass of police M is large enough. Under this scenario, $u(\mathbf{q})=0$.

Proposition 1 shows that the criminal activity in a certain location $p_k(\mathbf{q})$ increases with the perceived productivity of the area A_k and decreases with both its apprehension risk R_k and the number of police officers in place $q_k M$. This proposition allows us to determine the patterns of displacement of criminal activity across locations as the public authority changes the initial police allocation. We elaborate next on this description.

Displacement. Let $Q \equiv \left\{\mathbf{q} \in \Delta^K : \sum_{k\in K}\theta_k/q_k < 1\right\}$ be the set of all possible vectors of police allocations $\mathbf{q}$ such that $\sum_{k\in K}\theta_k/q_k < 1$ (i.e., some people choose the outside option, or $p_0(\mathbf{q}) > 0$), and assume this set is nonempty.[8] For all police distribution $\mathbf{q} \in Q$ and each area $k \in K$, we get that overall expected utility of an offender in location k is $\mathcal{U}(k, p_k(\mathbf{q}), q_k) = 0$ and the fraction of people who decide to commit a crime is given by $p_k(\mathbf{q}) = \theta_k/q_k$. That is, when the mass of police M is large enough, the outside option (not to commit a crime) regulates the second-stage equilibrium payoffs of potential offenders, and the level of criminal activity in each area p_k depends only on the proportion of police assigned to that specific area q_k (rather than on the whole vector of police allocation $\mathbf{q}$). This means that, if q_k increases, then location k becomes less attractive to potential offenders and its criminal activity p_k decreases. Increasing q_k in and of itself does not induce any initial displacement of criminality from Area k to the other areas. However, we do observe an increase in crime in other areas due to the removal of police from the latter. In other words, the displacement effect occurs in our model because, in order to increase the police force in Area k, the law enforcement agency has to reduce it in other areas, which then experience an increase in crime rates. This displacement mechanism changes when either the outside option displays congestion effects or there are complementarities in criminal decisions. We evaluate these possibilities in sections 5, and state an important implication of these alternative specifications for the econometric analysis of the effect of police on criminal activity.

As we mentioned before, the literature on criminality defines a hot spot as an area with above-average level of crime relative to the entire space. Using our notation, this means that Area k is defined as a hot spot if $p_k(\mathbf{q})N/S_k > (1/K)\sum_{k\in K}p_k(\mathbf{q})N/S_k$ (i.e., its crime density d_k is above average). Then, assuming some people choose to work in a legal activity (i.e., $p_0(\mathbf{q}) > 0$), we get from Proposition 1 that Area k is a so-called hot spot if and only if

$$A_k/R_k e_k > (1/K)\sum_{k\in K}A_k/R_k e_k.$$

Thus, in our model, whether Area k is a hot spot depends on both the productivity-to-risk ratio of the area A_k/R_k and its density of police e_k. Sherman (1995) writes

> Drake Place was a "hot spot" of crime. It was so hot that the police said they stayed away from it as much as possible, unless they got a call.

The structural characterization of hot spots we offer revises the causality of this expression. The next section evaluates the crime-minimizing and egalitarian police allocations.

Crime-Minimizing and Egalitarian Police Assignments

The last subsection described the behavior of potential offenders given different police assignments. Using this result, we now characterize the optimal distribution of police $\mathbf{q}^* \equiv \left(q_k^*\right)_{k\in K}$ for a public authority whose goal is to minimize the overall rate of criminal activity. We then compare the optimal policy with one that targets hot spots until they disappear.

Proposition 2. **(Efficient Allocation).** Let $M > \underline{M}$. Then, for each $k \in K$,

[8] It can be easily shown that Q is a convex set.

$$q_k^* = \sqrt{\theta_k} / \sum_{k \in K} \sqrt{\theta_k} \quad \text{and} \quad p_k^* = \sqrt{\theta_k} \sum_{k \in K} \sqrt{\theta_k}.$$

REMARK. $\underline{M} \equiv \left(\sum_{k \in K} S_k \sqrt{A_k / R_k} \right)^2 / Nf$ is the minimum mass of police such that some people opt not to commit a crime (i.e., $p_0(\mathbf{q}) > 0$). We assume $M > \underline{M}$ as, otherwise, the problem of the enforcement agency is trivial. As in Proposition 1, we define $\theta_k \equiv (S_k)^2 A_k / R_k MNf$, for all $k \in K$.

Proposition 2 indicates that the optimal amount of police in each area q_k^* depends on both the productivity-to-risk ratio A_k/R_k and the size of the area S_k. Specifically, at equilibrium, the ratio of police densities of Area k to Area l is given by

$$e_k^* / e_l^* = \sqrt{A_k / R_k} / \sqrt{A_l / R_l}.$$

Thus, the optimal police allocation q_k^* indeed involves placing higher efforts in potentially high crime locations. However, this proposition also implies that the extra efforts in potentially more dangerous areas do not fully offset the impact of their initial structural differences. Therefore, areas that are a priori more attractive to offenders remain so in the crime-minimizing police allocation. In other words, our model contains hot spots as an equilibrium outcome. As we described in section 3, Area k is defined as a hot spot if it has an above-average crime density d_k^*. We next formalize this idea.

Hot Spots

Let $M > \underline{M}$. Area k is a hot spot at the crime-minimizing equilibrium if and only if

$$\sqrt{A_k / R_k} > (1/K) \sum_{k \in K} \sqrt{A_k / R_k}.$$

We next examine the case in which the goal of the enforcement agency is to obtain an even distribution of criminal activity across all areas (i.e., $d_k^{**} = d_l^{**}$ for all $k, l \in K$). To this end, we assume that the available mass of police M is large enough to induce some people to choose the outside option at the egalitarian allocation. This result holds for all $M > \underline{M}' \equiv \left(\sum_{k \in K} S_k \sum_{k \in K} S_k A_k / R_k \right) / Nf$. Under this assumption, we obtain the following allocation, for each $k \in K$,

$$q_k^{**} = (\theta_k / S_k) / \sum_{k \in K} (\theta_k / S_k).$$

Comparing the egalitarian policy with the crime-minimizing allocation strategy, we obtain

$$e_k^{**} / e_l^{**} = (A_k / R_k) / (A_l / R_l) > \sqrt{(A_k / R_k)} / \sqrt{(A_l / R_l)} = e_k^* / e_l^*,$$

whenever $(A_k / R_k) > (A_l / R_l)$. This inequality means that the ratio of police density of Area k to Area l is larger with the egalitarian policy e_k^{**} / e_l^{**} than with the efficient policy e_k^* / e_l^*. That is, the egalitarian approach targets areas that are a priori more attractive to offenders more intensively than does a public authority who aims to minimize overall crime levels. This leads to our next proposition.

PROPOSITION 3. **(Opportunity Cost of Egalitarian Policy).** *Let* $M > \underline{M}'$. The opportunity cost of equity in terms of overall crime levels is given by

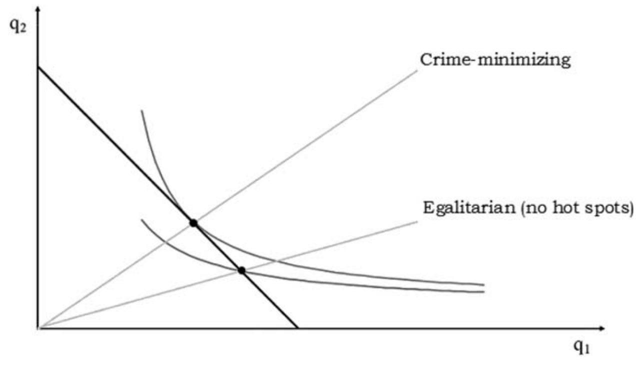

Figure 1. Crime-Minimizing Versus Egalitarian Police Allocation

$$p_0^* - p_0^{**} = (1/MNf)\sum_{k<l} S_k S_l \left(\sqrt{A_k/R_k} - \sqrt{A_l/R_l}\right)^2.$$

Proposition 3 indicates that the opportunity cost (in terms of criminal activity) of the egalitarian policy strategy increases with the variability of the productivity-to-risk ratio A_k/R_k across locations. Thus, while our results suggest that some hot spot policing can indeed reduce the criminal activity, they also suggest that this strategy should be carefully implemented as its extreme implementation implies a higher overall crime rate (i.e., $p_0^* \geq p_0^{**}$).

The next example illustrates our results so far and anticipates the analysis in the next section.

Example 1. Let $K = \{1, 2\}$, $A_1 = 0.8$, $A_2 = 0.4$, $R_1 = 0.1$, $R_2 = 0.2$, $S_1 = S_2 = 1$, $f = 1$ and $NM > 20$. Thus, Area 1 is both more productive ($A_1 = 0.8 > A_2 = 0.4$) and less risky than Area 2 ($R_1 = 0.1 < R_2 = 0.2$). Furthermore,

$$\mathcal{U}(1, p_1, q_1) = 0.8/Np_1 - 0.1Mq_1 \quad \text{and} \quad \mathcal{U}(2, p_2, q_2) = 0.4/Np_2 - 0.2Mq_2.$$

Given that $MN > 20$, by Proposition 1, the crime-minimizing policy solves

$$\max_{q_1, q_2}\{1 - (\theta_1/q_1 + \theta_2/q_2) : 0 \leq q_1 \leq 1, 0 \leq q_2 \leq 1, q_1 + q_2 = 1\} \tag{1}$$

with $\theta_1 = 8/MN$ and $\theta_2 = 2/MN$. Figure 1 exhibits a graphical representation of this result. In this figure, the constraint set is denoted by the bold line. The two curves can be thought of as different indifference curves: each displays combinations of police assignments q_1 and q_2 that induce the same level of criminal activity and higher indifference curves are associated with lower crime levels.

We define a crime-minimizing allocation as one that occurs whenever the marginal efficacy of police resources is the same across areas. After a simple calculation, we obtain that this holds whenever

$$q_2 = \sqrt{\theta_2/\theta_1}\, q_1.$$

Using the fact that $q_1^* + q_2^* = 1$, we get $q_1^* = 2/3$ and $q_2^* = 1/3$. This police assignment corresponds to the upper-left intersection in Figure 1. Under this allocation get $p_1^* = 12/NM > 6/NM = p_2^*$, meaning that Area 1 is a hot spot at equilibrium.

In contrast to the crime-minimizing allocation strategy, the egalitarian policy satisfies the following condition

$$q_2 = (\theta_2/\theta_1)q_1.$$

Using the constraint set, we obtain $q_1^{**}=4/5$ and $q_2^{**}=1/5$. This police assignment corresponds to the lower-right intersection in Figure 1. Note that these two intersections coincide if only if $\theta_1=\theta_2$. Under the egalitarian allocation, $p_1^{**}=p_2^{**}=10/NM$. Thus, by construction, there are no remaining hot spots. However, as Figure 1 shows, the egalitarian allocation is on a lower indifference curve. The opportunity cost of this policy in terms of crime level is $2/NM$.

Note that, if we increase either the penalty in case of being caught f or the amount of police M, then both θ_1 and θ_2 decrease by the same percentage. Thus, while in these two scenarios the police allocation remains the same, the indifference curves get re-leveled with the induced criminal activity curves shifting downward. Alternatively, if we reduce θ_1 by reducing productivity A_1 and/or increasing riskiness R_1, then the criminal activity shifts downward by $1/q_1^*$ [this follows by applying the envelope theorem to Expression 1]. This change flattens the indifference curves, so that now both the crime-minimizing and the egalitarian allocations entail a lower q_1 and a higher q_2. This means that structural changes in Area 1 have beneficial spillover effects on crime levels in Area 2 via subsequent police reallocation. The discussion in the next section elaborates on this argument. ■

4. Modifying the Attributes of the Areas

This section extends our model to consider an enforcement agency that aims to change the characteristics of places that give rise to criminal opportunities while sustaining an optimal police allocation. Specifically, we are interested in two questions: (i) What is the effect of changing the attributes of an area on the overall rate of criminal activity? and (ii) What is the impact on the criminal level of the areas not directly benefited by such policy?

The question of how to best fight crime has received both academic and practical consideration. For example, Braga and Weisburd (2010, p. 185) state that the aim of place-based policy strategies should go beyond hot spot policing. In their own words,

> Since crime hot spots generate a bulk of urban crime problems, it seems commonsensical to address the conditions and situations that give rise to the criminal opportunities that sustain high-activity crime places.

Similarly, public authorities have instrumented a number of area changes to increase apprehension risk R_k or decrease productivity potential A_k. These measures include, for instance, improving the lighting in dark areas or inking store merchandise.

To illustrate such initiatives, notice that by Proposition 1 (and assuming $M > \underline{M}$) the problem of the enforcement agency regarding the allocation of police resources $\mathbf{q}$ is as follows

$$\max_{\mathbf{q}} \{p_0 = 1 - \textstyle\sum_{k\in K} \theta_k/q_k : \mathbf{q} \in \Delta^K\}. \tag{2}$$

Note that lowering the productivity-to-risk ratio in Area k (A_k/R_k) is similar to decreasing $\theta_k \equiv (S_k)^2 A_k/R_k MNf$. By applying the envelope theorem on Equation 2, we then obtain

$$\partial p_0^*/\partial(-\theta_k)=1/q_k^*.$$

Thus, slightly reducing θ_k (e.g., via lowering the productivity-to-risk ratio A_k/R_k) increases the fraction of people who opt not to commit a crime p_0^* by $1/q_k^*$. We next elaborate on the mechanism by which this change happens.

Specifically, there are two forces behind the last result that reinforce each other. First, the target Area k becomes less attractive to potential offenders and, therefore, its criminal activity p_k^* naturally diminishes (i.e., $\partial p_k^*/\partial(-\theta_k)=-1/2<0$). Second, using Proposition 2 we get that, for each $l \neq k$,

$$\partial q_l^*/\partial(-\theta_k)=(1/2)\left(q_l^*\right)^2/\sqrt{\theta_l\theta_k}>0 \quad \text{and} \quad \partial p_l^*/\partial(-\theta_k)=-(1/2)p_l^*/p_k^*<0.$$

The first derivative is positive, implying that the police force is optimally reallocated from Area k to all the others. The second derivative is negative, which implies that the criminal activity in those other areas diminishes as well. We conclude by saying that structural changes in a certain area have beneficial spillover effects on all other locations via subsequent optimal police reallocations.

5. Extensions of the Model

In this section, we consider three natural extensions of our initial framework. While the first two modify the way in which we model the outside option, the last one changes the nature of the interaction effects among potential offenders.

Outside Option and the Labor Market

In our model, the expected payoff of the outside option (not to commit a crime) $\mathcal{U}(0)$ is assumed to be 0. This restriction simplifies our exposition without changing the two main implications, namely, the nature of the displacement of criminal activity and the characterization of hot spots. Nevertheless, it impacts both the effectiveness of the public authority in reducing the overall crime rate and the optimal police allocation. To formalize this effect, we extend Proposition 2 (Efficient Allocation) with $\mathcal{U}(0)\equiv c$, so that c measures, for instance, the benefits from working in a legal job. This specification leads to the next result.

PROPOSITION 4. Let $M>\underline{M}/\left(1+c\sum_{k\in K}\gamma_k\right)$ with $\gamma_k=S_k/R_kMf$. In addition, let $\underline{M}$, q_k^*, and p_k^* be defined as in Proposition 2. Then, for each $k\in K$,

$$q_k^{***}=q_k^*\left(1+c\textstyle\sum_{k\in K}\gamma_k\right)-c\gamma_k \quad \text{and} \quad p_k^{***}=p_k^*/\left(1+c\textstyle\sum_{k\in K}\gamma_k\right).$$

When the outside option is interpreted as the possibility to work in a legal activity, the number of people who choose not to commit a crime p_0N comprises the labor supply in the economy. Thus, an increase in c could correspond, for instance, to a decrease in the unemployment rate or an increase in the minimum wage. Proposition 4 says that higher benefits from working in a legal job c facilitate the condition under which the fraction of people who decide not to commit a crime is strictly positive (i.e., $p_0(\mathbf{q})>0$). This happens because now the minimum amount of police that ensures that some people will not commit a crime is $\underline{M}/\left(1+c\sum_{k\in K}\gamma_k\right)$, which is strictly less than $\underline{M}$. Furthermore, as expected, improvements in the labor market reduce the level of criminality p_k^{***} in all locations because $p_k^{***} < p_k^*$ for all k.[9]

Congestion Effects in the Outside Option

The model described in section 2 rules out the possibility of congestion effects in the outside option. However, we can imagine a simple mechanism by which the opposite is true. For instance, when the number of people searching for a legal job increases, salaries may be pushed down or unemployment increased, thereby making this outside option less attractive. This possibility reduces the effectiveness of the police force to fight crimes.

In this section, we incorporate congestion effects in the outside option by assuming $\mathcal{U}(0,p_0) \equiv A_0/d_0$, where A_0 captures the attractiveness of the outside option (e.g., the expected benefits from a legal job), and $d_0 = p_0 N$ indicates the number of persons who opt not to commit a crime. That is, we incorporate congestion effects in the outside option by assuming that the overall payoff of deciding not to commit a crime $\mathcal{U}(0,p_0)$ decreases with the fraction of people who choose to do so (p_0). Under this specification, the model does not have a closed-form solution either for people's choices conditional on police assignments or for the optimal police allocation. Nevertheless, it still delivers relevant information regarding both the displacement mechanism and the characterization of hot spots at the optimal allocation of police resources. We start by describing the implications of congestion effects on the displacement mechanism.

Proposition 5. Let $\mathcal{U}(0,p_0) \equiv A_0/d_0$ and $Q \equiv \left\{\mathbf{q} \in \Delta^K : q_k > 0, k \in K\right\}$. For all $\mathbf{q} \in Q$, all $k \in K$ and all $m \in K$ with $m \neq k$, we get

$$\partial p_k(\mathbf{q})/\partial q_k \leq 0 \quad \text{and} \quad \partial p_m(\mathbf{q})/\partial q_k \geq 0.$$

Proposition 5 states that increasing police resources q_k in Area k reduces its criminal activity $p_k(\mathbf{q})$ (first derivative), but it also increments the criminal level $p_m(\mathbf{q})$ in all other locations (second derivative). The reason is as follows: When q_k increases, location k becomes less attractive to potential offenders, and this pushes some criminals to the outside option. When the value of this outside option $\mathcal{U}(0,p_0)$ is independent of the number of people who decide not to commit a crime, there are no further consequences. However, when the outside option displays congestion effects, the value of not to commit a crime decreases, incentivizing people to commit crimes in other locations. This has the undesired effect of shifting $p_m(\mathbf{q})$ up in all other areas. We next describe an econometric challenge raised by Proposition 5.

Estimates of Crime-Reducing Effects of Police

Academics have long studied the relationship between the scale of policing and the level of criminal activity by using panel data. The first few studies on this issue did not find evidence of a strong causal effect of police on crimes. As Levitt and Miles (2007) explain, one of the reasons behind such disappointing results is that early studies did not take into account an endogeneity bias. Namely, jurisdictions with higher crime rates react by hiring more police, and this response

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[9] See, for instance, the empirical evidence reported by Imrohoroglu, Merlo, and Rupert (2004).

induces a positive cross-sectional correlation between police and crimes. Marvell and Moody (1996) and Levitt (1997) address this difficulty using an approach based on Granger causality, Lazzati (2015) proposes a partial identification approach that relies on the use of police resources as monotone instrumental variables, and Worrall and Kovandzic (2010) propose an instrumental variables approach using two types of federal law enforcement grants as instruments.[10]

Proposition 5 poses another identification challenge. Under congestion effects in the outside option, the crime rate in each area depends not only on the police resources assigned to that location q_k but also on the whole vector of police allocation $\mathbf{q}$. That is, if congestion effects prevail, then any study that uses panel data to evaluate the effect of police on crimes should implement a simultaneous equations approach.

The next result shows that some hot spots still remain at the optimal police allocation under the presence of congestion effects in the outside option. It also states that whether an area is a hot spot depends on its productivity-to-risk ratio (A_k/R_k) in the same way as when $\mathcal{U}(0) \equiv 0$.

Proposition 6. Let $\mathcal{U}(0, p_0) \equiv A_0/d_0$. Area k is a hot spot at the crime-minimizing equilibrium if and only if

$$\sqrt{A_k/R_k} > (1/K)\textstyle\sum_{k\in K}\sqrt{A_k/R_k}.$$

This proposition corroborates that equilibrium hot spots are a robust feature of our model.

Complementarities in Criminal Activity

In the model described in section 2, the game induced in the second stage displays negative interaction effects among potential offenders. We next evaluate the consequences of an alternative specification.

The overall expected utility of an offender in location k is given by

$$\mathcal{U}(k, p_k, q_k) = \mathcal{Y}(k, p_k, q_k) - \mathcal{P}(k, p_k, q_k)f.$$

Differentiating this expression with respect to p_k, we obtain

$$\partial\mathcal{U}(k, p_k, q_k)/\partial p_k = \partial\mathcal{Y}(k, p_k, q_k)/\partial p_k - (\partial\mathcal{P}(k, p_k, q_k)/\partial p_k)f. \tag{3}$$

We expect both derivatives on the right-hand side of (3) to be weakly negative. Specifically, we expect congestion effects in criminal activity (i.e., $\partial\mathcal{Y}(k, p_k, q_k)/\partial p_k \le 0$) as the higher the fraction of people who commit a crime in the area p_k, the lower the piece of the pie for each offender. Congestion effects in costs (i.e., $\partial\mathcal{P}(k, p_k, q_k)/\partial p_k \le 0$) are also expected as one police officer cannot be at two different places at the same time. Therefore, the higher the number of criminals in a given area, the lower the probability that any one of them is apprehended (Sah 1991; Freeman, Grogger, and Sonstelie 1996). Thus, the sign of the total effect will depend on the relative size of these two forces. That is, for each $k \in K$,

$$\partial\mathcal{U}(k, p_k, q_k)/\partial p_k \ge (\le) 0 \quad \text{if and only if}$$

[10] See also McCrary and Chalfin (2012).

$$\partial\mathcal{Y}(k,p_k,q_k)/\partial p_k \geq (\leq)(\partial\mathcal{P}(k,p_k,q_k)/\partial p_k)f.$$

In the model described in section 2, the second term dominates the first one (i.e., $\partial\mathcal{U}(k,p_k,q_k)/\partial p_k < 0$), thereby inducing a congestion game among potential offenders. The reason is that, while the first term takes a strictly negative value

$$\partial\mathcal{Y}(k,p_k,q_k)/\partial p_k = -A_k/p_k d_k < 0,$$

the second term takes the value of 0

$$\partial\mathcal{P}(k,p_k,q_k)/\partial p_k = 0.$$

This last result stems from the fact that, in our initial model, overall apprehension $\mathcal{A}(k,p_k,q_k)$ is proportional to p_k. Thus, p_k affects both the numerator and denominator of $\mathcal{P}(k,p_k,q_k)=\mathcal{A}(k,p_k,q_k)/d_k S_k$ in such a way that it cancels out. If we alternatively assume overall apprehension $\mathcal{A}(k,p_k,q_k)$ increases in p_k at a sufficiently large decreasing rate, so that $\partial\mathcal{P}(k,p_k,q_k)/\partial p_k$ is strictly negative and large in absolute value (as in Example 2 below), we get

$$|\partial\mathcal{Y}(k,p_k,q_k)/\partial p_k| < |\partial\mathcal{P}(k,p_k,q_k)/\partial p_k|f$$

and $\partial\mathcal{U}(k,p_k,q_k)/\partial p_k > 0$. In this context, the second-stage game displays strategic complementarities. Such games often display multiple equilibria and involve coordination problems. In our case, all offenders may coordinate in the same option and police allocation choices can easily affect the selected one. The possibility that policy interventions may affect equilibrium selection is well-described by Blume (2006) for a discrimination model. The next example applies this phenomenon to our model of crimes.

Example 2. Let $K=\{1,2\}$, $N=1$, and $M=1$. We further assume that $\mathcal{U}(0)\equiv 0$ and

$$\mathcal{U}(1,p_1,q_1)=1/p_1-(q_1+1/2)/(p_1)^2 \quad \text{and} \quad \mathcal{U}(2,p_2,q_2)=1/p_2-4(q_2+1/2)/(p_2)^2.$$

In this example, for all $q_1=q_2$ and $p_1=p_2$, we get

$$4(q_2+1/2)/(p_2)^2 > (q_1+1/2)/(p_1)^2.$$

Thus, Area 2 has greater apprehension risk than Area 1. In addition,

$$\partial\mathcal{U}(1,p_1,q_1)/\partial p_1 = 1/(p_1)^2(-1+2(q_1+1/2)/p_1) > 0 \text{ and}$$
$$\partial\mathcal{U}(2,p_2,q_2)/\partial p_2 = 1/(p_2)^2(-1+8(q_2+1/2)/p_2) > 0$$

which means that the higher the number of people who commit a crime in a certain location, the higher the incentives for other people to commit a crime there as well. This means that the second stage of the game displays strategic complementarities.

In particular, when $q_1=q_2=1/2$, the second-stage game has two Nash equilibria: $\mathbf{p}(1/2,1/2)\in\{(1,0,0),(0,1,0)\}$. While these two equilibria imply the same level of utility for people (zero), the last one is much riskier for potential offenders. The reason is that the first equilibrium guarantees each person a payoff of zero independently of what other people choose. Alternatively, the second equilibrium gives each offender a payoff of zero if and only if all other people follow the same equilibrium strategy and select to commit a crime in Area 1. Otherwise, the payoff is negative. Thus, choosing not to commit a crime is a dominant strategy and, therefore, it is reasonable to predict that everyone will choose this option.

Let us now assume the public authority assigns all police force to the riskier area, so that $q_1=0$ and $q_2=1$. Although the equilibrium set does not change, the two predictions differ regarding expected payoffs. While the payoff of coordinating not to commit a crime is zero, the payoff of coordinating to commit a crime in Area 1 is 1/2 for each offender. Thus, it may now be reasonable to predict that people will coordinate on the second equilibrium. ■

Whether crime decisions are substitutes or complements is ultimately an empirical question with relevant policy implications. De Paula and Tang (2012) and Aradillas-Lopez and Gandhi (2012) provide theoretical results on identification of signs of interaction effects in games. Their work could be very useful in addressing whether crime decisions are complements or substitutes. The answer to this fundamental question would provide relevant insights for developing further theoretical and empirical research on crime. An alternative approach would consist in developing policies that are robust to those two possibilities.

6. Final Discussion

Crime rates fell sharply in the United States during the 1990s, including both violent and property crimes. In NYC, the fall was so strong that the media often refers to this phenomenon as the New York "miracle." This drop in crimes generated deep debates among crime experts, and hot spot policing appears as one of the most cited explanations.[11] The main argument against focusing police resources on hot spots is that it would simply displace criminal activity from one area to another. We evaluate hot spot policing via a game theoretic approach with a special emphasis on the displacement mechanism. We provide specific conditions under which the displacement effect does and does not take place. We find that areas that are initially more attractive for potential offenders should, indeed, receive more police resources. Nevertheless, in our model, some hot spots still remain at the crime-minimizing police allocation. Thus, further hot spot policing strategies should be carefully studied in terms of ultimate objectives, as they might have the unintentional effect of increasing the overall crime rate. We finally study alternative place-based policies that display attractive properties in terms of geographic spillovers of crime reduction. The mechanism by which the spillovers take place is particularly interesting: By making a target area less attractive to potential offenders, the public authority directly reduces its criminal activity. The spillover effect is due to the subsequent optimal police reallocation from the improved area to the other ones, where the criminal activity diminishes as well.

[11] Levitt (2004) evaluates frequently cited reasons for the crime decline in articles in major newspapers over the 1990s. He presents a list of six factors, which has innovative policing strategies at the top and increased number of police as the least cited factor among the six. While he finds innovative policing strategies do not appear to have played an important role in the drop in crime, he suggests increased number of police may have been an important determinant.

Appendix: Proofs

Proof of Proposition 1. By Sandholm (2001), $\mathbf{p}(\mathbf{q})$ is a Nash equilibrium in the second stage of the game if it satisfies the Kuhn–Tucker conditions for the Lagrangian

$$\mathcal{L}(\mathbf{p},\mathbf{q})=\int_0^{p_0}\mathcal{U}(0)dt+\sum\nolimits_{k\in K}\int_\varepsilon^{p_k}\mathcal{U}(k,t,q_k)dt+\sum\nolimits_{k\in K_0}\varphi_k p_k+\lambda\Big(1-\sum\nolimits_{k\in K_0}p_k\Big).$$

That is, if $(\mathbf{p}(\mathbf{q}),\varphi(\mathbf{q}),\lambda(\mathbf{q}))$ satisfies the following conditions:

$$\begin{gathered}A_kS_k/Np_k(\mathbf{q})-R_kMfq_k/S_k=-\varphi_k(\mathbf{q})+\lambda(\mathbf{q}),\ \text{for all}\ k\in K,\\ 0=-\varphi_0(\mathbf{q})+\lambda(\mathbf{q})\\ \varphi_k(\mathbf{q})\geq 0,p_k(\mathbf{q})\geq 0\quad\text{and}\quad\varphi_k(\mathbf{q})p_k(\mathbf{q})=0,\ \text{for all}\ k\in K_0,\\ \lambda(\mathbf{q})\geq 0\quad\text{and}\quad\Big(1-\sum\nolimits_{k\in K_0}p_k(\mathbf{q})\Big)=0.\end{gathered}$$

It is readily verified that the non-negativity constraints are non-binding, that is, $\varphi_k^*=0$, for all $k\in K$. Thus, the previous conditions reduce to

$$\begin{gathered}A_kS_k/Np_k(\mathbf{q})-R_kMfq_k/S_k=\lambda(\mathbf{q})\ \text{for all}\ k\in K\\ \varphi_0(\mathbf{q})=\lambda(\mathbf{q})\\ \varphi_0(\mathbf{q})\geq 0,p_0(\mathbf{q})\geq 0\quad\text{and}\quad\varphi_0(\mathbf{q})p_0(\mathbf{q})=0\\ p_k(\mathbf{q})\geq 0\quad\text{for all}\quad k\in K\\ \lambda(\mathbf{q})\geq 0\quad\text{and}\quad\sum\nolimits_{k\in K_0}p_k(\mathbf{q})=1.\end{gathered}$$

As a consequence, we need to consider only two cases, namely, $\varphi_0(\mathbf{q})>0$ and $\varphi_0(\mathbf{q})=0$. We first suppose $\varphi_0(\mathbf{q})>0$. Then $p_0(\mathbf{q})=0$ and $p_k(\mathbf{q})=S_kA_k/N(\lambda(\mathbf{q})+R_kq_kMf/S_k)$, for all $k\in K$. However, this is possible if and only if there exists $\lambda(\mathbf{q})>0$, such that $\sum_{k\in K}p_k(\mathbf{q})=1$. Note that$\sum_{k\in K}p_k(\mathbf{q})$ is decreasing in $\lambda(\mathbf{q})$ and $\sum_{k\in K}p_k(\mathbf{q})\rightarrow 0$ as $\lambda(\mathbf{q})\rightarrow\infty$. Thus, by the intermediate value theorem, this condition holds if and only if $\sum_{k\in K}\theta_k/q_k\geq 1$.We now suppose $\varphi_0(\mathbf{q})=0$. This yields the following equation:

$$p_k(\mathbf{q})=(S_k)^2A_k/NR_kq_kMf=\theta_k/q_k,$$

for all $k\in K$. Since $p_0(\mathbf{q})\geq 0$, then $\sum_{k\in K}\theta_k/q_k\leq 1$ with strict inequality if $p_0(\mathbf{q})>0$. Uniqueness follows as the objective function is strictly concave in $\mathbf{p}$ on Δ^{K_0}. ■

Proof of Proposition 2. For M large enough, any crime-minimizing police allocation satisfies the following condition:

$$\mathcal{U}(k,p_k(\mathbf{q}),q_k)=0.$$

Thus, for all $k\in K$,

$$p_k(\mathbf{q})=\theta_k/q_k.$$

The problem of the public authority can then be posed as

$$\max{}_{\mathbf{q}}\Big\{1-\sum\nolimits_{k\in K}\theta_k/q_k\ :\mathbf{q}\in\Delta^K\Big\}.\tag{A1}$$

In Expression A1, the objective function is differentiable and strictly concave for all $\mathbf{q}$ in the interior of Δ^K. Thus, the solution to Expression A1 exists and is unique. Moreover, $\mathbf{q}^*$ is an equilibrium if it satisfies the Kuhn–Tucker conditions for the Lagrangian:

$$\mathcal{L}(\mathbf{q})=1-\sum\nolimits_{k\in K}\theta_k/q_k-\sum\nolimits_{k\in K}\varphi_kq_k-\lambda\Big(\sum\nolimits_{k\in K}q_k-1\Big).$$

In this case, $(\mathbf{q}^*, \varphi^*, \lambda^*)$ satisfies the following conditions:

$$\theta_k/\left(q_k^*\right)^2 - \varphi_k^* = \lambda^*, \text{ for all } k \in K,$$
$$\varphi_k^* \geq 0, q_k^* \geq 0 \quad \text{and} \quad \varphi_k^* q_k^* = 0, \text{ for all } k \in K,$$
$$\lambda^* \geq 0 \quad \text{and} \quad \left(1 - \sum\nolimits_{k \in K} q_k^*\right) = 0.$$

It is readily verified that the non-negativity constraints are non-binding, that is, $\varphi_k^* = 0$ for all $k \in K$. The characterization of $\mathbf{q}^*$ follows through a simple calculation.

To find $\underline{M}$, note that $\sum_{k \in K} \theta_k / q_k^* < 1$ needs to hold for $p_0^* > 0$ to be true. That is,

$$\left(\sum\nolimits_{k \in K} S_k \sqrt{A_k/R_k}\right)^2 / NMf < 1.$$

It follows that $\underline{M} = \left(\sum_{k \in K} S_k \sqrt{A_k/R_k}\right)^2 / Nf$. ■

Proof of Proposition 3. We know that

$$p_0^* = 1 - \left(\sum\nolimits_{k \in K} \sqrt{\theta_k}\right)^2 \quad \text{and} \quad p_0^{**} = 1 - \sum\nolimits_{k \in K} S_k \sum\nolimits_{k \in K} \theta_k / S_k.$$

Then,

$$p_0^* - p_0^{**} = \sum\nolimits_{k \in K} S_k \sum\nolimits_{k \in K} \theta_k / S_k - \left(\sum\nolimits_{k \in K} \sqrt{\theta_k}\right)^2.$$

By expanding the first term and applying the Multinomial Theorem to the second term, the last expression takes the form of

$$\begin{aligned} p_0^* - p_0^{**} &= \sum\nolimits_{k \in K} \theta_k + \sum\nolimits_{k,l \in K, k \neq l} (S_l/S_k)\theta_k - \sum\nolimits_{k \in K} \theta_k - \sum\nolimits_{k,l \in K, k \neq l} \sqrt{\theta_k}\sqrt{\theta_l} \\ &= \sum\nolimits_{k,l \in K, k \neq l} (S_l/S_k)\theta_k - \sum\nolimits_{k,l \in K, k \neq l} \sqrt{\theta_k}\sqrt{\theta_l}. \end{aligned}$$

Since $\theta_k \equiv (S_k)^2 A_k / R_k MNf$, then

$$\begin{aligned} p_0^* - p_0^{**} &= (1/MNf) \sum\nolimits_{k,l \in K, k \neq l} S_k S_l \left[(A_k/R_k) - \sqrt{A_k/R_k}\sqrt{A_l/R_l}\right] \\ &= (1/MNf) \sum\nolimits_{k,l \in K, k < l} S_k S_l \left(\sqrt{A_k/R_k} - \sqrt{A_l/R_l}\right)^2. \end{aligned}$$

which completes the proof. ■

Proof of Proposition 4. The proof of this result is very similar to the proofs of Propositions 1 and 2, thus we omit it. ■

Proof of Proposition 5. Under this specification, for all $\mathbf{q} \in Q$, people will redistribute across options till the utility obtained in each of them is the same. Therefore, $p_0 = S_0 A_0 / Nu(\mathbf{q})$ and, for each $k \in K$, we have

$$p_k(\mathbf{q}) = S_k A_k / N[u(\mathbf{q}) + R_k q_k f M / S_k] \tag{A2}$$

where $u(\mathbf{q})$ is the constant that solves $\sum_{k \in K_0} p_k(\mathbf{q}) = 1$. Differentiating Equation A2 $p_k(\mathbf{q})$ and $p_m(\mathbf{q})$ with respect to q_k we get

$$\partial p_k(\mathbf{q})/\partial q_k = -\left\{S_k A_k / N[u(\mathbf{q}) + R_k q_k f M/S_k]^2\right\}[\partial u(\mathbf{q})/\partial q_k + R_k f M/S_k]$$
$$\partial p_m(\mathbf{q})/\partial q_k = -\left\{S_k A_k / N[u(\mathbf{q}) + R_k q_k f M/S_k]^2\right\} \partial u(\mathbf{q})/\partial q_k.$$

By the Implicit Function Theorem applied to $\sum_{k \in K_0} p_k(\mathbf{q})=1$, we get

$$\partial u(\mathbf{q})/\partial q_k = -\frac{\left\{S_k A_k / N[u(\mathbf{q})+R_k q_k fM/S_k]^2\right\} R_k fM/S_k}{\sum_{k \in K}\left\{S_k A_k / N[u(\mathbf{q})+R_k q_k fM/S_k]^2\right\} + S_0 A_0 / Nu(\mathbf{q})^2} \leq 0.$$

Substituting the last expression in the previous two, we obtain that $\partial p_k(\mathbf{q})/\partial q_k \leq 0$ and $\partial p_m(\mathbf{q})/\partial q_k \geq 0$. ■

Proof of Proposition 6. At the second stage equilibrium, $p_0 = S_0 A_0 / Nu(\mathbf{q})$. Thus, maximizing p_0 is the same as selecting the vector $\mathbf{q}$ that minimizes $u(\mathbf{q})$. It follows that any optimal $\mathbf{q}^*$ must satisfy, for all $k, m \in K$,

$$\partial u(\mathbf{q})/\partial q_k = \partial u(\mathbf{q})/\partial q_k.$$

Using intermediate results from the proof of Proposition 5 we get that, for all $k, m \in K$,

$$\sqrt{A_k R_k}/[u(\mathbf{q})+R_k q_k fM/S_k] = \sqrt{A_m R_m}/[u(\mathbf{q})+R_m q_m fM/S_m] = H.$$

Thus, for each $k \in K$, we have

$$Np_k(\mathbf{q})/S_k = d_k = \sqrt{A_m/R_m}H$$

and the result follows immediately. ■

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