A Dynamic Model of Firm Valuation

Natalia Lazzati*

University of California Santa Cruz

Amilcar A. Menichini

Graduate School of Business and Public Policy, Naval Postgraduate School

Abstract

We propose a dynamic version of the dividend discount model, solve it in closed form, and assess its empirical validity. The valuation method is tractable and can be easily implemented. We find that our model produces equity value forecasts that are very close to market prices, and explains a large proportion of the observed variation in share prices. Moreover, we show that a simple portfolio strategy based on the difference between market and estimated values earns considerably positive returns. These returns cannot be simply explained either by the Fama-French three-factor model (even after adding a momentum factor) or the Fama-French five-factor model.

Keywords: firm valuation, dividend discount model, Gordon Growth Model, dynamic programming

JEL Classifications: G31, G32

Corresponding author: Department of Economics, University of California Santa Cruz, 1156 High St., Santa Cruz, CA 95064; Phone: (831) 459-5697; Fax: (831) 459-5697; E-mail: nlazzati@ucsc.edu.

1. Introduction

We derive a dynamic model of the firm in closed form and show that it can be used for actual firm valuation. To test its empirical validity, we price firms in COMPUSTAT in the period 1980–2015 and evaluate the results from three perspectives. First, we find that the model produces consistent forecasts of stock prices in the sense that model predicted values are very close to the actual market values, on average. Second, we show that the model explains a large fraction (around 92%) of the variation in current market prices. Third, we find that the temporary or short-run deviations between market prices and model estimates can be economically exploited. Overall, we believe these results suggest our model is a valuable pricing tool that may enhance current approaches to firm valuation.

We use dynamic programming to develop a model of the firm in which the latter chooses investment, labor, and how to finance its assets in every period. While this type of model has been used extensively in corporate finance to explain firm behavior, we introduce three fundamental features that make our model particularly useful for asset pricing purposes. First, we invoke the two-fund separation principle, which shows that, as long as we discount future cash flows with an appropriately risk-adjusted discount rate, we do not need to specify shareholders' utility functions in the valuation process beyond the assumptions discussed in Cass and Stiglitz (1970). Second, we allow the firm to grow in the long run, which could be interpreted as the firm facing a market size that increases over time, independent of the short-term fluctuations generated by the business cycle. Third, we introduce risky debt to our model and find an analytic solution, which, to the best of our knowledge, is novel among existing discrete-time dynamic investment models of the firm. In particular, debt in our model is protected by a positive net-worth covenant and, in the event of bankruptcy, the firm pays the bankruptcy costs, is reorganized under Chapter 11 of the U.S. Bankruptcy Code, and continues its operations. This modeling strategy generates a debt behavior that is in line with the empirical evidence. For instance, survey results from Graham and Harvey (2001) suggest that most firms have a target leverage. Consistently, the firm in our model chooses debt in every period following a target leverage that depends on its own characteristics.

As mentioned above, an important advantage of our model regarding valuation is that we solve it analytically. Closed-form equations are strongly preferred to numerical approximations because the former yield extremely accurate values at very low computing time. Indeed, a usual problem with the numerical solution of dynamic programming models is the so-called Bellman's curse of dimensionality. This problem arises from the discretization of continuous state and decision variables, since the computer time and space needed increase exponentially with the number of points in the discretization (Rust, 1997, 2008). Thus, more accurate firm valuations imply necessarily exponentially longer periods of computing time. In addition, explicit solutions allow the user to estimate model parameters with ease.

After presenting the firm model, we evaluate its actual pricing performance. We first compute the ratio of the actual market prices to the values predicted by our model and find that its mean value is around 1. This result means that our model yields equity value estimates that are, on average, very close to market values. We regress the market value of equity on the value estimated by our model and find that we cannot reject the null hypothesis that the intercept is 0 and the slope is 1, which suggests that our model produces unbiased estimates of stock prices. We also show that our model predictions can explain a large fraction of the observed variability of the stock prices. Specifically, the R^2 coefficient is around 92%. This outcome turns out to be better than the results reported by related papers (described below), and implies a strong linkage between model forecasts and market values over time. We implement this last regression which includes fixed effects at the industry level and find that, jointly they are not statistically different from zero with a p-value close to 0.67. Moreover, we run an analogous regression at the firm level and find that we cannot reject the null hypothesis that the intercept is 0 and the slope is 1 for about 71% of the firms at the 5% significance level and for 83% of the firms at the 1% significance level.

While we show that our estimates are very close to market values, we also find temporary deviations between stock prices and model estimates. We implement a simple spread strategy to test whether we can take economic advantage of these deviations. The spread strategy consists in ranking firms based on their ratios of market prices to model estimates, then forming quintiles based on those ratios, and finally buying the firms in the lowest quintile and selling the firms in the highest quintile. Our results show that this strategy earns, on average, around 20%, 34%, 41%, 44%, and 48% returns after one, two, three, four, and five years of portfolio formation, respectively. We also study the returns of the spread strategy in the context of the Fama-French three-factor model (Fama and French, 1993) and find a positive alpha that is statistically different from zero. This means that the positive returns of our spread strategy cannot be simply accounted for by these factors. We obtain similar results when we add a momentum factor to the Fama-French three-factor model and when we employ the Fama-French five-factor model (Fama and French, 2015). As benchmark, we calculate the returns of the spread strategy based on three well-known financial ratios: market-to-book, price-earnings, and price-dividend. Based on our model, we find that the spread strategy estimates consistently outperforms the ones using these ratios.

To do the previous analyses, we estimate the structural parameters at the firm level for all firms in our data set. In all our estimations, we perform a forward-looking exercise in the sense that we use data available prior to the valuation period to make out-of-sample predictions. Doing so is important because this procedure replicates the situation a user would face when performing actual valuation.

This paper values public firms in COMPUSTAT. However, our valuation method could also be implemented with firms for which market prices do not exist, as long as financial statements are available for parameter estimation. These cases include,

among others, private companies such as Koch Industries and Cargill, initial public offerings such as Facebook in 2012 and Alibaba Group Holding in 2014, and firms' new investment projects.

1.1. Literature review

Our paper contributes to two strands of literature in finance, namely, discrete-time dynamic investment models of the firm and firm valuation models.

Several papers in corporate finance use different dynamic programming models of the firm to explain firm choices (for a comprehensive review of this large litera-ture, see Strebulaev and Whited, 2012). For instance, Moyen (2004, 2007), Hennessy and Whited (2005, 2007), Hennessy, Levy and Whited (2007), Tserlukevich (2008), Riddick and Whited (2009), and Hennessy, Livdan and Miranda (2010) use dynamic programming models to rationalize a large number of stylized facts about firm be-havior. We show that, after introducing some new features, this type of model can also be used successfully for firm valuation. While this possibility was suggested by Dixit and Pindyck (1994), we believe our paper is one of the first attempts in this direction. As we mention earlier, a fundamental feature of our model is that we do not need to specify shareholders' utility function in the valuation process (beyond the assumptions described in Cass and Stiglitz, 1970) by assuming the two-fund separa-tion principle holds. Another important feature regarding valuation is the possibility of firm growth in the long run. As documented by Lazzati and Menichini (2015), secular growth can account for more than 30% of the value of the firm. This is of particular importance for certain industries, such as manufacturers of chemical prod-ucts and industrial machinery, and providers of communication services (Jorgenson and Stiroh, 2000). In addition, as we explain above, we contribute to the literature by introducing risky debt in closed form to our model. The fact that we obtain analytic solutions is very important for the accuracy of the model predictions.

While our tests show that our model produces successful valuation results, we also find that it performs likewise in some regards and better in some oth-ers, when compared to other valuation models. Kaplan and Ruback (1995) and Copeland, Weston and Shastri (2005), who implement the discounted cash flow model (DCFM), show that their implementations produce value forecasts that are, as in our case, roughly equal to market prices. With respect to the explanation of the variation in current market prices, our model seems to outperform the results in some related studies. Bernard (1995) compares the ability of the dividend discount model (DDM) and the residual income model (RIM) to explain the observed variation in stock prices. He finds that the RIM explains 68% of the variability in market values and outperforms the DDM, which can only explain 29% of such variation.[1] In a similar

---

[1] The DDM, the DCFM, and the RIM are theoretically equivalent, but they differ with respect to the information used in their practical implementation. The DDM uses the future stream of expected dividend

study, Frankel and Lee (1998) test the RIM empirically and find that the model estimates explain around 67% of the variability in current stock prices. More recently, Spiegel and Tookes (2013) use a dynamic model of oligopolistic competition to perform cross-sectional firm valuation and find that their model explains around 43% of the variation in market values. While in all these papers the samples differ in terms of firm composition and periods, we find that our model can explain a higher fraction of the variability of the stock prices (around 92%).

The paper is organized as follows. In Section 2, we derive a dynamic version of the DDM in closed form and explain its main parts. The empirical evaluation of the performance of our model is in Section 3. Section 4 concludes. Appendix A contains the proofs and Appendix B presents additional robustness checks regarding the spread strategy returns.

2. A dynamic DDM

We derive a dynamic version of the standard DDM in closed form. We solve the problem of the firm (i.e., share price maximization) using discrete-time, infinite-horizon, stochastic dynamic programming. We obtain the solution within the context of the adjusted present value (APV) method introduced by Myers (1974), which has been used extensively with dynamic models of the firm (e.g., Leland, 1994; Goldstein, Ju and Leland, 2001; Strebulaev, 2007). All parameters in our model are firm specific. Thus, to simplify notation, we avoid using firm subscripts in the specification of the model. (We will add firm-specific subindices later, in Subsection 3.2, when we study the performance of the model.)

2.1. The problem of the firm

The life horizon of the firm is infinite, which implies that shareholders believe it will run forever. The CEO makes investment, labor, and financing decisions at the end of every period (e.g., month, quarter, or year) such that the market value of equity is maximized. (In this paper, we write a tilde on X (i.e., $\widetilde{X}$) to indicate that the variable is growing over time.) Variable $\widetilde{K}_t$ represents the book value of assets while variable $\widetilde{L}_t$ indicates the amount of labor used by the firm in period t. In each period, installed capital depreciates at constant rate $\delta > 0$ and firm investment is endogenously determined by the capital decision as $\widetilde{I}_{t+1} = \widetilde{K}_{t+1} - (1 - \delta)\widetilde{K}_t$. The debt of the firm in period t, $\widetilde{D}_t$, matures in one period and is rolled over at the end of every period. We assume debt is issued at par by letting the coupon rate c_B equal the market cost of debt r_B. In turn, this implies that book value of debt $\widetilde{D}_t$ equals the market value of debt $\widetilde{B}_t$. The amount of outstanding debt $\widetilde{B}_t$ will increase or decrease

---

payments to shareholders. The DCFM is based on some measure of future cash flows, such as free cash flows. Finally, the RIM uses accounting data (e.g., current and future book value of equity and earnings).

over time, according to financing decisions. We let debt be risky, which implies that the firm goes into bankruptcy when profits are sufficiently low. Following Brennan and Schwartz (1984), we assume the debt contract includes a protective covenant consisting in a positive net-worth restriction. In the event of bankruptcy, the firm has to pay bankruptcy costs $\xi \widetilde{K}_t$ (with $\xi > 0$), such as lawyer fees and other costs of the bankruptcy proceedings. Furthermore, we assume the bankrupt firm is reorganized and continues its operations after filing for protection under Chapter 11 of the U.S. Bankruptcy Code. This assumption is consistent with the empirical evidence showing that the majority of firms emerge from Chapter 11, and only a few firms are actually liquidated under Chapter 7 (see, e.g., Morse and Shaw, 1988; Gilson, John and Lang, 1990; Weiss, 1990). Finally, as in Hennessy and Whited (2007), we let bankruptcy costs be proportional to the level of assets.

We introduce randomness into the model through the profit shock z_t. Following Fama and French (2000) and Zhang (2005), we let profits be mean-reverting by assuming that random shocks follow an AR(1) process in logs

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \sigma x_t, \tag{1}$$

where $\rho \in (0, 1)$ is the autoregressive parameter that defines the persistence of profit shocks. In other words, a high ρ makes periods of high profit innovations (e.g., economic expansions) and low profit shocks (e.g., recessions) last more on average, and vice versa. The innovation term x_t is an independent and identically distributed (*iid*) standard normal random variable and, since we present the pricing model at the firm level (including the estimates of the corresponding parameters), our specification allows for any kind of association among the innovation terms across firms. The innovation term is scaled by constant $\sigma > 0$, which defines the volatility of profits over time. Finally, constant $c > 0$ defines the mean profitability level of the firm and captures efficiency differences across firms in the market due to, for instance, innate technology, management competence, and input quality.[2]

Gross profits in period t are defined by the following function:

$$\widetilde{Q}_t = (1 + g)^{t[1-(\alpha_K + \alpha_L)]} z_t \widetilde{K}_t^{\alpha_K} \widetilde{L}_t^{\alpha_L}, \tag{2}$$

where z_t is the profit shock in period t, $\alpha_K \in (0, 1)$ represents the elasticity of capital, and $\alpha_L \in (0, 1)$ indicates the elasticity of labor (we further assume $\alpha_K + \alpha_L < 1$). Constant g represents the growth rate of the firm and, with factor $(1 + g)^{t[1-(\alpha_K+\alpha_L)]}$, profits, costs and firm size grow proportionately over time. Factor $(1 + g)^{t[1-(\alpha_K+\alpha_L)]}$ allows us to use a standard normalization of growing variables that is required to solve the problem of the firm (see, e.g., Manzano, Perez and Ruiz, 2005). According

---

[2] See Ackerberg, Benkard, Berry and Pakes (2007) for an alternative way to introduce efficiency differences across firms.

to Equation (2), gross profits depend on a Cobb-Douglas production function with decreasing returns to scale in capital and labor inputs.[3]

Every period, the firm pays operating costs $f\widetilde{K}_t$ (with $f > 0$) and labor wages $\omega\widetilde{L}_t$ (with $\omega > 0$), while corporate earnings are taxed at rate $\tau \in (0, 1)$. Therefore, the firm's net profits in period t are

$$\widetilde{N}_t = (\widetilde{Q}_t - f\widetilde{K}_t - \delta\widetilde{K}_t - \omega\widetilde{L}_t - r_B\widetilde{B}_t)(1 - \tau). \tag{3}$$

With all the previous information, we can state the dividend that the firm pays to equity-holders in period t as

$$\widetilde{Y}_t = \widetilde{N}_t - [(\widetilde{K}_{t+1} - \widetilde{K}_t) - (\widetilde{B}_{t+1} - \widetilde{B}_t)] - \Theta\xi\widetilde{K}_t. \tag{4}$$

According to Equation (4), the dividend paid to shareholders in period t equals net profits minus the change in equity and, in the event of bankruptcy, minus the bankruptcy costs. The indicator function Θ equals 1 if the firm goes into bankruptcy. Given the positive net-worth covenant included in the debt contract, the event of bankruptcy occurs when

$$\left(zK^{\alpha_K}L^{\alpha_L} - fK - \delta K - \omega L - r_B\ell K\right)(1 - \tau) + K - \ell K < 0, \tag{5}$$

that is, when the after-shock book value of equity becomes negative.

We let rate r_S represent the market cost of equity and rate r_A denote the market cost of capital. We also assume the secular growth rate is lower than the market cost of capital (i.e., $g < r_A$). Finally, as it is common with other valuation models (e.g., the Black-Scholes formula), we do not introduce transaction or adjustment costs to our model.

Before we characterize the firm problem, we need to convert it into stationary, for which we normalize the growing variables by the gross growth rate: $X_t = \widetilde{X}_t/(1 + g)^t$, with $X_t = \{K_t, L_t, B_t, Q_t, N_t, Y_t\}$. We let E_0 indicate the expectation operator given the information at $t = 0$, (K_0, L_0, B_0, z_0). Then, the problem of the firm is to make optimal capital, labor, and financing decisions, such that the market value of equity is maximized. Using the normalized variables and modifying the payoff function accordingly, the maximized market value of equity can be expressed as

$$S(K_0, L_0, B_0, z_0) = \max_{\{K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}} \left\{0, E_0 \sum_{t=0}^{\infty} \frac{(1 + g)^t}{\prod_{j=0}^{t}(1 + r_{S_j})}Y_t\right\}. \tag{6}$$

Equation (6) says that the stock price is the summation of the maximized expected discounted future dividends of the firm. We solve the firm problem by separating investment (and labor) from financing decisions, as shown by Modigliani and

---

[3] Equation (2) can take on only positive values. However, the model can be easily extended to allow for negative values of gross profits by subtracting a positive constant as a proportion of assets (e.g., $a\widetilde{K}_t$) in Equation (2). We prefer not to include this term as we find that only 0.35% of the observations in our sample has negative gross profits.

Miller (1958). This separation is possible in our dynamic model because debt is fully adjusted in each period of time.

2.2. Model solution

Proposition 1 displays the closed form for the stock price when the firm does not go into bankruptcy. It also shows the probability of bankruptcy and the optimal leverage ratio analytically. (To help streamline the exposition, we solve the problem of the firm—Equation [6]—in Appendix A.)

Proposition 1. The market value of equity is

$$
S\left(K_t^*, L_t^*, B_t^*, z_t\right) = \left[z_t K_t^{*\alpha_K} L_t^{*\alpha_L} - fK_t^* - \delta K_t^* - \omega L_t^* - r_B B_t^*\right](1-\tau)
$$
$$
+ K_t^* - B_t^* + G(z_t), \tag{7}
$$

where the going-concern value is $G_t(z_t) = M(z_t)P^*$. Function $M(z_t)$ is given by

$$
M(z_t) = e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} \left\{ \left(\frac{1+g}{1+r_A}\right) E\left[z_{t+1}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] \right.
$$
$$
\left. + \left(\frac{1+g}{1+r_A}\right)^2 E\left[z_{t+2}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] + \cdots \right\} \tag{8}
$$

with the general term

$$
E\left[z_{t+n}^{1/[1-(\alpha_K+\alpha_L)]} | z_t\right] = \left(c^{\frac{1-\rho^n}{1-\rho}} z_t^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{(1-\rho^{2n})}{(1-\rho^2)} \frac{1}{[1-(\alpha_K+\alpha_L)]}}\right)^{\frac{1}{1-(\alpha_K+\alpha_L)}}, \quad n = 1, 2, \ldots \tag{9}
$$

and variable P^* takes the form

$$
P^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^*\right)(1-\tau) - r_A\Phi_1^*
$$
$$
+ \left(\frac{1+r_A}{1+r_B}\right)(r_B\tau\ell^* - \lambda^*\xi)\Phi_1^* \tag{10}
$$

with

$$
\Phi_1^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau}+f+\delta}\right)^{1-\alpha_L} \left(\frac{\alpha_L}{\omega}\right)^{\alpha_L}\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \tag{11}
$$

and

$$
\Phi_2^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau}+f+\delta}\right)^{\alpha_K} \left(\frac{\alpha_L}{\omega}\right)^{1-\alpha_K}\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}. \tag{12}
$$

The probability of bankruptcy is

$$
\lambda^* = \int_{-\infty}^{x_c^*} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz, \tag{13}
$$

where

$$x_c^* = -\sigma - \sqrt{2\left\{\sigma^2 + \ln\left\{\frac{\left[1 + \frac{1}{r_B(1-\tau)}\right]\xi}{\sqrt{2\pi}\,\tau\,\sigma\,\Phi_1^{*\alpha_K-1}\Phi_2^{*\alpha_L}}\right\}\right\}}.$$ (14)

The optimal book leverage ratio is given by

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c^* - \frac{1}{2}\sigma\right)}\Phi_1^{*\alpha_K-1}\Phi_2^{*\alpha_L} - f - \delta - \omega\frac{\Phi_2^*}{\Phi_1^*}\right](1-\tau) - \xi}{1 + r_B(1-\tau)}.$$ (15)

Finally, the optimal decisions are

$$K_{t+1}^* = (1+g)E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\Phi_1^*,$$

$$L_{t+1}^* = (1+g)E\left[z_{t+1}|z_t\right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}\Phi_2^*, \quad and \quad B_{t+1}^* = \ell^*K_{t+1}^*.$$ (16)

The market value of equity shown in Equation (7) represents an analytic solution of the Gordon Growth Model (Gordon, 1962) in the dynamic and stochastic setting. The first three terms in Equation (7) represent the after-shock book value of equity, while the last term, $G(z_t)$, is the going-concern value. The latter depends on function $M(z_t)$, which captures the effect of the infinite sequence of expected profit shocks. This sequence converges for each given z_t as long as $g < r_A$ and $\rho < 1$ (the proof is in Appendix A). Function $G(z_t)$ also depends on variable P^*, which denotes the dollar return on capital minus the dollar cost of capital at the optimum (including the interest tax shields and bankruptcy costs as financing side effects). The going-concern value shows that, using only information about the current state (i.e., assets, labor, debt, and gross profits), our model solves systematically for the full sequence of expected future dividends (and, thus, does not require the calculation of a terminal value). Our model also projects automatically the value of the real options (such as the option to expand the business, extend the life of current projects, shrink firm size, or even postpone investments) by allowing the firm to optimize decisions over time.[4]

As expected, optimal capital K_{t+1}^* decreases with the market cost of capital r_A, operating costs f, depreciation δ, and labor costs ω. On the contrary, optimal assets increase with the growth rate g, the efficiency parameter c, and the volatility of innovations σ because they increment the expected profitability of capital via Equation (3). The effects of α_K, α_L, and ρ depend on current profit shock z_t, but they are generally positive for standard values of the parameters. The sensitivity of optimal labor L_{t+1}^*, with respect to the characteristics of the firm, is analogous to that

---

[4] In a simpler version of this model, Lazzati and Menichini (2015) show that the value of the real options can easily represent more than 8% of the stock price and is particularly important for certain industries, such as in Oil and Gas Extraction. Thus, the inclusion of managerial flexibility is a key advantage of our model over the static ones.

of optimal assets. Furthermore, all the previous primitive features of the firm have the same directional effects on optimal debt B_{t+1}^*. Finally, the income tax rate τ has a negative effect on optimal assets and labor because the latter become less profitable as the former is higher. It also has a negative effect on optimal debt for the great majority of parameter values, including those used in this paper.

In a survey, Graham and Harvey (2001) provide empirical evidence suggesting that most firms actually follow some form of target leverage. Consistently, our model produces an optimal debt that is a constant proportion of optimal assets, with the factor of proportionality given by ℓ^* in Equation (15). This optimal ratio can be interpreted as the target leverage of the firm. It is readily verified that ℓ^* is strictly less than 1 and bounded below by 0, decreases in nondebt tax deductions (e.g., operating costs f and depreciation δ) and the market cost of debt r_B, and is an increasing function of the income tax rate τ.

Finally, our model also yields a constant probability of bankruptcy, which is in line with the findings of Kisgen (2006, 2009), who suggests that firms aim to maintain their debt within certain credit ratings. Ou (2011) and Vazza and Kraemer (2015) show that each credit rating implies a certain and stable default probability. As expected, this probability increases with the operating costs f, depreciation δ, the market cost of debt r_B, and the income tax rate τ.

3. Testing the performance of the valuation model

We study two fundamental aspects of our dynamic model. First, we analyze the consistency between the model estimates and the market prices. That is, we address how close the predicted values by our model are to the actual market prices. Complementing this analysis, we examine how much of the observed variation in contemporaneous share prices is explained by the model estimates. Second, we investigate the possibility to use our model to economically exploit short-term differences between actual and estimated stock values.

3.1. Sample and estimation procedure

We value firms in the COMPUSTAT database during the period 1980–2015.[5] We construct the sample using two data sources. Historical accounting data are obtained from the COMPUSTAT annual files, while the corresponding stock price data are obtained from the CRSP monthly files.

In all our empirical analyses, we ensure that accounting data are known at the time the stock price is set in the exchange. Thus, we use the share price observed five months later than the fiscal-year-end of the firm. For instance, we match the

---

[5] Because we value COMPUSTAT firms during the period 1980–2015, our sample does not suffer from the survivorship bias problem.

accounting data of a December year-end firm with its closing stock price at the end of May of the following year. Because our objective is to do out-of-sample predictions, we estimate parameter values using the existing historical information up to the year prior to the valuation period. All parameter estimates are firm specific.

We employ the method of moments and least squares to estimate parameters c, ρ, σ, α_K, α_L, f, δ, ω, τ, r_B, and ξ for each firm.[6] We identify parameters f, δ, ω, τ, r_B, and ξ for each firm by taking advantage of the variation over time of the following ratios. The average of the ratio Selling, General, and Administrative Expense (XSGA) to Total Assets (AT) helps us pin down the operating costs parameter (f); the average of the fraction Depreciation and Amortization (DP) to AT is informative about the capital depreciation rate (δ); the average of the ratio Total Staff Expense (XLR) to Number of Employees (EMP) helps us identify the labor wages (ω); and, the average of the fraction Total Income Taxes (TXT) to Pretax Income (PI) helps us pin down the corporate income tax rate (τ). Regarding the financing side, the average of the proportion Total Interest and Related Expense (XINT) to Total Liabilities (LT) is informative about the market cost of debt (r_B), while the average of the fraction LT to AT in Equation (15) helps us identify the costs of bankruptcy (ξ).

Based on the procedure employed by Moyen (2004), we obtain parameters c, ρ, σ, α_K and α_L using the firm's autoregressive profit shock process, $\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \sigma x_t$, and the gross profits function, $\widetilde{Q}_t = (1+g)^{t[1-(\alpha_K+\alpha_L)]} z_t \widetilde{K}_t^{\alpha_K} \widetilde{L}_t^{\alpha_L}$. The COMPUSTAT data items we use with these equations are Gross Profit (GP), AT, and EMP. We regress (log) gross profits on (log) assets and (log) labor, which is informative about the elasticity parameters α_K and α_L.[7] Firms with lower curvature of the production function should have less responsive profits to changes in capital and labor inputs. We employ the residuals from that regression to recover the fundamental characteristics of the driving process of z using the first-order autoregression described above. Accordingly, the intercept from that regression helps us identify the efficiency parameter (c). The coefficient estimate on $\ln(z_{t-1})$ provides information about the persistence parameter (ρ), while the variability of the residuals is informative about the volatility of profit shocks (σ).

We use the Fama-French three-factor model to calculate the market cost of equity, r_S, for each firm. We obtain the three monthly factors (i.e., $R_M - R_f$, *SMB*, and *HML*) from Ken French's Web site and monthly stock price data from CRSP. We use the 10-year T-Bond yields as the risk-free interest rate. Lee, Myers and Swaminathan (1999) and Francis, Olsson and Oswald (2000) also use this factor

---

[6] In a simpler but related version of our model, Lazzati and Menichini (2015) perform a sensitivity analysis of the stock price with respect to model parameters and find that firm primitives such as the efficiency parameter (c), the persistence of profit shocks (ρ), the curvature of the production function with respect to capital (α_K), and the market cost of capital (r_A) are the ones with the largest impact on the stock price. This type of analysis could help practitioners improve the implementation of the model.

[7] This is a standard procedure in the empirical economics literature (see, e.g., Balistreri, McDaniel and Wong, 2003; Fox and Smeets, 2011; Young, 2013).

model to calculate discount rates. However, while they implement it at the industry level, we do so at the firm level. We then use r_S and r_B to calculate the market cost of capital, r_A, by computing the before-tax, weighted average cost of capital, as required by the APV method. Finally, following Lakonishok, Shleifer and Vishny (1994), we calculate the growth rate, g, for each firm by averaging the yearly percent change in Sales/Turnover (SALE) during the five years prior to the valuation period.

To improve the accuracy of the estimation, we use all the historical data available for each firm and require at least 20 observations before the valuation period. As we described before, we use the data available prior to the valuation period in order to make out-of-sample predictions. We also eliminate observations with missing data and trim the ratios at the lower and upper one-percentiles to diminish the impact of outliers and errors in the data. The final sample includes 11,318 firm-year observations.

3.2. Explanation of contemporaneous stock prices

We study the consistency between our model estimates and market prices. To this end, for each firm-year observation in the period 1980–2015, we construct the market-to-value ratio (P_{it}/S_{it}), which is the market value of equity (P_{it}) divided by the equity value estimated by our dynamic model (S_{it}), for firm i in period of time t (i.e., year t). Figure 1 shows the evolution of the cross-sectional average market-to-value ratio for each period of time. It shows that the mean ratio is close to the value of 1 for the majority of years. The figure also shows that the ratio moves away from 1 during periods of strong market movements, such as before and after the Great Recession of 2008, returning toward 1 in the subsequent periods.

Complementing the previous analysis, Figure 2 shows the time series average market-to-value ratio for each firm in the sample. Likewise the previous figure, it shows that the mean ratio is close to the value of 1 for the great majority of firms.

Panel A in Table 1 reports the summary statistics for the market-to-value ratio for all firms. The table shows that the unconditional mean of *P/S* is around 1.03, which implies that the mean observation of the market value of equity is almost equal to the mean model estimate. The median of *P/S* for all firms and years has a value of 0.98. We believe the difference between the mean and the median is reasonable because the market-to-value ratio is bounded below at 0 but unbounded above, which creates a distribution of *P/S* that is skewed to the right. Panel B in Table 1 presents different measures of central tendency that further corroborate our first result. For instance, it shows that around 60% of the model estimates are within ±15% of the observed values, which is in line with Kaplan and Ruback (1995) and Liu, Nissim and Thomas (2002).

We compute the time series average *P/S* ratio for each firm and, consistently with Figure 2, we find that around 85% of those time series averages are within
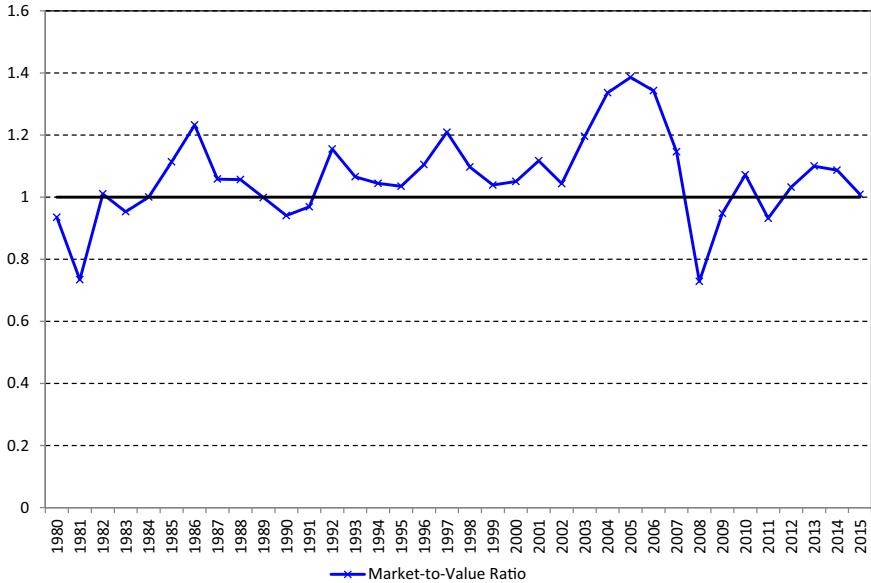
Figure 1

Evolution of the market-to-value ratio

The figure displays the evolution over time of the cross-sectional mean market-to-value ratio for the sample firms in the period 1980–2015. The market-to-value ratio is the market value of equity divided by the value estimated by the model.

$\pm 15\%$ of the value of 1. To investigate this observation further, we test the null hypothesis that the time series mean P/S ratio is equal to 1 for each individual firm in the sample. We find that we cannot reject the null hypothesis for about 87% of the firms at the 5% significance level and around 95% of the firms at the 1% significance level. We also study whether the fraction of firms for which we cannot reject that null hypothesis changes significantly across industries, according to the Standard Industry Classification (SIC). As Table 2 suggests, we do not find evidence of this sort of variation in our sample. Across industries, the fraction of firms for which we cannot reject the null hypothesis is fairly stable.[8]

The previous results show that market values and model estimates are very close to each other. We now study the level of linear association between those two variables, as well as the proportion of the variation in current prices that is explained by the predictions of our model. This analysis highlights the goodness of fit of

---

[8] There are no firms in the sample belonging to Agriculture, Forestry, and Fishing, or Construction industries. For that reason, we did not include those industries in Table 2.
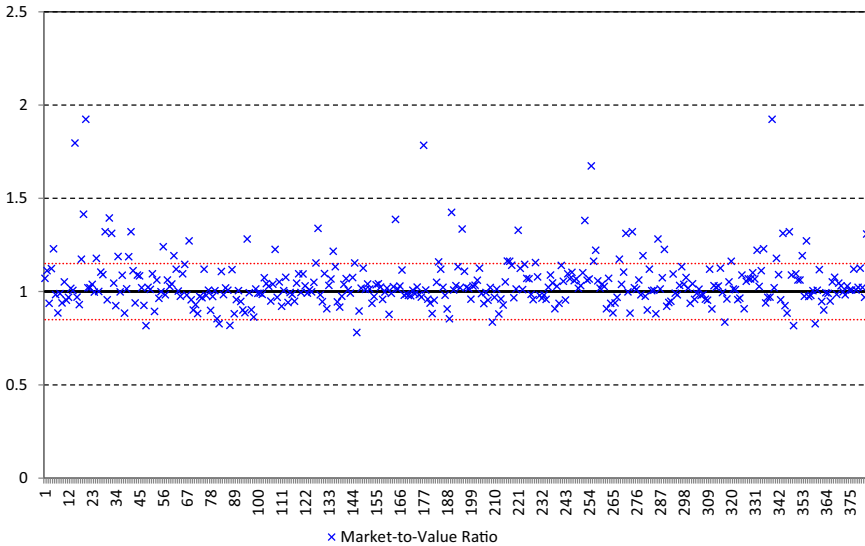
Figure 2

Average market-to-value ratio across firms

The figure displays the time series average of the market-to-value ratio across firms in the sample. The sample period is 1980–2015. The market-to-value ratio is the market value of equity divided by the value estimated by the model. The dotted lines are located at $\pm 15\%$ from the value of 1.

our valuation model. Accordingly, we start estimating the following fixed effects regression model

$$P_{it} = \alpha + \alpha_j + \beta S_{it} + \epsilon_{it}, \tag{17}$$

where t indexes the period (i.e., year), ϵ_{it} is an *iid* random term, and α_j indicates the SIC industry (aggregated at the SIC group level) to which the firm belongs. In theory, an intercept, α, of 0 and a slope, β, of 1 would suggest that our model produces unbiased estimates of market values. In addition, we should reject the possibility that the fixed effects are different from zero. Table 3 shows the results from this regression. We find that we cannot reject the null hypothesis that the intercept is 0 and the slope is 1 with a *p*-value of around 0.16. In addition, the industry-level fixed effects are jointly not statistically different from zero with a *p*-value close to 0.67. Finally, with an R^2 value of 92.4%, the dynamic DDM explains a large proportion of the variation in current stock prices. In untabulated results, we find that omitting fixed effects in the previous specification of the model generates almost identical results.

The R^2 coefficient generated by our model is larger than the ones found by related valuation studies, such as Bernard (1995), Frankel and Lee (1998), and Spiegel and

Table 1

Valuation results

The table shows the valuation results of the dynamic model for the sample in the period 1980–2015. *P/S* is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). The first line in Panel B shows the percentage of times that the value estimated by the model is within 15% of the market value of equity. The second line in Panel B shows the median value of the absolute difference between the equity value estimated by the model and the market value of equity (in percentage). The third line in Panel B shows the median value of the squared difference between the value estimated by the model and the market value of equity (in percentage). Standard errors are in parentheses.

	P/S
Panel A: Summary statistics	
Mean	1.03
	(0.50)
Median	0.98
Interquartile range	0.41
Panel B: Performance measures	
Percentage within 15%	59.98%
Median absolute error	20.86%
Median squared error	4.35%

Table 2

Significance of the mean market-to-value ratio by industry

The table shows the proportion of firms for which the null hypothesis that the mean market-to-value ratio, *P/S*, is equal to 1 cannot be rejected for different SIC industries, aggregated at the division level. The null hypothesis is tested for each individual firm. Results are shown for significance levels of 5% and 1%. The market-to-value ratio is the market value of equity divided by the value estimated by the model.

SIC industry	5% Level	1% Level
Mining	85.71%	85.71%
Manufacturing	86.21%	94.25%
Transportation and Public Utilities	90.91%	100.00%
Wholesale Trade	83.33%	100.00%
Retail Trade	85.71%	95.24%
Finance, Insurance & Real Estate	93.55%	96.77%
Services	100.00%	100.00%
Sample	87.40%	94.66%

Tookes (2013)—their R^2 coefficients are 68%, 67%, and 43%, respectively. As a benchmark, Table 3 also displays the results from regressing the market value of equity on the book value of equity, net earnings, and dividends. In each of the three cases, the R^2 is below the one we obtain using S_{it} as the regressor.

Table 3

Regression of the market value of equity

The table shows the results from different regressions of the market value of equity, including fixed effects at the industry level. In column 1, the regressor is the equity value estimated by the model; in column 2, the regressor is the book value of equity; in column 3, the regressor is net income; and in column 4, the regressor is dividends. The sample is composed of COMPUSTAT firms in the period 1980–2015. Standard errors are in parentheses, computed using the Newey-West (1987) procedure with four lags. Row-labeled Joint test: $\alpha_j = 0$ shows the p-values of a joint test of the null hypothesis that all industry fixed effects are zero. Row-labeled Joint test: $\alpha = 0$ and $\beta = 1$ shows the p-values of a joint test of the null hypothesis that the intercept is 0 and the slope is 1.

Variable	(1)	(2)	(3)	(4)
Intercept	81.12	1587.71	1679.14	3192.15
	(79.77)	(146.13)	(130.75)	(151.83)
Equity value estimate	1.00			
	(0.024)			
Book value of equity		1.34		
		(0.139)		
Net earnings			10.20	
			(0.649)	
Dividends				24.48
				(1.507)
Joint test: $\alpha_j = 0$	0.666	0.000	0.000	0.000
Joint test: $\alpha = 0$ and $\beta = 1$	0.159			
R^2	0.924	0.711	0.767	0.677

As before, we provide further corroboration that our model works well at the individual firm level. To this end, we test the null hypothesis that $\alpha_i = 0$ and $\beta_i = 1$ for the regression model

$$P_{it} = \alpha_i + \beta_i S_{it} + \epsilon_{it} \tag{18}$$

individually for each firm i, where t indexes the period (i.e., year) and ϵ_{it} is an *iid* random term. We find that we cannot reject the null hypothesis for about 71% of the firms at the 5% significance level and around 83% of the firms at the 1% significance level.

While we have used our model to predict prices, it can be easily adapted to predict firm returns. Specifically, the expected (gross) return of firm i in the next period $t + 1$ can be expressed as follows:

$$E[r_{it+1}] = \frac{S_{it+1}}{P_{it}}, \tag{19}$$

where P_{it} is the observed market price of firm i in period of time t and S_{it+1} is the price of the same firm predicted by our model for one year in the future (with the available data till year t). In particular, we calculate S_{it+1} by first determining the state of the model in $t + 1$, $(K_{t+1}^*, L_{t+1}^*, B_{t+1}^*, E[z_{t+1}|z_t])$, and then using it in Equation (7).

As we did with prices, we can compare the expected returns predicted by our model with the observed stock returns. To do so, the realized (gross) stock return is given by

$$r_{it+1} = \frac{P_{it+1}}{P_{it}}. \tag{20}$$

With these returns, $E[r_{it+1}]$ and r_{it+1}, we then repeat all the tests described above for the relation between predicted prices, S_{it}, and observed prices, P_{it}. We find almost identical results regarding intercept, slope, and industry fixed effects. The sole difference is that the R^2 of the regression is now around 35%.

Overall, the results in this subsection suggest that our dynamic DDM produces equity value estimates that are consistent with market prices and explain a large part of the variation in current stock prices. We next explore the possibility to use the model to economically exploit short-run differences between market values and model estimates.

3.3. Portfolio strategy returns

We just showed that our dynamic DDM produces value estimates that are very close to contemporaneous share prices. However, our results also suggest that the linear association is not perfect, which means that there are temporary or short-run deviations between market prices (*P*) and estimated values (*S*) for individual stocks in certain years. We show that these differences can be economically exploited. Subsequently, we evaluate the risk of the resulting returns.

We use the ratio of market prices to model predictions *P/S* across firms to implement a spread strategy that consists of buying stocks that seem undervalued— according to our model—and selling stocks that seem overvalued—again, according to our model. Specifically, on June 1 of each year, we first rank the firms in the sample based on their demeaned *P/S* ratio. We form quintiles based on that *P/S* ratio, where lower quintiles include firms with low *P/S* and higher quintiles include firms with high *P/S*. The last step consists of implementing a simple spread strategy (which we call *Q*1–*Q*5) by taking a long position in the bottom quintile (*Q*1) and a short position in the top quintile (*Q*5). For this study, we take an equal weighted position in each firm. The resulting spread strategy has zero cost.

The motivation for the spread strategy is quite simple: Firms in the lower quintiles have market prices that are low relative to our model predictions. And, we believe these firms will experience higher future stock returns than firms in the higher quintiles. The opposite reasoning holds for firms in the higher quintiles. As we just described, we construct the quintiles using demeaned measures of the *P/S* ratios. The objective of this demeaning is to fully exploit short-term deviations between stock prices and model estimates. Appendix B compares the returns of the spread strategy with and without demeaning, ascertaining the relevance of this step.

We evaluate our spread strategy using firm data from 1980 through 2015. Thus, we are able to implement the spread strategy 35 times (i.e., years). We refer to each implementation as a portfolio. We track the cumulative returns of each portfolio over the following 60 months after formation (or the longest possible period for the most recent four years).

Panel A in Table 4 displays the outcomes of these portfolios. The column labeled $Q1$–$Q5$ shows that the spread strategy earns 20.26%, 33.67%, 40.51%, 44.07%, and 47.88% on average over the 12, 24, 36, 48, and 60 months following portfolio formation, respectively. The last column, % Winners, reports the percentage of periods in which the spread strategy earns positive cumulative returns. Specifically, it shows that, after 12 months of portfolio formation, this strategy obtains positive cumulative results 98.08% of the time, and that after 24 months of portfolio formation the strategy produces positive cumulative returns in 100% of the implementations.

To appreciate the magnitude of the results of our strategy, we compare them with the $Q1$–$Q5$ spread strategies that use demeaned ratios of market-to-book (*P/B*), price-earnings (*P/E*), and price-dividend (*P/D*) to construct the quintiles. These three are among the most well-known financial ratios that have been used for predicting stock returns (Fama and French, 1992) and to assess valuation outcomes (e.g., Frankel and Lee, 1998). Panels B–D of Table 4 show that the spread strategy using *P/S* outperforms the spread strategies based on *P/B*, *P/E*, and *P/D* in each of the five investment horizons. For example, over the period of 36 months, the *P/S* portfolios yield, on average, roughly 7%, 20%, and 20% more than the *P/B*, *P/E*, and *P/D* portfolios, respectively. Column 3 of Table 4 shows the annualized Sharpe ratios, suggesting that the better performance of the *P/S* portfolios still holds if we adjust the corresponding returns in terms of risk. Finally, the percentage of winner periods with the spread strategy based on our model estimates is larger than those based on the *P/B*, *P/E*, and *P/D* ratios.

Complementing our previous results, Figure 3 displays the evolution of the average returns of the $Q1$–$Q5$ spread strategy based on *P/S* over the 60 months after portfolio formation. The concavity, or flattening of the curve, suggests that the benefits from the information available at the moment of portfolio formation naturally diminish as time passes. As benchmark, Figure 3 also shows the lower average returns obtained by the spread strategies based on *P/B*, *P/E*, and *P/D* over time.

We finally ascertain the risk of the $Q1$–$Q5$ spread strategy returns by relating our results to the Fama-French three-factor model of stock returns (Fama and French, 1993). They find that the expected return on a stock in excess of the risk-free rate, $E(R) - R_f$, can be explained by these three factors:

1. The expected return on the market portfolio in excess of the risk-free rate, $E(R_M) - R_f$. This factor proxies for systematic risk.
2. The expected return on a portfolio of small stocks minus the expected return on a portfolio of big stocks (Small Minus Big [*SMB*]). As Fama and French (1993) suggests, this variable might be associated with a common risk factor

Table 4

Cumulative *Q*1–*Q*5 spread strategy returns

The table presents the average cumulative returns of four strategies. The strategies are based on the ratios: the market-to-value ratio (*P/S*), the market-to-book ratio (*P/B*), the price-earnings ratio (*P/E*), and the price-dividend ratio (*P/D*). Each strategy ranks firms based on their corresponding ratios and groups them in quintiles. The spread strategy (*Q*1–*Q*5) consists in buying firms in the bottom quintile (*Q*1) and selling firms in the top quintile (*Q*5). *P/S portfolios* refers to the strategy based on the *P/S* ratio; *P/B portfolios* refers to the strategy based on the *P/B* ratio; *P/E portfolios* refers to the strategy based on *P/E* ratio; and *P/D portfolios* refers to the strategy based on the *P/D* ratio. Rows *Ret12*, *Ret24*, *Ret36*, *Ret48*, and *Ret60* show 12-, 24-, 36-, 48-, and 60-month magnitudes, respectively. Column *Q*1–*Q*5 shows average cumulative returns. Column SE shows annualized standard errors. Column Sharpe ratio shows annualized Sharpe ratios. Column % Winners shows the percentage of winning periods.

	*Q*1–*Q*5	SE	Sharpe ratio	% Winners
Panel A: P/S portfolios				
Ret12	20.26%***	0.16	1.22	98.08%
Ret24	33.67%***	0.24	1.33	100.00%
Ret36	40.51%***	0.28	1.40	100.00%
Ret48	44.07%***	0.34	1.25	100.00%
Ret60	47.88%***	0.40	1.17	100.00%
Panel B: P/B portfolios				
Ret12	15.02%***	0.14	1.01	92.86%
Ret24	27.11%***	0.22	1.17	98.21%
Ret36	33.06%***	0.26	1.23	96.43%
Ret48	36.37%***	0.31	1.13	94.64%
Ret60	39.46%***	0.38	1.01	94.64%
Panel C: P/E portfolios				
Ret12	10.00%***	0.19	0.47	80.36%
Ret24	11.23%**	0.38	0.27	82.14%
Ret36	20.22%***	0.39	0.48	82.14%
Ret48	24.57%***	0.58	0.40	80.36%
Ret60	28.69%***	0.65	0.43	78.57%
Panel D: P/D portfolios				
Ret12	9.40%***	0.23	0.36	76.79%
Ret24	18.41%***	0.32	0.53	76.79%
Ret36	20.05%***	0.35	0.54	76.79%
Ret48	26.24%***	0.42	0.60	82.14%
Ret60	30.86%***	0.52	0.57	82.14%

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

that explains the observed negative relation between firm size and average return.

3. The expected return on a portfolio of value stocks (high book-to-market ratio stocks) minus the expected return on a portfolio of growth stocks (low
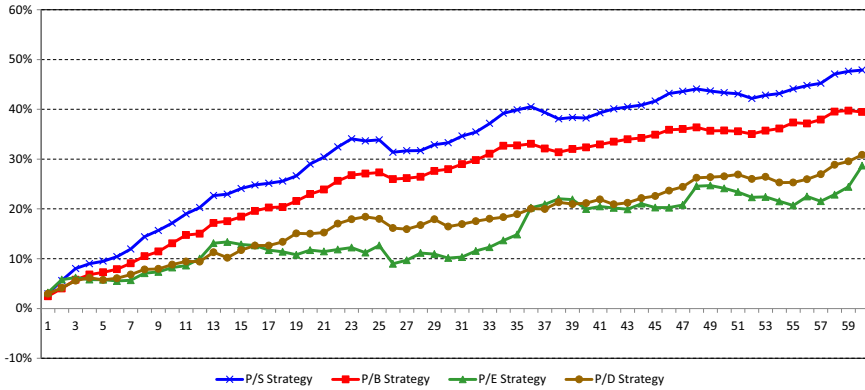
Figure 3

Cumulative spread strategy returns

The figure displays the cumulative returns of the spread strategies for the *P/S, P/B, P/E,* and *P/D* ratios during the 60 months following portfolio formation. These strategies are constructed with the sample firms in the period 1980–2015. *P/S* is the market-to-value ratio (i.e., the market value of equity divided by the equity value estimated by the model). *P/B* is the market-to-book ratio (i.e., the market value of equity divided by the book value of equity). *P/E* is the price-earnings ratio (i.e., the market value of equity divided by the firm's net income). *P/D* is the price-dividend ratio (i.e., the market value of equity divided by the firm's dividend). The spread strategies are formed by sorting firms into quintiles according to their *P/S, P/B, P/E,* and *P/D* ratios. The spread strategy consists in buying firms in the bottom quintile and selling firms in the top quintile.

book-to-market ratio stocks; High Minus Low [*HML*]). The same authors suggest that this variable might be associated with a common risk factor that explains the observed positive relation between the book-to-market ratio and average return.

We then study whether the positive returns shown in Table 4 for the $Q1$–$Q5$ spread strategy based on the *P/S* ratio can be explained by the exposure to the risk factors proposed by the Fama-French model. Accordingly, we estimate the next model,

$$R_{st} - R_{f_t} = \alpha + \beta_1(R_{M_t} - R_{f_t}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{st}, \qquad (21)$$

where we regress the excess (monthly) returns of the spread portfolios on the three factors. In this equation, *s* indicates the specific portfolio from 1 to 35, *t* denotes the month of the return, and ϵ_{st} is an *iid* random term. The first row in Table 5 contains the results from Equation (21). The alpha takes a value of 0.011 and is statistically significantly different from zero (*p*-value $= 0$). This means that the positive returns of our spread strategy cannot be accounted for by the factors in the Fama-French three-factor model.

Table 5

Risk exposure of the Q1–Q5 spread strategy

The table shows the results from regressions of the spread strategy' excess returns on three factor models. *FF3* refers to the Fama-French three-factor model, *FF3 plus momentum* refers to the Fama-French three-factor model adding a momentum factor, and *FF5* refers to the Fama-French five-factor model. The regressor is the excess return of the *P/S* spread strategy, which is based on the ranking of the market-to-value ratio (*P/S*). *Rm-Rf* is the excess return on the market, *SMB* (Small Minus Big) is the return on a portfolio of small stocks minus the return on a portfolio of big stocks, *HML* (High Minus Low) is the return on a portfolio of value stocks minus the return on a portfolio of growth stocks, *MOM* (Momentum) is the return on a portfolio of firms with high prior return minus the return on a portfolio of firms with low prior return, *RMW* (Robust Minus Weak) is the return on a portfolio of firms with robust operating profitability minus the return on a portfolio of firms with weak operating profitability, and *CMA* (Conservative Minus Aggressive) is the return on a portfolio of firms with conservative investment minus the return on a portfolio of firms with aggressive investment. The sample is composed of the strategy's returns during the period 1980–2015. *p*-Values are in parentheses, computed using Newey-West (1987) errors with four lags.

Specification	*Alpha*	*Rm-Rf*	*SMB*	*HML*	*MOM*	*RMW*	*CMA*
FF3	0.011***	−0.010	−0.098***	0.055			
	(0.000)	(0.621)	(0.002)	(0.147)			
FF3 plus momentum	0.011***	−0.011	−0.099***	0.049	−0.023		
	(0.000)	(0.591)	(0.002)	(0.212)	(0.292)		
FF5	0.012***	−0.014	−0.128***	0.054		−0.143**	−0.025
	(0.000)	(0.534)	(0.000)	(0.339)		(0.020)	(0.761)

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

To shed more light on the robustness of our results, we study the returns of the stocks in each of the five quintiles. For this analysis, we assume that we purchase the firms included in each of those quintiles and hold the resulting portfolios for up to 60 months. Thus, for each quintile, we have 35 portfolios (from 1980 through 2015), and we hold each of them for 60 months. Recall that firms are first ranked according to their *P/S* ratios and then grouped in quintiles. Thus, if low (high) *P/S* ratios are indeed capturing temporary undervaluation (overvaluation), then we should observe that average returns decrease across quintiles. Table 6, which exhibits the average return for each quintile over the 12, 24, 36, 48, and 60 months after portfolio formation, shows this is in fact the case. The returns across quintiles (from *Q*1 through *Q*5) decrease monotonically for all the time horizons considered. For instance, they go from a positive average return of 16.34% for *Q*1 after one year of portfolio formation to an average return of −3.92% for *Q*5 over the same period.

We also analyze the returns of the five quintiles in the context of the Fama-French three-factor model. To this end, we reproduce Equation (21) for each quintile and show the results in Table 7. Across quintiles, the alphas exhibit a diminishing pattern, going from a positive value of $\alpha = 0.010$ for *Q*1 to a negative value of $\alpha = -0.002$ for *Q*5. This result is in line with our previous finding about average returns in

Table 6

Cumulative quintile portfolio returns

The table presents the quintile portfolio average cumulative returns. The quintiles *Q*1 through *Q*5 are constructed based on the ranking of the market-to-value ratio (*P/S*). *Ret12*, *Ret24*, *Ret36*, *Ret48*, and *Ret60* are the average 12-, 24-, 36-, 48-, and 60-month portfolio returns, respectively. The sample is composed of the quintile portfolios' returns during the period 1980–2015.

	*Q*1	*Q*2	*Q*3	*Q*4	*Q*5
Ret12	16.34%	9.62%	4.65%	1.27%	−3.92%
Ret24	27.28%	15.72%	6.06%	0.24%	−6.39%
Ret36	32.58%	15.93%	6.34%	−0.82%	−7.94%
Ret48	37.10%	17.31%	6.44%	−1.67%	−6.98%
Ret60	40.49%	17.72%	6.62%	−0.78%	−7.39%

Table 7

Risk exposure of the quintile portfolios: Fama-French three-factor model

The table shows the results from regressions of the quintile portfolios' excess returns on the factors in the Fama-French three-factor model. The quintiles *Q*1 through *Q*5 are constructed based on the ranking of the market-to-value ratio (*P/S*). The regressor is the excess return of each quintile portfolio. *Rm-Rf* is the excess return on the market, *SMB* (Small Minus Big) is the return on a portfolio of small stocks minus the return on a portfolio of big stocks, and *HML* (High Minus Low) is the return on a portfolio of value stocks minus the return on a portfolio of growth stocks. The sample is composed of the quintile portfolios' returns during the period 1980–2015. *p*-Values are in parentheses, computed using Newey-West (1987) errors with four lags. Row-labeled Joint test: $\alpha = 0$ shows the *p*-values of a joint test of the null hypothesis that all alphas are zero.

Quintile	*Alpha*	*Rm-Rf*	*SMB*	*HML*
*Q*1	0.010***	−0.019	−0.116***	0.113**
	(0.000)	(0.406)	(0.001)	(0.016)
*Q*2	0.004***	−0.019	−0.039	0.087**
	(0.000)	(0.301)	(0.205)	(0.016)
*Q*3	0.002**	0.001	−0.021	0.086***
	(0.042)	(0.935)	(0.353)	(0.002)
*Q*4	0.001	−0.006	−0.029	0.066**
	(0.491)	(0.677)	(0.174)	(0.011)
*Q*5	−0.002**	−0.002	−0.033	0.085***
	(0.032)	(0.891)	(0.130)	(0.001)
Joint test: $\alpha = 0$	0.000			

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

Table 6. Finally, we also reject the null hypothesis that the five alphas are jointly zero with a *p*-value of 0. (Note also that the *p*-values for the individual alphas have an inverted U-shape across quintiles. This means that the extremal quintiles [i.e., *Q*1 and *Q*5] are statistically more different from zero. This is also consistent with our prediction above.)

In Appendix B, we extend the previous analyses in two directions. First, we show that we obtain similar results if we evaluate the risk of the spread strategy and the five quintiles in the context of the Fama-French three-factor model plus a momentum factor and the Fama-French five-factor model. Second, we analyze the impact of demeaning the ratios used in the implementation of the spread strategy.

4. Conclusion

We derive a dynamic version of the DDM in closed form and evaluate its empirical performance. We find that our model forecasts stock prices consistently in the sense that model estimates are very close to market prices. In addition, the model explains a large proportion of the observed variability in current stock prices. We also find that the observed temporary differences between market prices and model estimates can be economically exploited. In particular, a spread strategy based on the ranking of the ratio of stock prices to model forecasts earns positive returns over the five following years (e.g., an average of around 20%, 34%, 41%, 44%, and 48% cumulative returns after one, two, three, four, and five years of portfolio formation, respectively). Finally, we show that those portfolio returns cannot be simply explained either by the Fama-French three-factor model (even after we add a momentum factor) or the Fama-French five-factor model.

Appendix A: Proofs

The proof of Proposition 1 requires an intermediate result that we present next.

Lemma 1. The maximum level of book leverage in each period is given by

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c' - \frac{1}{2}\sigma\right)}\Phi_1^{*\alpha_K - 1}\Phi_2^{*\alpha_L} - f - \delta - \omega\frac{\Phi_2^*}{\Phi_1^*}\right](1 - \tau) - \xi}{1 + r_B(1 - \tau)}. \tag{A1}$$

Proof. Given the value of x_c' (i.e., for an arbitrary probability of bankruptcy $\lambda(x_c')$), the firm goes into bankruptcy when

$$\left(z_c' K'^{\alpha_K} L'^{\alpha_L} - fK' - \delta K' - \omega L' - r_B \ell K'\right)(1 - \tau) + K' - \ell K' - \xi K' < 0, \tag{A2}$$

where $z_c' = cz^\rho e^{\sigma x_c'}$ is the cut-off value of z such that the probability of $z' < z_c'$ is $\lambda(x_c')$. The maximum book leverage ratio consistent with probability of bankruptcy $\lambda(x_c')$ then satisfies

$$\left(z_c' K'^{\alpha_K} L'^{\alpha_L} - fK' - \delta K' - \omega L' - r_B \ell^* K'\right)(1 - \tau) + K' - \ell^* K' - \xi K' = 0. \tag{A3}$$

Working on the previous expression (and using the optimal policies derived in Equation [A15]), the maximum level of book leverage is

$$\ell^* = \frac{1 + \left[e^{\sigma\left(x_c' - \frac{1}{2}\sigma\right)}\Phi_1^{*\alpha_K - 1}\Phi_2^{*\alpha_L} - f - \delta - \omega\frac{\Phi_2^*}{\Phi_1^*}\right](1 - \tau) - \xi}{1 + r_B(1 - \tau)}, \tag{A4}$$

as shown in Equation (A1).   □

Proof of Proposition 1. The market value of equity can be expressed as

$$S_0(K_0, L_0, B_0, z_0) = \max_{\{K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{(1+g)^t}{\prod_{j=0}^{t}(1+r_{S_j})} Y_t. \quad (A5)$$

Because we use the APV method of firm valuation, we solve the problem of the firm in Equation (A5) in three steps. First, we determine the value of the unlevered firm, $S_{u_0}(K_0, L_0, z_0)$. Second, we solve for optimal debt and compute the present value of the financing side effects. Finally, we obtain the value of the levered firm in Equation (A5).

The market value of equity for the unlevered firm can be expressed as

$$S_{u_0}(K_0, L_0, z_0) = \max_{\{K_{t+1}, L_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{1+g}{1+r_A}\right)^t Y_{u_t}. \quad (A6)$$

where $Y_{u_t} = N_{u_t} - (K_{t+1} - K_t)$ and $N_{u_t} = (z_t K_t^{\alpha_K} L_t^{\alpha_L} - fK_t - \delta K_t - \omega L_t)(1 - \tau)$. We let normalized variables with primes indicate values in the next period and normalized variables with no primes denote current values. Then, the Bellman equation for the problem of the firm in Equation (A6) is given by

$$S_u(K, L, z) = \max_{K', L'} \left\{ (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1 - \tau) - (1+g)K' + K \right.$$

$$\left. + \frac{(1+g)}{(1+r_A)} E\left[S_u(K', L', z') | z\right] \right\}. \quad (A7)$$

We use the guess and verify method as the proof strategy. Thus, we start by guessing that the solution is given by

$$S_u(K, L, z) = (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1 - \tau) + K + M(z)P_u^*, \quad (A8)$$

where

$$M(z) = e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K + \alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} \sum_{n=1}^{\infty} \left\{ \left(\frac{1+g}{1+r_A}\right)^n \left(c^{\frac{1-\rho^n}{1-\rho}} z^{\rho^n} e^{\frac{1}{2}\sigma^2 \frac{(1-\rho^{2n})}{(1-\rho^2)} \frac{1}{[1-(\alpha_K+\alpha_L)]}} \right)^{\frac{1}{1-(\alpha_K+\alpha_L)}} \right\}, \quad (A9)$$

$$P_u^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^* \right)(1 - \tau) - r_A \Phi_1^*, \quad (A10)$$

$$\Phi_1^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta} \right)^{1-\alpha_L} \left(\frac{\alpha_L}{\omega} \right)^{\alpha_L} \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}, \quad \text{and} \quad (A11)$$

$$\Phi_2^* = \left[\left(\frac{\alpha_K}{\frac{r_A}{1-\tau} + f + \delta} \right)^{\alpha_K} \left(\frac{\alpha_L}{\omega} \right)^{1-\alpha_K} \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}}. \tag{A12}$$

We obtain this initial guess as the solution of Equation (A7) by the backward induction method.

We now verify our guess. To this end, let us write

$$S_u(K, L, z) = \max_{K', L'} \{F(K', L', K, L, z)\} \tag{A13}$$

with F defined as the objective function in Equation (A7).

The First Order Condition (FOC) for this problem is

$$\partial F(K', L', K, L, z)/\partial K' = -(1+g)$$

$$+ \frac{(1+g)}{(1+r_A)} \left[\left(E\left[z'|z \right] \alpha_K K^{*\alpha_K - 1} L^{*\alpha_L} - f - \delta \right)(1-\tau) + 1 \right] = 0$$

$$\partial F(K', L', K, L, z)/\partial L' = \frac{(1+g)}{(1+r_A)} \left(E\left[z'|z \right] K^{*\alpha_K} \alpha_L L^{*\alpha_L - 1} - \omega \right)(1-\tau) = 0 \tag{A14}$$

and optimal capital and labor turn out to be

$$K^* = E\left[z'|z \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^* \quad \text{and} \quad L^* = E\left[z'|z \right]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_2^*, \tag{A15}$$

where Φ_1^* and Φ_2^* are as in Equations (A11) and (A12), respectively.

Finally, the market value of equity for the unlevered firm becomes

$$S_u(K, L, z) = (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1-\tau) - (1+g)K^* + K$$

$$+ \frac{(1+g)}{(1+r_A)} \left[(E[z'|z]K^{*\alpha_K} L^{*\alpha_L} - fK^* - \delta K^* - \omega L^*)(1-\tau) \right.$$

$$+ K^* + E\left[M(z')|z \right] P_u^* \right]$$

$$= (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1-\tau) + k$$

$$- (1+g)E[z'|z]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^* + \frac{(1+g)}{(1+r_A)} \left\{ E[z'|z]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \right.$$

$$\left[\left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f\Phi_1^* - \delta\Phi_1^* - \omega\Phi_2^* \right)(1-\tau) + \Phi_1^* \right]$$

$$+ E[M(z')|z]P_u^* \right\}$$

$$= (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1-\tau) + K$$

$$+ \frac{(1+g)}{(1+r_A)} \left(e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} E\left[z'^{\frac{1}{1-(\alpha_K+\alpha_L)K}} |z \right] + E\left[M(z')|z \right] \right) P_u^*$$

$$= (zK^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L)(1-\tau) + K + M(z)P_u^*, \tag{A16}$$

which is equivalent to our initial guess in Equation (A8).

Next, we obtain optimal risky debt. We first re-express debt as fraction of capital: $B = \ell K$. In each period, the firm solves the following problem:

$$\max_{\ell', x'_c} \left\{ \ell' K' - \frac{1}{(1 + r_B)} \left\{ \ell' K' [1 + r_B(1 - \tau)] - \lambda(x'_c)\xi K' \right\} \right\}. \quad \text{(A17)}$$

where x'_c is the cut-off value of the standard normal random variable x such that the probability of $x' < x'_c$ is $\lambda(x'_c)$.

The above problem can be solved in two steps. In the first step, given the value of x'_c (i.e., for an arbitrary probability of bankruptcy $\lambda(x'_c)$), the firm chooses optimal book leverage ℓ^*. Defining F as the objective function in Equation (A17), the FOC turns out to be $\partial F(K', \ell', x'_c)/\partial \ell' = \frac{1}{(1+r_B)} r_B \tau K' > 0$. Because $r_B > 0$, $\tau > 0$, and $K' > 0$ the firm increases debt as much as possible to maximize the tax benefits of debt. Then, optimal debt is $B^* = \ell^* K^*$ where

$$\ell^* = \frac{1 + \left[e^{\sigma \left(x'_c - \frac{1}{2}\sigma \right)} \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L} - f - \delta - \omega \frac{\Phi_2^*}{\Phi_1^*} \right] (1 - \tau) - \xi}{1 + r_B(1 - \tau)} \quad \text{(A18)}$$

as shown in Lemma 1. In the second step, the firm selects the value of x'_c that maximizes the present value of the financing side effects. Accordingly, the problem of the firm becomes

$$\max_{x'_c} \left\{ \ell^* \left(x'_c \right) K' - \frac{1}{(1 + r_B)} \left\{ \ell^* \left(x'_c \right) K' [1 + r_B(1 - \tau)] - \lambda \left(x'_c \right) \xi K' \right\} \right\}. \quad \text{(A19)}$$

After solving the previous problem, the optimal probability of bankruptcy is given by

$$\lambda^* = \int_{-\infty}^{x_c^*} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz, \quad \text{(A20)}$$

where

$$x_c^* = -\sigma - \sqrt{2 \left\{ \sigma^2 + \ln \left\{ \frac{\left[1 + \frac{1}{r_B(1-\tau)} \right] \xi}{\sqrt{2\pi} \tau \sigma \Phi_1^{*\alpha_K - 1} \Phi_2^{*\alpha_L}} \right\} \right\}}. \quad \text{(A21)}$$

Finally, the present value of the financing side effects turns out to be

$$Q(z) = \left(\frac{1 + g}{1 + r_A} \right) \left\{ \left(\frac{1 + r_A}{1 + r_B} \right) \left(r_B \tau \ell^* - \lambda^* \xi \right) K^* + E \left[Q \left(z' \right) | z \right] \right\}$$

$$= M(z) \left(\frac{1 + r_A}{1 + r_B} \right) \left(r_B \tau \ell^* - \lambda^* \xi \right) \Phi_1^*, \quad \text{(A22)}$$

where $M(z)$ is as in Equation (A9). Under this financial policy, the amount of debt and interest payments will vary with the future asset cash flows (i.e., they depend on future firm performance). Then, because the financing side effects will have a level

of risk in line with that of the firm cash flows, we use the cost of capital, r_A, as the discount rate. This feature of the model is consistent with Kaplan and Ruback (1995).

The third step consists in obtaining the market value of equity for the levered firm that does not go into bankruptcy. If we assume the firm used debt B in the previous period, and now has to pay interest $r_B B(1 - \tau)$, then the stock price for the levered firm is

$$S(K, L, B, z) = S_u(K, L, z) + M(z) \left(\frac{1+r_A}{1+r_B} \right) \left(r_B \tau \ell^* - \lambda^* \xi \right) \Phi_1^* - B - r_B B(1-\tau)$$

$$= (z K^{\alpha_K} L^{\alpha_L} - fK - \delta K - \omega L - r_B B)(1-\tau) + K - B + G(z), \quad \text{(A23)}$$

where $G(z) = M(z)P^*$ and variable P^* takes the form

$$P^* = \left(\Phi_1^{*\alpha_K} \Phi_2^{*\alpha_L} - f \Phi_1^* - \delta \Phi_1^* - \omega \Phi_2^* \right)(1-\tau) - r_A \Phi_1^*$$

$$+ \left(\frac{1+r_A}{1+r_B} \right) \left(r_B \tau \ell^* - \lambda^* \xi \right) \Phi_1^*. \quad \text{(A24)}$$

Finally, the optimal decisions of the firm are given by

$$K_{t+1}^* = (1 + g) E [z_{t+1}|z_t]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_1^*,$$

$$L_{t+1}^* = (1 + g) E [z_{t+1}|z_t]^{\frac{1}{1-(\alpha_K+\alpha_L)}} \Phi_2^*, \quad \text{and} \quad B_{t+1}^* = \ell^* K_{t+1}^* \quad \text{(A25)}$$

and the market value of equity is

$$S \left(K_t^*, L_t^*, B_t^*, z_t \right) = \left[(1 + g)^{t[1-(\alpha_K+\alpha_L)]} z_t K_t^{*\alpha_K} L_t^{*\alpha_L} - fK_t^* - \delta K_t^* \right.$$

$$\left. - \omega L_t^* - r_B B_t^* \right](1 - \tau) + K_t^* - B_t^* + G(z_t) \quad \text{(A26)}$$

as shown in Proposition 1.   □

Lemma 2. Suppose $g < r_A$ and $\rho < 1$. Then $M(z_t)$ converges for each given z_t.

Proof. Function $M(z_t)$ is given by

$$M(z_t) = e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}} \left\{ \left(\frac{1+g}{1+r_A} \right) E \left[z_{t+1}^{1/[1-(\alpha_K+\alpha_L)]}|z_t \right] \right.$$

$$\left. + \left(\frac{1+g}{1+r_A} \right)^2 E \left[z_{t+2}^{1/[1-(\alpha_K+\alpha_L)]}|z_t \right] + \cdots \right\}. \quad \text{(A27)}$$

The first factor, $e^{-\frac{1}{2}\sigma^2 \frac{(\alpha_K+\alpha_L)}{[1-(\alpha_K+\alpha_L)]^2}}$, is a positive constant. In addition, $E[z_{t+n}^{1/[1-(\alpha_K+\alpha_L)]}|z_t]$ is bounded for all n (including $n \to \infty$) if $\rho < 1$. Thus,

$$\left(\frac{1+g}{1+r_A} \right) E \left[z_{t+1}^{1/[1-(\alpha_K+\alpha_L)]}|z_t \right] + \left(\frac{1+g}{1+r_A} \right)^2 E \left[z_{t+2}^{1/[1-(\alpha_K+\alpha_L)]}|z_t \right] + \cdots \quad \text{(A28)}$$

converges if $\frac{1+g}{1+r_A} < 1$—which is the same as to say $g < r_A$.   □

Appendix B: Robustness checks

In this Appendix, we perform further analyses to ascertain the robustness of our results in Subsection 3.3. First, we study the returns of the $Q1$–$Q5$ spread strategy and the five quintiles in the context of the Fama-French three-factor model plus a momentum factor and the Fama-French five-factor model. Second, we ascertain the importance of the demeaning of the ratios used to implement the spread strategy.

We test the performance of the spread strategy based on *P/S* against the Fama-French three-factor model plus a momentum factor. The latter, denoted as *MOM*, is given by the return on a portfolio of firms with high prior year return minus the return on a portfolio of firms with low prior year return. This factor attempts to explain the observed positive relation between prior year return and current year return (Jegadeesh and Titman, 1993). Accordingly, we perform the following regression:

$$R_{st} - R_{f_t} = \alpha + \beta_1(R_{M_t} - R_{f_t}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \epsilon_{st}, \quad \text{(B1)}$$

where we regress the excess (monthly) returns of the $Q1$–$Q5$ spread strategy portfolios on the four factors. We let s indicate the specific portfolio from 1 to 35, t denote the month of the return, and ϵ_{st} be an *iid* random term. We show the results from this regression in the second row in Table 5. The alpha, with a value of 0.011, is statistically significantly different from zero (*p*-value = 0), implying that the spread strategy yields significantly positive returns that cannot be explained by the Fama-French three-factor model plus momentum.

Next, we reproduce the previous analysis in the context of the Fama-French five-factor model. The latter extends the Fama-French three-factor model by adding two more factors: *RMW* and *CMA*. Factor *RMW* (Robust Minus Weak) represents the return on a portfolio of firms with robust (i.e., high) operating profitability minus the return on a portfolio of firms with weak (i.e., low) operating profitability. Fama and French (2015) suggests this variable might explain the expected positive relation between operating profitability and average return. Factor *CMA* (Conservative Minus Aggressive) is the return on a portfolio of firms with conservative (i.e., low) investment minus the return on a portfolio of firms with aggressive (i.e., high) investment. According to Fama and French (2015), this variable might explain the expected negative relation between investment and average return. Thus, we perform the next regression

$$R_{st} - R_{f_t} = \alpha + \beta_1(R_{M_t} - R_{f_t}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t$$
$$+ \beta_5 CMA_t + \epsilon_{st}, \quad \text{(B2)}$$

where we regress the excess (monthly) returns of the $Q1$–$Q5$ spread strategy portfolios on the five factors. As before, s indexes the specific portfolio from 1 to 35, t indicates the month of the return, and ϵ_{st} is an *iid* random term. The third row in Table 5

Table B1

Risk exposure of the quintile portfolios: Fama-French three-factor model plus momentum

The table shows the results from regressions of the quintile portfolios' excess returns on the factors in the Fama-French three-factor model plus momentum. The quintiles $Q1$ through $Q5$ are constructed based on the ranking of the market-to-value ratio (P/S). The regressor is the excess return of each quintile portfolio. *Rm-Rf* is the excess return on the market, *SMB* (Small Minus Big) is the return on a portfolio of small stocks minus the return on a portfolio of big stocks, *HML* (High Minus Low) is the return on a portfolio of value stocks minus the return on a portfolio of growth stocks, and *MOM* (Momentum) is the return on a portfolio of firms with high prior year return minus the return on a portfolio of firms with low prior year return. The sample is composed of the quintile portfolios' returns during the period 1980–2015. *p*-Values are in parentheses, computed using Newey-West (1987) errors with four lags. Row-labeled Joint test: $\alpha = 0$ shows the *p*-values of a joint test of the null hypothesis that all alphas are zero.

Quintile	*Alpha*	*Rm-Rf*	*SMB*	*HML*	*MOM*
Q1	0.011***	−0.020	−0.119***	0.101**	−0.042
	(0.000)	(0.370)	(0.001)	(0.016)	(0.170)
Q2	0.004***	−0.020	−0.040	0.082**	−0.018
	(0.000)	(0.285)	(0.194)	(0.025)	(0.389)
Q3	0.002**	0.000	−0.022	0.080**	−0.022
	(0.023)	(0.985)	(0.320)	(0.014)	(0.206)
Q4	0.001	−0.007	−0.029	0.062**	−0.013
	(0.390)	(0.650)	(0.163)	(0.018)	(0.353)
Q5	−0.002*	−0.003	−0.034	0.079***	−0.020
	(0.059)	(0.847)	(0.119)	(0.002)	(0.243)
Joint test: $\alpha = 0$	0.000				

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

shows the results from this analysis. Likewise Equation (B1), the alpha is statistically significantly different from zero, with a value of 0.012 and a *p*-value of 0.

Overall, the previous results suggest that the spread strategy based on *P/S* yields significantly positive returns that cannot be accounted for by exposure to the factors in the Fama-French three-factor model plus momentum or the Fama-French five-factor model.

We also analyze the returns of the quintile portfolios in the context of the Fama-French three-factor model plus momentum and the Fama-French five-factor model. As Table B1 shows, adding momentum to the Fama-French three-factor model yields almost identical results to those of Table 7 in the sense that the alphas exhibit a diminishing pattern across quintiles (from $\alpha = 0.011$ for $Q1$ to $\alpha = -0.002$ for $Q5$) and their statistical significance increases for the extremal quintiles (i.e., $Q1$ and $Q5$). In addition, with a *p*-value of 0, we strongly reject the null hypothesis that the five alphas are jointly zero.

The results regarding the returns of the quintile portfolios and the Fama French five-factor model are shown in Table B2 and the main conclusions are roughly

Table B2

Risk exposure of the quintile portfolios: Fama-French five-factor model

The table shows the results from regressions of the quintile portfolios' excess returns on the factors in the Fama-French five-factor model. The quintiles $Q1$ through $Q5$ are constructed based on the ranking of the market-to-value ratio (P/S). The regressor is the excess return of each quintile portfolio. Rm-Rf is the excess return on the market, SMB (Small Minus Big) is the return on a portfolio of small stocks minus the return on a portfolio of big stocks, HML (High Minus Low) is the return on a portfolio of value stocks minus the return on a portfolio of growth stocks, RMW (Robust Minus Weak) is the return on a portfolio of firms with high operating profitability minus the return on a portfolio of firms with low operating profitability, and CMA (Conservative Minus Aggressive) is the return on a portfolio of firms with low investment minus the return on a portfolio of firms with high investment. The sample is composed of the quintile portfolios' returns during the period 1980–2015. p-Values are in parentheses, computed using Newey-West (1987) errors with four lags. Row-labeled Joint test: $\alpha = 0$ shows the p-values of a joint test of the null hypothesis that all alphas are zero.

Quintile	Alpha	Rm-Rf	SMB	HML	RMW	CMA
Q1	0.011***	−0.020	−0.171***	0.088	−0.262***	−0.001
	(0.000)	(0.386)	(0.000)	(0.143)	(0.000)	(0.993)
Q2	0.005***	−0.013	−0.096***	0.022	−0.276***	0.080
	(0.000)	(0.492)	(0.007)	(0.688)	(0.000)	(0.268)
Q3	0.002***	0.009	−0.060**	0.022	−0.192***	0.092
	(0.009)	(0.587)	(0.010)	(0.555)	(0.000)	(0.130)
Q4	0.001	−0.002	−0.059**	0.027	−0.146***	0.051
	(0.264)	(0.883)	(0.018)	(0.446)	(0.000)	(0.318)
Q5	−0.001*	−0.000	−0.066**	0.054	−0.158***	0.033
	(0.094)	(0.996)	(0.013)	(0.165)	(0.000)	(0.580)
Joint test: $\alpha = 0$	0.000					

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

equal to those we just described for the Fama-French three-factor model plus momentum.

Finally, we evaluate the impact of the demeaning step in the $Q1$–$Q5$ spread strategy implementation by comparing the results in Table 4 with those of the same strategy without demeaning. In other words, we repeat the same exercise without demeaning each of the four ratios. Consistently with our previous results, Table B3 shows that the returns of the spread strategy using the P/S ratio are positive (37.07% return after 60 months) and outperform those of the strategies based on the P/B, P/E and P/D ratios (30.71%, 22.84%, and 4.49% returns after 60 months, respectively). More importantly, by comparing the results in Table B3 with those in Table 4, we find that the strategy returns of the four ratios are higher with the demeaning step. In other words, our results suggest that the demeaning step plays an important role in the implementation of the spread strategies. We believe this finding might result particularly useful for practitioners.

Table B3

Cumulative *Q*1–*Q*5 spread strategy returns with no demeaning

The table presents the average cumulative returns of four strategies. The strategies are based on the ratios: the market-to-value ratio (*P/S*), the market-to-book ratio (*P/B*), the price-earnings ratio (*P/E*), and the price-dividend ratio (*P/D*). Each strategy ranks firms based on their corresponding ratios and groups them in quintiles. The spread strategy (*Q*1–*Q*5) consists in buying firms in the bottom quintile (*Q*1) and selling firms in the top quintile (*Q*5). *P/S portfolios* refers to the strategy based on the *P/S* ratio; *P/B portfolios* refers to the strategy based on the *P/B* ratio; *P/E portfolios* refers to the strategy based on the *P/E* ratio; and *P/D portfolios* refers to the strategy based on the *P/D* ratio. Rows *Ret12*, *Ret24*, *Ret36*, *Ret48*, and *Ret60* show 12-, 24-, 36-, 48-, and 60-month magnitudes, respectively. Column *Q*1–*Q*5 shows average cumulative returns. Column SE shows annualized standard errors. Column Sharpe ratio shows annualized Sharpe ratios. Column % Winners shows the percentage of winning periods.

	*Q*1–*Q*5	SE	Sharpe ratio	% Winners
Panel A: P/S portfolios				
Ret12	15.23%***	0.12	1.15	96.15%
Ret24	24.50%***	0.22	1.08	96.15%
Ret36	30.93%***	0.21	1.43	100.00%
Ret48	34.03%***	0.26	1.28	100.00%
Ret60	37.07%***	0.31	1.15	100.00%
Panel B: P/B portfolios				
Ret12	11.78%***	0.22	0.49	76.79%
Ret24	18.76%***	0.27	0.66	76.79%
Ret36	24.73%***	0.29	0.82	85.71%
Ret48	28.06%***	0.37	0.72	83.93%
Ret60	30.71%***	0.48	0.62	80.36%
Panel C: P/E portfolios				
Ret12	7.72%**	0.21	0.32	76.79%
Ret24	11.64%***	0.28	0.38	75.00%
Ret36	16.56%***	0.31	0.49	71.43%
Ret48	18.93%***	0.43	0.42	73.21%
Ret60	22.84%***	0.51	0.43	71.43%
Panel D: P/D portfolios				
Ret12	4.45%	0.20	0.17	64.29%
Ret24	7.20%**	0.25	0.25	71.43%
Ret36	4.88%	0.28	0.14	66.07%
Ret48	5.25%	0.28	0.15	64.29%
Ret60	4.49%	0.31	0.11	60.71%

***, **, * indicate statistical significance at the 0.01, 0.05 and 0.10 level, respectively.

References

Ackerberg, D.A., C.L. Benkard, S. Berry, and A. Pakes, 2007. Econometric tools for analyzing market outcomes, in: J. Heckman and E. Leamer, eds., *The Handbook of Econometrics*, vol. 6A (North-Holland, Amsterdam).

Balistreri, E.J., C.A. McDaniel, and E.V. Wong, 2003. An estimation of US industry-level capital-labor substitution elasticities: Support for Cobb–Douglas, *North American Journal of Economics and Finance* 14, 343–356.

Bernard, V.L., 1995. The Feltham–Ohlson framework: Implications for empiricists, *Contemporary Accounting Research* 11, 733–747.

Brennan, M.J. and E.S. Schwartz, 1984. Optimal financial policy and firm valuation, *Journal of Finance* 39, 593–607.

Cass, D. and J.E. Stiglitz, 1970. The structure of investor preferences and asset returns, and separability in portfolio allocation, *Journal of Economic Theory* 2, 122–160.

Copeland, T.E., J.F. Weston, and K. Shastri, 2005. *Financial Theory and Corporate Policy*, 4th ed. (Pearson Addison Wesley, New York).

Dixit, A.K. and R.S. Pindyck, 1994. *Investment under Uncertainty*, 1st ed. (Princeton University Press, Princeton, NJ).

Fama, E.F. and K.R. French, 1992. The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, E.F. and K.R. French, 1993. Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, E.F. and K.R. French, 2000. Forecasting profitability and earnings, *Journal of Business* 73, 161–175.

Fama, E.F. and K.R. French, 2015. A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fox, J.T. and V. Smeets, 2011. Does input quality drive measured differences in firm productivity? *International Economic Review* 52, 961–989.

Francis, J., P. Olsson, and D.R. Oswald, 2000. Comparing the accuracy and explainability of dividend, free cash flow, and abnormal earnings equity value estimates, *Journal of Accounting Research* 38, 45–70.

Frankel, R. and C.M.C. Lee, 1998. Accounting valuation, market expectation, and cross-sectional stock returns, *Journal of Accounting and Economics* 25, 283–319.

Gilson, S.C., K. John, and L.H.P. Lang, 1990. Troubled debt restructurings, *Journal of Financial Economics* 27, 315–353.

Goldstein, R., N. Ju, and H. Leland, 2001. An EBIT-based model of dynamic capital structure, *Journal of Business* 74, 483–512.

Gordon, M.J., 1962. The savings investment and valuation of a corporation, *The Review of Economics and Statistics* 44, 37–51.

Graham, J.R. and C.R. Harvey, 2001. The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics* 60, 187–243.

Hennessy, C.A., A. Levy, and T.M. Whited, 2007. Testing Q theory with financing frictions, *Journal of Financial Economics* 83, 691–717.

Hennessy, C.A., D. Livdan, and B. Miranda, 2010. Repeated signaling and firm dynamics, *Review of Financial Studies* 23, 1981–2023.

Hennessy, C.A. and T.M. Whited, 2005. Debt dynamics, *Journal of Finance* 60, 1129–1165.

Hennessy, C.A. and T.M. Whited, 2007. How costly is external financing? Evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.

Jegadeesh, N. and S. Titman, 1993. Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jorgenson, D.W. and K.J. Stiroh, 2000. U.S. economic growth at the industry level, *American Economic Review* 90, 161–167.

Kaplan, S.N. and R.S. Ruback, 1995. The valuation of cash flow forecasts: an empirical analysis, *Journal of Finance* 50, 1059–1093.

Kisgen, D.J., 2006. Credit ratings and capital structure, *Journal of Finance* 61, 1035–1072.

Kisgen, D.J., 2009. Do firms target credit ratings or leverage levels? *Journal of Financial and Quantitative Analysis* 44, 1323–1344.

Lakonishok, J., A. Shleifer, and R.W. Vishny, 1994. Contrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541–1578.

Lazzati, N. and A.A. Menichini, 2015. A dynamic approach to the dividend discount model, *Review of Pacific Basin Financial Markets and Policies* 18, 1–36.

Lee, C.M.C., J. Myers, and B. Swaminathan, 1999. What is the intrinsic value of the Dow? *Journal of Finance* 54, 1693–1741.

Leland, H.E., 1994. Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Liu, J., D. Nissim, and J. Thomas, 2002. Equity valuation using multiples, *Journal of Accounting Research* 40, 135–172.

Manzano, B., R. Perez, and J. Ruiz, 2005. Identifying optimal contingent fiscal policies in a business cycle model, *Spanish Economic Review* 7, 245–266.

Modigliani, F. and M.H. Miller, 1958. The cost of capital, corporation finance, and the theory of investment, *American Economic Review* 48, 261–297.

Morse, D. and W. Shaw, 1988. Investing in bankrupt firms, *Journal of Finance* 43, 1193–1206.

Moyen, N., 2004. Investment-cash flow sensitivities: Constrained versus unconstrained firms, *Journal of Finance* 59, 2061–2092.

Moyen, N., 2007. How big is the debt overhang problem? *Journal of Economic Dynamics and Control* 31, 433–472.

Myers, S.C., 1974. Interactions of corporate financing and investment decisions-implications for capital budgeting, *Journal of Finance* 29, 1–25.

Newey, W.K. and K.D. West, 1987. A simple positive-definite heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Ou, S., 2011. Corporate default and recovery rates, 1920-2010, *Moody's Investors Service* February 28, 1–66.

Riddick, L.A. and T.M. Whited, 2009. The corporate propensity to save, *Journal of Finance* 64, 1729–1766.

Rust, J., 1997. Using randomization to break the curse of dimensionality, *Econometrica* 65, 487–516.

Rust, J., 2008. Dynamic programming, in: S.N. Durlauf and L.E. Blume, eds., *The New Palgrave Dictionary of Economics*, 2nd ed. (Palgrave Macmillan, Basingstoke, UK).

Spiegel, M. and H. Tookes, 2013. Dynamic competition, valuation, and merger activity, *Journal of Finance* 68, 125–172.

Strebulaev, I.A., 2007. Do tests of capital structure theory mean what they say? *Journal of Finance* 62, 1747–1787.

Strebulaev, I.A. and T.M. Whited, 2012. Dynamic models and structural estimation in corporate finance, *Foundations and Trends in Finance* 6, 1–163.

Tserlukevich, Y., 2008. Can real options explain financing behavior? *Journal of Financial Economics* 89, 232–252.

Vazza, D. and N.W. Kraemer, 2015. 2014 Annual global corporate default study and rating transitions, *Standard & Poor's Ratings Direct* April 30, 1–126.

Weiss, L.A., 1990. Bankruptcy resolution, *Journal of Financial Economics* 27, 285–314.

Young, A.T., 2013. U.S. elasticities of substitution and factor augmentation at the industry level, *Macroeconomic Dynamics* 17, 861–897.

Zhang, L., 2005. The value premium, *Journal of Finance* 60, 67–103.