Practice for Exam 2

1. Use the definition of the derivative to find the derivative of $f(x)$ at the indicated point:
   a) $f(x) = x^2 - x$ at $x = 3$.
   b) $f(x) = \sqrt{x}$ at $x = 16$.

2. Find the equation for the tangent line to the graph of $y = f(x)$:
   a) $f(x) = \frac{1}{2}x^2 - 3x + 2$ when $x = 2$.
   b) $f(x) = e^{2x-1}$ when $x = 1$.
   c) $f(x) = \tan(4x)$ when $x = \frac{\pi}{16}$

3. Find the derivative:
   a) $y = e^x \sin x$
   b) $g(t) = \frac{t + \sec t}{t^3}$
   c) $y = [\cos(x^5 - 4x^2 + 2)^7$
   d) $f(x) = (\ln x)^3$

4. Show that the tangent line to the curve $y = x^2$ at the point $(1,1)$ passes through the point $(0, -1)$

5. $\lim_{x \to 0} \frac{e^{2x} - 1}{5x}$

6. Let $f(x) = \sqrt{5 + x^2}, x \geq 0$. Find $\frac{d}{dx} f^{-1}(3)$, note that $f(2) = 3$. 
7. Find the linear approximation for $f(x) = \frac{1}{1-x}$ at $a = 0$.

8. Find the first three derivatives of $f(x) = \tan 3x$.

9. Assume the radius $r$ and the volume $V = \frac{4}{3}\pi r^3$ of a sphere are differentiable functions of time $t$. Express $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$.

10. Find $\frac{dy}{dx}$ by implicit differentiation.
    a) $y = x^2 + xy$
    
    b) $x^{\frac{3}{4}} + y^{\frac{3}{4}} = 1$

11. Find the equation for the tangent line and the normal line to the curve given by $y^2 = x^2 - x^4$ at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$