1. Benjamin is fortunate enough to be the only producer in the market. He faces a demand curve:

\[ P = 20 - 3Q \]

and his costs are given by

\[ C = 2Q. \]

a. Suppose Benjamin behaves competitively. Find the equilibrium industry price and quantity.

b. Benjamin realizes he is the only producer and decides to start behaving like a monopolist. Find the monopolist’s choice of price and quantity.

c. On a graph, draw the supply, demand, and marginal revenue curves, and label the equilibrium prices and quantities under perfect competition and monopoly. Calculate the producer and consumer surplus in each case. What is the deadweight loss associated with the monopoly?

d. Suppose the government wants to induce Benjamin to produce the efficient quantity by introducing a quantity tax. Each unit he produces are taxed at \( t \), so that his profit function is given by \([P(Q)-t]Q - 2Q\), where \( P(Q) \) is the market demand curve. Find Benjamin’s production as a function of the tax. What tax leads to the same quantity as in the competitive equilibrium? What is the intuition for this result?

2. Game theory
Two firms are deciding simultaneously whether to enter a market. If neither firm enters, they make zero profits. If both firms enter, they each make profits of -1, since the market is too small for two firms. If only one enters, that firm makes high profits. This game is summarized in the following matrix:

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<tr>
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<tbody>
<tr>
<td>Firm 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enter</td>
<td>-1, -1</td>
<td>10, 0</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>0, 5</td>
<td>0, 0</td>
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a. What are the pure-strategy Nash Equilibria of this game? (3 points)

Now assume that firm 1 can enter the market with probability \( p_1 \) and firm 2 can enter the market with probability \( p_2 \).

b. Write down the expected utility of each firm as a function of the strategy of the other player, and find the best response correspondence for firms 1 and 2.
c. Graph these best response correspondences and find the Nash equilibria in mixed strategies.

Now suppose instead that firm 1 makes its entry decision first, and firm 2 follows. Find the equilibrium outcome of this game.

d. Write down the game tree that describes the payoffs to this game.
e. What are the subgame perfect equilibria?
f. Why is the equilibrium of this game different that what you found in part a?

3. Perfect competition, monopoly, and oligopoly

In this problem you will be given a market demand curve, and a firm cost function function, and you will be asked to find the outcomes of price and quantity under different assumptions regarding the number of firms. The cost function of an individual firm is given by $C(y) = cy$, where the parameter $c>0$ and $y$ is the production of the individual firm. Demand is assumed to be linear, with $p=a-bY$, where $Y$ is the total industry quantity.

a. Is the production function of an individual firm
b. First assume that the market is characterized by perfect competition. What is average cost and marginal cost? What is the supply curve for an individual firm? How about market supply?
c. What is the equilibrium price and quantity?
d. Now assume that one firm is a monopolist. What is this firm’s profit function? Maximize this with respect to output. What is the quantity produced, and what is the price?
e. Now assume that there is a duopoly – there are two firms $i=1,2$. Write the profit function of firm $i$. Maximize this with respect to the firm’s production, and express the production of firm $i$, $y_i$, as a function of the production of the other firm.
f. Find the Nash equilibrium level of production for each firm, $y_1^*$ and $y_2^*$. How much is produced in the industry in total, $Y=y_1^*+y_2^*$? What is the price? Compare the duopoly price and output with the perfect competition and monopoly case.
g. Now assume that there are $N$ firms in the market. Write down the profit maximization of a particular firm, $i$. Find the first-order condition of the firm.
h. We now want to find the solution to this general oligopoly case. To do so, you need to assume a symmetric solution, which means that all firms will produce the same quantity. We know this will be the outcome here since all firms have the same cost function. Use this assumption in the first-order condition you found in part g to obtain the Nash equilibrium quantity. What is the industry production and the industry price?
i. We can now derive what happens to industry output and price as the number of firms varies. Using your answer to h, what is industry price and production
if \(N=1\)? \(N=2\)? \(N=\infty\)? How does this compare to what you found in earlier parts of the problem?