In this chapter we studied the supply decision of a profit-maximizing firm. Our general goal was to show how such a firm responds to price signals from the marketplace. In addressing that question, we developed a number of analytical results.

• In order to maximize profits, the firm should choose to produce at that output level for which marginal revenue (the revenue from selling one more unit) is equal to marginal cost (the cost of producing one more unit).

• If a firm is a price taker then its output decisions do not affect the price of its output, so marginal revenue is given by this price. If the firm faces a downward-sloping demand for its output, however, then it can sell more only at a lower price. In this case marginal revenue will be less than price and may even be negative.

• Marginal revenue and the price elasticity of demand are related by the formula

\[ MR = P \left(1 + \frac{1}{e_P}\right) \]

where \( P \) is the market price of the firm's output and \( e_P \) is the price elasticity of demand for its product.

• The supply curve for a price-taking, profit-maximizing firm is given by the positively-sloped portion of its marginal cost above the point of minimum average variable cost (AVC). If price falls below minimum AVC, the firm's profit-maximizing choice is to shut down and produce nothing.

• The firm's reactions to changes in the various prices it faces can be studied through use of its profit function.

The profit function can also be used to calculate changes in producer surplus.

PROBLEMS

11.1
John's Lawn Moving Service is a small business that acts as a price taker (i.e., MR = P). The prevailing market price of lawn mowing is $20 per acre. John's costs are given by

\[ TC = 0.1Q^2 + 10Q + 50 \]

where \( Q \) is the number of acres John chooses to cut a day.

a. How many acres should John choose to cut in order to maximize profit?
b. Calculate John's maximum daily profit.
c. Graph these results and label John's supply curve.

11.2
Would a lump-sum profits tax affect the profit-maximizing quantity of output? How about a proportional tax on profits? How about a tax assessed on each unit of output? How about a tax on labor input?

11.3
This problem concerns the relationship between demand and marginal revenue curves for a few functional forms.

a. Show that, for a linear demand curve, the marginal revenue curve bisects the distance between the vertical axis and the demand curve for any price.
b. Show that, for any linear demand curve, the vertical distance between the demand and marginal revenue curves is \( 1/k \cdot e \), where \( k < 0 \) is the slope of the demand curve.
c. Show that, for a constant elasticity demand curve of the form \( q = aP^\gamma \), the vertical distance between the demand and marginal revenue curves is a constant ratio of the height of the demand curve, with this constant depending on the price elasticity of demand.
d. Show that, for any downward-sloping demand curve, the vertical distance between the demand and marginal revenue curves at any point can be found by using a linear approximation to the demand curve at that point and applying the procedure described in part (b).
e. Graph the results of parts (a)-(d) of this problem.
PROBLEMS

12.1 Suppose there are 100 identical firms in a perfectly competitive industry. Each firm has a short-run total cost function of the form

\[ C(q) = \frac{1}{360}q^3 + 0.2q^2 + 4q + 10. \]

a. Calculate the firm’s short-run supply curve with \( q \) as a function of market price \( P \).

b. On the assumption that there are no interaction effects among costs of the firms in the industry, calculate the short-run industry supply curve.

c. Suppose market demand is given by \( Q = -200P + 8,000 \). What will be the short-run equilibrium price-quantity combination?

12.2 Suppose there are 1,000 identical firms producing diamonds. Let the total cost function for each firm be given by

\[ C(q, w) = q^2 + wq, \]

where \( q \) is the firm’s output level and \( w \) is the wage rate of diamond cutters.

a. If \( w = 10 \), what will be the firm’s (short-run) supply curve? What is the industry’s supply curve? How many diamonds will be produced at a price of 20? How many more diamonds would be produced at a price of 21?

b. Suppose the wages of diamond cutters depend on the total quantity of diamonds produced and suppose the form of this relationship is given by

\[ w = 0.002Q, \]

where \( Q \) represents total industry output, which is 1,000 times the output of the typical firm.

In this situation, show that the firm’s marginal cost (and short-run supply curve) depends on \( Q \). What is the industry supply curve? How much will be produced at a price of 20? How much more will be produced at a price of 21? What do you conclude about the shape of the short-run supply curve?

12.3 A perfectly competitive market has 1,000 firms. In the very short run, each of the firms has a fixed supply of 100 units. The market demand is given by

\[ Q = 160,000 - 10,000P. \]

a. Calculate the equilibrium price in the very short run.

b. Calculate the demand schedule facing any one firm in this industry.

c. Calculate what the equilibrium price would be if one of the sellers decided to sell nothing or if one seller decided to sell 200 units.

d. At the original equilibrium point, calculate the elasticity of the industry demand curve and the elasticity of the demand curve facing any one seller.

Suppose now that, in the short run, each firm has a supply curve that shows the quantity the firm will supply \( (q) \) as a function of market price. The specific form of this supply curve is given by

\[ q = -200 + 50P. \]

Using this short-run supply response, supply revised answers to (a)-(d).

12.4 A perfectly competitive industry has a large number of potential entrants. Each firm has an identical cost structure such that long run average cost is minimized at an output of 20 units \( (q_1 = 20) \). The minimum average cost is $10 per unit. Total market demand is given by

\[ Q = 1,500 - 50P. \]

a. What is the industry’s long-run supply schedule?

b. What is the long-run equilibrium price \( (P^*) \)? The total industry output \( (Q^*) \)? The output of each of the firms \( (q_1) \)? The profit of each firm?

c. The short-run total cost function associated with each firm’s long-run equilibrium output is given by

\[ C(q) = 0.5q^2 - 10q + 200. \]

Calculate the short-run average and marginal cost function. At what output level does short-run average cost reach a minimum?

d. Calculate the short-run supply function for each firm and the industry short-run supply function.

e. Suppose now that the market demand function shifts upward to \( Q = 2,000 - 50P \). Using this new demand curve, answer part (b) for the very short run when firms cannot change their outputs.

f. In the short run, use the industry short-run supply function to recalculate the answers to (b).

g. What is the new long-run equilibrium for the industry?

12.5 Suppose that the demand for stilts is given by

\[ Q = 1,500 - 50P \]

and that the long-run total operating costs of each stilts-making firm in a competitive industry are given by

\[ C(q) = 0.5q^2 - 10q \]

where \( w \) is the annual wage paid. Suppose also that each stilts-making firm requires one (and only one) entrepreneur (hence, the quantity of entrepreneurs hired is equal to the number of firms). Long-run total costs for each firm are then given by

\[ C(q, w) = 0.5q^2 - 10q + w. \]

a. What is the long-run equilibrium quantity of stilts produced? How many stilts are produced by each firm? What is the long-run equilibrium price of stilts? How many firms will there be? How many entrepreneurs will be hired, and what is their wage?