1. NS 9.2, 9.5

2. A firm’s production function for hand-squeezed orange juice uses two inputs, oranges and labor. Let $x_1$ denote the kilograms of oranges used and let $x_2$ denote the hours of labor used in squeezing. The production function for orange juice is

$$y = x_1^{1/3} x_2^{1/3}$$

where $y$ is measured in liters of orange juice. Let $w_1$ denote the cost of a kilogram of oranges and let $w_2$ denote the wage for an hour of labor.

(a) Show whether this production function exhibits CRS, DRS, or IRS.
(b) Find the marginal product of each input, $x_1$ and $x_2$. Find the technical rate of substitution. Explain in words what the TRS tells you.
(c) Minimize the firm’s cost of using $x_1$ and $x_2$ subject to producing $y$. What are the conditional factor demand functions of $x_1$ and $x_2$?
(d) What is the cost function that minimizes the cost of producing $y$ at prices $w_1$ and $w_2$? What is average cost? What is marginal cost?
(e) Suppose the price of input 1 is given by $w_1=10$ and the price of input 2 is $w_2=30$. Write the firm’s profit function. Find the quantity that maximizes profit as a function of price.

3. Consider a firm that produces sweaters using two inputs: capital $K$ (in machine hours) and labor $L$ (in hours). The production function is $Y = K^{1/2} L^{1/2}$. The input prices this firm faces are $r=4$ and $w=36$, where $r$ is the cost of capital and $w$ is the wage. Suppose the firm expects to produce $Y = 300$ units of output.

(a) Calculate the cost minimizing input mix. What is the total cost of production, and what is the average cost per unit of output?

Suppose the output target unexpectedly rises to 450 units after the capital has been installed.

(b) In the short run the capital stock is fixed and cannot be changed. What is the firm’s short-run production function? Does it display IRS, CRS, or DRS?
(c) What amount of labor should the firm use to produce the 450 units at minimum costs? What are the variable costs of production? What are total costs (including the cost of the capital)? What is the cost per unit?
(d) Repeat part (c) assuming the output target falls to 200 in the short-run. Plot the three points on this firm’s short-run average cost function.