Econ 100A: Intermediate Microeconomics  
Answer Key to Problem Set 1  
Fall, 2011

Note: This answer key does not contain graphs. For all graphs, refer to the supplement to this answer key that has been provided alongside.

1. Supply and Demand

Demand Curve: \( Q^D = 20 - 2P \)
Supply Curve: \( Q^S = 2P \)

a) The inverse demand curve can be obtained by expressing the demand curve in terms of \( P \). Therefore, we get:

Inverse Demand Curve: \( P = 10 - \frac{1}{2}Q^D \)

The marginal willingness to pay for any unit is just the price of that unit on the demand curve. Therefore, we can plug \( Q=5 \) in the inverse demand curve to get:

Marginal willingness to pay = \( 10 - 2.5 = 7.5 \)

b) To get the equilibrium price, set the quantity demanded equal to the quantity supplied:

At equilibrium: \( 20-2P = 2P \)
\[ \Rightarrow 20 = 4 \]
\[ \therefore P^* = 5 \]

Plug the equilibrium value of \( P \) into either the demand or the supply function to get the equilibrium quantity. Plugging \( P^* = 5 \) into the supply function yields:

\[ Q^* = 2 \times 5 \]
\[ Q^* = 10 \]

c) Since the demand curve is linear, the price elasticity of demand is given by \(-m\frac{P}{Q}\), where \( m \) is the slope of the demand curve.

\[ \therefore \epsilon = 2 \frac{P}{Q} \]

At equilibrium, this takes the value: \( \epsilon = \frac{2 \times 5}{10} = 1 \)
Therefore, demand is neither elastic, nor inelastic at this point.

d) Alternative Supply Curve: \( Q^S = 5 \)

Notice that the quantity supplied is fixed at 5 units. This implies that the market clearing price is the price which sets the quantity demanded to also be 5. Therefore, we can plug \( Q^D = 5 \) into the inverse demand curve from (a) to get:
\[ P = 10 - \frac{1}{2} \times 5 \]
\[ \therefore P^* = 7.5 \]

e) Alternative Demand Curve: \( Q^D = 30 - 2P \)
At equilibrium with the original supply curve: \( 30 - 2P = 2P \)
\[ \Rightarrow P^* = 7.5 \]

At the original equilibrium in part (a), the market clearing price was 5. With the alternative specification, the demand curve is shifted out/up, implying that the consumers are demanding more at each price. For instance, a vertical intercept of 20 in the original demand curve corresponds to the quantity demanded being zero at a price of 20. Under the new specification, however, the price needs to go up to 30 for the demand to fall to 0. A potential reason for such a shift could be an increase in income.

Since the consumers are now demanding more at each price; for a given supply curve, the market will clear at a higher price. Hence, the shift from 5 to 7.5.

\[ e.2. \] At equilibrium with the alternative supply curve: \( 30 - 2P = 5 \)
\[ \Rightarrow P^* = 12.5 \]

Here, too, the equilibrium price is higher than the one we got in part (d). The reason for this is identical to what was explained above in e.1.

Also, notice that the equilibrium price in this case is higher than what we got in e.1 with the original supply specification. This is because the alternative supply curve from (d) is perfectly inelastic - the quantity supplied is fixed at 5, and the suppliers are unable to increase it in response to an increase in demand. As a result, the price must climb even higher to accommodate the increase in demand. With the original supply curve, the equilibrium quantity also goes up (to 15), leading to a smaller price increase.

2. Preferences Problem

\[ U(x_1, x_2) = \ln(x_1) + 2x_2 \]

a) Differentiate the utility function with respect to \( x_1 \) and \( x_2 \) to get the respective marginal utilities:
\[ MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{x_1} \]
\[ MU_2 = \frac{\partial U}{\partial x_2} = 2 \]

For a person to prefer more to less, his marginal utility must be positive, i.e., consuming more should add to his total utility.

We know that, \( MU_1 = \frac{1}{x_1} > 0 \), for positive values of \( x_1 \).

Also, \( MU_2 = 2 > 0 \)

Therefore, this person prefers more to less.

We can take the second derivative of the utility function to ascertain if the marginal utility is diminishing:
This utility function exhibits diminishing marginal utility with respect to good 1, and constant marginal utility with respect to good 2.

\[
\frac{\partial^2 U}{\partial x_1^2} = -\frac{1}{x_1^2} < 0 \\
\frac{\partial^2 U}{\partial x_2^2} = 0
\]

b) \( \text{MRS}_{x_1, x_2} = \frac{M_{U_{x_1}}}{M_{U_{x_2}}} = \frac{\frac{1}{x_1}}{\frac{1}{2x_1}} = \frac{1}{2} \)

\[
\frac{\partial \text{MRS}_{x_1, x_2}}{\partial x_1} = -\frac{1}{x_1} < 0
\]

Therefore, the marginal rate of substitution of good 1 for good 2 is diminishing as the consumption of good 1 is increasing.

c) Note that \( x_1 \) appears in the utility function as its natural log. We know that the log of 0 if not defined - this implies that the consumer always has to consume a positive amount of \( x_1 \). Therefore, the indifference curves cannot intercept the \( x_2 \) axis.

We have no such restriction on the consumption of \( x_2 \). Therefore, we can have an \( x_1 \) intercept.

For a plot of the typical indifference curve, refer to the supplementary document containing graphs.

3. Constrained Optimization Problems:

(Note: There are two alternative ways of doing these problems. This key demonstrates both methods for part (a), and only 1 for all the problems thereafter. You can choose to do either on the exam.)

Budget Constraint: \( 2F + 4C \leq 100 \)

a) \( U(F, C) = F^{\frac{1}{4}}C^{\frac{3}{4}} \)

Method 1:
Set up the Lagrange for optimization:

\[
\mathcal{L} = \max_{F, C, \lambda} F^{\frac{1}{4}}C^{\frac{3}{4}} + \lambda(100 - 2F - 4C)
\]

First Order Conditions:

\[
F: \frac{1}{4}F^{-\frac{3}{4}}C^{\frac{3}{4}} - 2\lambda = 0 \quad (1) \\
C: \frac{3}{4}F^{\frac{1}{4}}C^{-\frac{1}{4}} - 4\lambda = 0 \quad (2) \\
\lambda: 100 - 2F - 4C = 0 \quad (3)
\]

Express equation 3 in terms of \( F \):

\[
F = 50 - 2C \quad (4)
\]
Substitute the value for \( F \) in equations 1 and 2, and express in terms of \( \lambda \) to get:

\[
\frac{1}{8} (50 - 2C)^{-\frac{3}{4}} C^{\frac{3}{4}} = \lambda \quad (1a) \\
\frac{3}{16} (50 - 2C)^{\frac{1}{4}} C^{-\frac{1}{4}} = \lambda \quad (2a)
\]

\[
\Rightarrow \frac{1}{8} \left( \frac{50 - 2C}{C} \right)^{-\frac{3}{4}} = \frac{3}{16} \left( \frac{50 - 2C}{C} \right)^{\frac{1}{4}} \quad (5)
\]

\[
\Rightarrow \frac{50 - 2C}{C} = \frac{2}{3} \\
\Rightarrow 150 - 6C = 2C \\
C = \frac{150}{8} = \frac{75}{4} \\
F = 50 - 2 \times \frac{150}{8} = \frac{50}{4} = \frac{25}{2}
\]

Method 2:

At the optimum, the consumer will set the MRS equal to the price ratio:

\[
MRS_{f,c} = \frac{\frac{1}{4} F^{\frac{3}{4}} C^{-\frac{1}{4}}}{\frac{3}{4} F^{\frac{1}{4}} C^{-\frac{1}{4}}} = \frac{1}{3} \frac{C}{F}
\]

\[
\frac{P_f}{P_c} = \frac{1}{2}
\]

At the optimum: \( \frac{1}{3} \frac{C}{F} = \frac{1}{2} \)

\[
\Rightarrow C = \frac{3}{2} F \\
\therefore C = \frac{3}{2} F
\]

Substitute into equation 3:

\[
100 - 2F - 4 \times \frac{3}{2} F = 0 \\
100 - 8F = 0
\]

\[
\Rightarrow F = \frac{25}{2} \\
C = \frac{3}{2} \times \frac{25}{2} = \frac{75}{4}
\]

b) \( U(F,C) = .25 \ln(F) + .75 \ln(C) \)

Set up the Lagrange for optimization:
\[ \mathcal{L} = \max_{F,C,\lambda} 25 \ln(F) + 0.75 \ln(C) + \lambda(100 - 2F - 4C) \]

First Order Conditions:

\[ F: \frac{25}{F} - 2\lambda = 0 \quad (1) \]
\[ C: \frac{75}{C} - 4\lambda = 0 \quad (2) \]
\[ \lambda: 100 - 2F - 4C = 0 \quad (3) \]

Express equation 3 in terms of F:

\[ F = 50 - 2C \quad (4) \]

Substitute the value for F in equation 1, and express in terms of \( \lambda \) to get:

\[ \frac{25}{2(50 - 2C)} = \lambda \quad (1a) \]
\[ \frac{75}{4C} = \lambda \quad (2a) \]

\[ \Rightarrow \frac{25}{100 - 4C} = \frac{75}{4C} \]
\[ \Rightarrow \frac{100 - 4C}{4C} = 3 \]
\[ \Rightarrow 4C = 300 - 12C \]
\[ C = \frac{150}{4} = \frac{75}{2} \]
\[ F = \frac{50}{2} = \frac{25}{2} \]

\( c) \ U(F,C) = \ln(F) + 4C \)

Set up the Lagrange for optimization:

\[ \mathcal{L} = \max_{F,C,\lambda} \ln(F) + 4C + \lambda(100 - 2F - 4C) \]

First Order Conditions:

\[ F: \frac{1}{F} - 2\lambda = 0 \quad (1) \]
\[ C: 4 - 4\lambda = 0 \quad (2) \]
\[ \lambda: 100 - 2F - 4C = 0 \quad (3) \]

Express equation 3 in terms of F:

\[ F = 50 - 2C \quad (4) \]

Equation 2 \( \Rightarrow \lambda = 1 \) \( (5) \)

Substitute (5) into (1) to get:

\[ F = \frac{1}{2} \]
Substitute the value of $F$ into equation 3 to get:

$$100 - 1 - 4C = 0$$

$$\Rightarrow C = \frac{99}{4}$$

d) $U(F,C) = F + 3C$

Notice that the two goods are perfect complements, and we will have a corner solution unless the MRS equals the slope of the price line.

$$MRS_{c,f} = \frac{MU_c}{MU_F} = 3$$
$$\frac{P_c}{P_f} = 2$$

$$\Rightarrow MRS_{c,f} > \frac{P_c}{P_f}$$

Therefore, this consumer will only consume clothing at the optimum, and will choose the quantity that exhausts his budget.

$$\therefore 4C = 100$$
$$\Rightarrow C = 25$$
$$F = 0$$

4. Demand

$U(F,C) = \ln(F) + \ln(C)$

Income: $I$

Price of food: $p_f$

Price of clothing: $p_c$

a) At the optimum, the consumer sets MRS equal to the price ratio, i.e.:

$$\frac{C}{F} = \frac{p_f}{p_c}$$

$$\Rightarrow p_f.F = p_c.C \quad (1)$$

The consumer’s budget constraint is given by:

$$I - p_f.F - p_c.C = 0$$

Substituting (1) into the budget constraint:
\[ I - 2p_f \cdot F = 0 \]
\[ \Rightarrow F = \frac{I}{2p_f} \quad \text{Demand Curve for Food} \]

Similarly, calculate demand curve for Clothing:
\[ C = \frac{I}{2p_c} \]

b) We know that the demand curve for food is given by:
\[ F = \frac{I}{2p_f} \]
\[ \varepsilon_{F,I} = \frac{\Delta F}{\Delta I} \cdot \frac{I}{F} = \frac{1}{2p_f} \cdot \frac{I}{I} = \frac{1}{2p_f} \]

c) We already derived the demand functions for F and C in part a:
\[ F = \frac{I}{2p_f} \]
\[ C = \frac{I}{2p_c} \]

We can substitute the values for income and prices into this demand function to derive the optimal basket at each income level:

1) \( I = 10 \)
\[ F = \frac{10}{2 \times 1} = 5 \]
\[ C = \frac{10}{2 \times 1} = 5 \]

2) \( I = 20 \)
\[ F = \frac{20}{2 \times 1} = 10 \]
\[ C = \frac{20}{2 \times 1} = 10 \]

3) \( I = 30 \)
\[ F = \frac{30}{2 \times 1} = 15 \]
\[ C = \frac{30}{2 \times 1} = 15 \]

For the income consumption curve and the engel curves based on this data, please refer to the supplementary document containing graphs.

d) \( I = 30; p_f = 1 \)

Note that the demand for food is a function of income and the price of food alone, both of which are constant. Therefore, as the price of clothing changes, the quantity demanded of food will stay constant at 15.

The optimal consumption value of clothing for different values of \( p_c \) is given by:
1) \( P_c = 1 \)
\[
C = \frac{30}{2 \times 1} = 15
\]

2) \( P_c = 2 \)
\[
C = \frac{30}{2 \times 2} = 7.5
\]

3) \( P_c = 3 \)
\[
C = \frac{30}{2 \times 3} = 5
\]

Therefore, the optimal consumption basket for different values of \( p_c \) is given by:

- \( p_c = 1 \): \{15,15\}
- \( p_c = 2 \): \{15,7.5\}
- \( p_c = 3 \): \{15,5\}

For the price consumption curve and the demand curves based on this data, please refer to the supplementary document containing graphs.
Typical Indifference Curves
for \( U = \ln (x_1) + 2x_2 \)
4(c) Income Consumption Curve

Engel Curves

(a) Food

(b) Clothing

In this case, the Engel curves for F & C are identical due to identical prices and the form of the utility function. This won't always be the case.
4(d) Price Consumption Curve.

Demand Curve for Clothing.