In particular, the set \( \{u_1, u_2\} \) is linearly independent, and hence is a basis for \( \mathbb{R}^2 \) since there are two vectors in the set.

2. When \( y = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \) and \( u = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \),

\[
\hat{y} = \frac{y \cdot u}{u \cdot u} u = \frac{20}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}
\]

This is the same \( \hat{y} \) found in Example 3. The orthogonal projection does not seem to depend on the \( u \) chosen on the line. See Exercise 31.

3. \( Uy = \begin{bmatrix} 1/\sqrt{2} & 2/3 \\ 1/\sqrt{2} & -2/3 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} -3\sqrt{2} \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -7/2 \end{bmatrix} \]

Also, from Example 6, \( x = \begin{bmatrix} \sqrt{2} \\ 3 \end{bmatrix} \) and \( Ux = \begin{bmatrix} 1/\sqrt{2} \\ -5/2 \end{bmatrix} \). Hence

\[
Ux \cdot Uy = 3 + 7/2 = 12, \quad \text{and} \quad x \cdot y = -6 + 18 = 12
\]

6.3 ORTHOGONAL PROJECTIONS

The orthogonal projection of a point \( y \) in \( \mathbb{R}^2 \) onto a line through the origin has an important analogue in \( \mathbb{R}^n \). Given a vector \( y \) and a subspace \( W \) in \( \mathbb{R}^n \), there is a vector \( \hat{y} \) in \( W \) such that (1) \( \hat{y} \) is the unique vector in \( W \) for which \( y - \hat{y} \) is orthogonal to \( W \), and (2) \( \hat{y} \) is the unique vector in \( W \) closest to \( y \). See Fig. 1. These two properties of \( \hat{y} \) provide the key to finding least-squares solutions of linear systems, mentioned in the introductory example for this chapter. The full story will be told in Section 6.5.

To prepare for the first theorem, we observe that whenever a vector \( y \) is written as a linear combination of vectors \( u_1, \ldots, u_n \) in a basis of \( \mathbb{R}^n \), the terms in the sum for \( y \) can be grouped into two parts so that \( y \) can be written as

\[
y = z_1 + z_2
\]

where \( z_1 \) is a linear combination of some of the \( u_i \) and \( z_2 \) is a linear combination of the rest of the \( u_i \). This idea is particularly useful when \( \{u_1, \ldots, u_n\} \) is an orthogonal basis. Recall from Section 6.1 that \( W^\perp \) denotes the set of all vectors orthogonal to a subspace \( W \).

**Example 1.** Let \( \{u_1, \ldots, u_5\} \) be an orthogonal basis for \( \mathbb{R}^5 \) and let

\[
y = c_1 u_1 + \cdots + c_5 u_5
\]

Consider the subspace \( W = \text{Span} \{u_1, u_2\} \), and write \( y \) as the sum of a vector \( z_1 \) in \( W \) and a vector \( z_2 \) in \( W^\perp \).
Solution

Write

\[ y = \frac{c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4 + c_5u_5}{z_1 + z_2} \]

where \( z_1 = c_1u_1 + c_2u_2 \) is in \( \text{Span}\{u_1, u_2\} \)

and \( z_2 = c_3u_3 + c_4u_4 + c_5u_5 \) is in \( \text{Span}\{u_3, u_4, u_5\} \).

To show that \( z_2 \) is in \( W^\perp \), it suffices to show that \( z_2 \) is orthogonal to the vectors in the basis \( \{u_1, u_2\} \) for \( W \). (See Section 6.1.) Using properties of the inner product, compute

\[ z_2 \cdot u_1 = (c_3u_3 + c_4u_4 + c_5u_5) \cdot u_1 \]
\[ = c_3u_3 \cdot u_1 + c_4u_4 \cdot u_1 + c_5u_5 \cdot u_1 \]
\[ = 0 \]

because \( u_1 \) is orthogonal to \( u_3, u_4, \) and \( u_5 \). A similar calculation shows that \( z_2 \cdot u_2 = 0 \). Thus \( z_2 \) is in \( W^\perp \).

The next theorem shows that the decomposition \( y = z_1 + z_2 \) in Example 1 can be computed without having an orthogonal basis for \( \mathbb{R}^n \). It is enough to have an orthogonal basis only for \( W \).

**Theorem 8**

**The Orthogonal Decomposition Theorem**

Let \( W \) be a subspace of \( \mathbb{R}^n \). Then each \( y \) in \( \mathbb{R}^n \) can be written uniquely in the form

\[ y = \hat{y} + z \quad (1) \]

where \( \hat{y} \) is in \( W \) and \( z \) is in \( W^\perp \). In fact, if \( \{u_1, \ldots, u_p\} \) is any orthogonal basis of \( W \), then

\[ \hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \cdots + \frac{y \cdot u_p}{u_p \cdot u_p} u_p \quad (2) \]

and \( z = y - \hat{y} \).

The vector \( \hat{y} \) in (1) is called the **orthogonal projection of \( y \) onto \( W \)** and often is written as \( \text{proj}_W y \). See Fig. 2. When \( W \) is a one-dimensional subspace, the formula for \( \hat{y} \) matches the formula given in Section 6.2.

![FIGURE 2](image.png) The orthogonal projection of \( y \) onto \( W \).
**THEOREM**  Let \( \{u_1, \ldots, u_p\} \) be an orthogonal basis for \( W \), and define \( \hat{y} \) by (2). Then \( \hat{y} \) is in \( W \) because \( \hat{y} \) is a linear combination of the basis \( u_1, \ldots, u_p \). Let \( z = y - \hat{y} \). Since \( u_1 \) is orthogonal to \( u_2, \ldots, u_p \), it follows from (2) that

\[
\begin{align*}
z \cdot u_1 &= (y - \hat{y}) \cdot u_1 = y \cdot u_1 - \left( \frac{y \cdot u_1}{u_1 \cdot u_1} \right) u_1 \cdot u_1 = 0 - \cdots - 0 \\
\hat{y} \cdot u_1 &= y \cdot u_1 - y \cdot u_1 = 0
\end{align*}
\]

Thus \( z \) is orthogonal to \( u_1 \). Similarly, \( z \) is orthogonal to each \( u_i \) in the basis for \( W \). Hence \( z \) is orthogonal to every vector in \( W \). That is, \( z \) is in \( W^\perp \).

To show that the decomposition in (1) is unique, suppose \( y \) can also be written as \( y = \hat{y}_1 + z_1 \), with \( \hat{y}_1 \) in \( W \) and \( z_1 \) in \( W^\perp \). Then \( \hat{y} + z = \hat{y}_1 + z_1 \) (since both sides equal \( y \)), and so

\[
\hat{y} - \hat{y}_1 = z_1 - z
\]

This equality shows that the vector \( v = \hat{y} - \hat{y}_1 \) is in \( W \) and in \( W^\perp \) (because \( z_1 \) and \( z \) are both in \( W^\perp \), and \( W^\perp \) is a subspace). Hence \( v \cdot v = 0 \), which shows that \( v = 0 \). This proves that \( \hat{y} = \hat{y}_1 \) and also \( z_1 = z \).

The uniqueness of the decomposition (1) shows that the orthogonal projection \( \hat{y} \) depends only on \( W \) and not on the particular basis used in (2).

**EXAMPLE 2** Let \( u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \), \( u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \), and \( y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \). Observe that \( \{u_1, u_2\} \) is an orthogonal basis for \( W = \text{Span} \{u_1, u_2\} \). Write \( y \) as the sum of a vector in \( W \) and a vector orthogonal to \( W \).

**Solution** The orthogonal projection of \( y \) onto \( W \) is

\[
\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2
\]

\[
= \frac{9}{30} \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} + \frac{3}{6} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = \frac{9}{30} \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} + \frac{15}{30} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix}
\]

Also

\[
y - \hat{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix} = \begin{bmatrix} 7/5 \\ 0 \\ 14/5 \end{bmatrix}
\]

Theorem 8 ensures that \( y - \hat{y} \) is in \( W^\perp \). To check the calculations, however, it is a good idea to verify that \( y - \hat{y} \) is orthogonal to both \( u_1 \) and \( u_2 \) and hence to all of \( W \). The

---

1 We may assume that \( W \) is not the zero subspace, for otherwise \( W^\perp = \mathbb{R}^n \) and (1) is simply \( y = 0 + y \).

The next section will show that any nonzero subspace of \( \mathbb{R}^n \) has an orthogonal basis.
6.3 Orthogonal Projections

The desired decomposition of \( \mathbf{y} \) is

\[
\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix} + \begin{bmatrix} 7/5 \\ 0 \\ 14/5 \end{bmatrix}
\]

A Geometric Interpretation of the Orthogonal Projection

When \( W \) is a one-dimensional subspace, the formula (2) for \( \text{proj}_W \mathbf{y} \) contains just one term. Thus, when \( \dim W > 1 \), each term in (2) is itself an orthogonal projection of \( \mathbf{y} \) onto a one-dimensional subspace spanned by one of the \( \mathbf{u} \)'s in the basis for \( W \). Figure 3 illustrates this when \( W \) is a subspace of \( \mathbb{R}^3 \) spanned by \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \). Here \( \hat{y}_1 \) and \( \hat{y}_2 \) denote the projections of \( \mathbf{y} \) onto the lines spanned by \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \), respectively. The orthogonal projection \( \hat{y} \) of \( \mathbf{y} \) onto \( W \) is the sum of the projections of \( \mathbf{y} \) onto one-dimensional subspaces that are orthogonal to each other. The vector \( \hat{y} \) in Fig. 3 corresponds to the vector \( \mathbf{y} \) in Fig. 4 of Section 6.2, because now it is \( \hat{y} \) that is in \( W \).

![Figure 3](image)

**FIGURE 3** The orthogonal projection of \( \mathbf{y} \) is the sum of its projections onto one-dimensional subspaces that are mutually orthogonal.

Properties of Orthogonal Projections

If \( \{\mathbf{u}_1, \ldots, \mathbf{u}_p\} \) is an orthogonal basis for \( W \) and if \( \mathbf{y} \) happens to be in \( W \), then the formula for \( \text{proj}_W \mathbf{y} \) is exactly the same as the representation of \( \mathbf{y} \) given in Theorem 5 in Section 6.2. In this case, \( \text{proj}_W \mathbf{y} = \mathbf{y} \).

If \( \mathbf{y} \) is in \( W = \text{Span} \{\mathbf{u}_1, \ldots, \mathbf{u}_p\} \), then \( \text{proj}_W \mathbf{y} = \mathbf{y} \).

This fact also follows from the next theorem.
The Best Approximation Theorem

Let $W$ be a subspace of $\mathbb{R}^n$, $y$ any vector in $\mathbb{R}^n$, and $\hat{y}$ the orthogonal projection of $y$ onto $W$. Then $\hat{y}$ is the closest point in $W$ to $y$, in the sense that

$$||y - \hat{y}|| < ||y - v||$$

for all $v$ in $W$ distinct from $y$.

The vector $\hat{y}$ in Theorem 9 is called the best approximation to $y$ by elements of $W$. In later sections, we will examine problems where a given $y$ must be replaced, or approximated, by a vector $v$ in some fixed subspace $W$. The distance from $y$ to $v$, given by $||y - v||$, can be regarded as the "error" of using $v$ in place of $y$. Theorem 9 says that this error is minimized when $v = \hat{y}$.

Equation (3) leads to a new proof that $\hat{y}$ does not depend on the particular orthogonal basis used to compute it. If a different orthogonal basis for $W$ were used to construct an orthogonal projection of $y$, then this projection would also be the closest point in $W$ to $y$, namely, $\hat{y}$.

**Proof** Take $v$ in $W$ distinct from $\hat{y}$. See Fig. 4. Then $\hat{y} - v$ is in $W$. By the Orthogonal Decomposition Theorem, $y - \hat{y}$ is orthogonal to $W$. In particular, $y - \hat{y}$ is orthogonal to $\hat{y} - v$ (which is in $W$). Since

$$y - v = (y - \hat{y}) + (\hat{y} - v)$$

the Pythagorean Theorem gives

$$||y - v||^2 = ||y - \hat{y}||^2 + ||\hat{y} - v||^2$$

(See the colored right triangle in Fig. 4. The length of each side is labeled.) Now $||\hat{y} - v||^2 > 0$ because $\hat{y} - v \neq 0$, and so the inequality in (3) follows immediately.

![The orthogonal projection of $y$ onto $W$ is the closest point in $W$ to $y$.](image)

**Example 3**

If $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $W = \text{Span} \{u_1, u_2\}$, as in Example 2, then the closest point in $W$ to $y$ is

$$\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2 = \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix}$$
**Example 4.** The distance from a point \( y \) in \( \mathbb{R}^n \) to a subspace \( W \) is defined as the distance from \( y \) to the nearest point in \( W \). Find the distance from \( y \) to \( W = \text{Span} \{ u_1, u_2 \} \), where
\[
y = \begin{bmatrix} -1 \\ -5 \\ 10 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}
\]

**Solution** By the Best Approximation Theorem, the distance from \( y \) to \( W \) is \( \| y - \hat{y} \| \), where \( \hat{y} = \text{proj}_W y \). Since \( \{ u_1, u_2 \} \) is an orthogonal basis for \( W \),
\[
\hat{y} = \frac{15}{30} u_1 + \frac{-21}{6} u_2 = \frac{1}{2} \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} - \frac{7}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 4 \end{bmatrix}
\]

\[
y - \hat{y} = \begin{bmatrix} -1 \\ -5 \\ 10 \end{bmatrix} - \begin{bmatrix} -8 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \\ 6 \end{bmatrix}
\]

\[
\| y - \hat{y} \|^2 = 7^2 + (-9)^2 + 6^2 = 45
\]

The distance from \( y \) to \( W \) is \( \sqrt{45} = 3\sqrt{5} \).

The final theorem of this section shows how formula (2) for \( \text{proj}_W y \) is simplified when the basis for \( W \) is an orthonormal set.

**Theorem 10** If \( \{ u_1, \ldots, u_p \} \) is an orthonormal basis for a subspace \( W \) of \( \mathbb{R}^n \), then
\[
\text{proj}_W y = (y \cdot u_1) u_1 + (y \cdot u_2) u_2 + \cdots + (y \cdot u_p) u_p
\]

(4)

If \( U = [ u_1 \quad u_2 \quad \cdots \quad u_p ] \), then
\[
\text{proj}_W y = U U^T y \quad \text{for all } y \in \mathbb{R}^n
\]

(5)

**Proof** Formula (4) follows immediately from (2). Also, (4) shows that \( \text{proj}_W y \) is a linear combination of the columns of \( U \) using the weights \( y \cdot u_1, \ y \cdot u_2, \ldots, \ y \cdot u_p \). The weights can be written as \( u_1^T y, \ u_2^T y, \ldots, \ u_p^T y \), showing that they are the entries in \( U^T y \) and justifying (5).

Suppose \( U \) is an \( n \times p \) matrix with orthonormal columns, and let \( W \) be the column space of \( U \). Then
\[
U^T U = I_p \quad \text{for all } x \in \mathbb{R}^p \quad \text{Theorem 6}
\]
\[
U U^T y = \text{proj}_W y \quad \text{for all } y \in \mathbb{R}^n \quad \text{Theorem 10}
\]

If \( U \) is an \( n \times n \) (square) matrix with orthonormal columns, then \( U \) is an orthogonal matrix, the column space \( W \) is all of \( \mathbb{R}^n \), and \( U U^T y = I y = y \) for all \( y \) in \( \mathbb{R}^n \).

Although formula (4) is important for theoretical purposes, in practice it usually involves calculations with square roots of numbers (in the entries of the \( u_i \)’s). Formula (2) is recommended for hand calculations.
13. \( z = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -3 \\ -1 \end{bmatrix} \)

14. \( z = \begin{bmatrix} 4 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 5 \\ 4 \\ -2 \\ -5 \end{bmatrix} \)

15. Let \( y = \begin{bmatrix} 5 \\ -9 \\ 5 \end{bmatrix}, \quad u_1 = \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \). Find the distance from \( y \) to the plane in \( \mathbb{R}^3 \) spanned by \( u_1 \) and \( u_2 \).

16. Let \( y, v_1, \) and \( v_2 \) be as in Exercise 12. Find the distance from \( y \) to the subspace of \( \mathbb{R}^4 \) spanned by \( v_1 \) and \( v_2 \).

17. Let \( y = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -2/3 \\ 1/3 \\ 1/3 \end{bmatrix} \), and \( W = \text{Span} \{ u_1, u_2 \} \). Compute \( U^T U \) and \( U U^T \).

18. Let \( y = \begin{bmatrix} 7 \\ 9 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}, \) and \( W = \text{Span} \{ u_1 \} \). Compute \( \text{proj}_W y \) and \( (U U^T)y \).

19. Let \( u_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}, \) and \( u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \). Note that \( u_1 \) and \( u_2 \) are orthogonal but that \( u_3 \) is not orthogonal to \( u_1 \) or \( u_2 \). It can be shown that \( u_3 \) is not in the subspace \( W \) spanned by \( u_1 \) and \( u_2 \). Use this fact to construct a nonzero vector \( v \) in \( \mathbb{R}^3 \) that is orthogonal to \( u_1 \) and \( u_2 \).

20. Let \( u_1 \) and \( u_2 \) be as in Exercise 19, and let \( u_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \). It can be shown that \( u_4 \) is not in the subspace \( W \) spanned by \( u_1 \) and \( u_2 \). Use this fact to construct a nonzero vector \( v \) in \( \mathbb{R}^3 \) that is orthogonal to \( u_1 \) and \( u_2 \).

In Exercises 21 and 22, all vectors and subspaces are in \( \mathbb{R}^n \). Mark each statement True or False. Justify each answer.

21. a. If \( z \) is orthogonal to \( u_1 \) and to \( u_2 \) and if \( W = \text{Span} \{ u_1, u_2 \} \), then \( z \) must be in \( W^\perp \).
   b. For each \( y \) and each subspace \( W \), the vector \( y - \text{proj}_W y \) is orthogonal to \( W \).
   c. The orthogonal projection \( \hat{y} \) of \( y \) onto a subspace \( W \) can sometimes depend on the orthogonal basis for \( W \) used to compute \( \hat{y} \).
   d. If \( y \) is in a subspace \( W \), then the orthogonal projection of \( y \) onto \( W \) is \( y \) itself.
   e. If the columns of an \( n \times p \) matrix \( U \) are orthonormal, then \( U U^T y \) is the orthogonal projection of \( y \) onto the column space of \( U \).

22. a. If \( W \) is a subspace of \( \mathbb{R}^n \) and if \( v \) is in both \( W \) and \( W^\perp \), then \( v \) must be the zero vector.
   b. In the Orthogonal Decomposition Theorem, each term in formula (2) for \( \hat{y} \) is itself an orthogonal projection of \( y \) onto a subspace of \( W \).
   c. If \( y = x_1 + x_2 \), where \( x_1 \) is in a subspace \( W \) and \( x_2 \) is in \( W^\perp \), then \( x_1 \) must be the orthogonal projection of \( y \) onto \( W \).
   d. The best approximation to \( y \) by elements of a subspace \( W \) is given by the vector \( y - \text{proj}_W y \).
   e. If an \( n \times p \) matrix \( U \) has orthonormal columns, then \( U U^T x = x \) for all \( x \) in \( \mathbb{R}^n \).

23. Let \( A \) be an \( m \times n \) matrix. Prove that every vector \( x \) in \( \mathbb{R}^n \) can be written in the form \( x = p + u \), where \( p \) is in Row \( A \) and \( u \) is in \( \text{Nul} A \). Also, show that if the equation \( A x = b \) is consistent, then there is a unique \( p \) in Row \( A \) such that \( A p = b \).

24. Let \( W \) be a subspace of \( \mathbb{R}^n \) with an orthogonal basis \( \{ w_1, \ldots, w_p \} \), and let \( \{ v_1, \ldots, v_q \} \) be an orthogonal basis for \( W^\perp \).
   a. Explain why \( \{ w_1, \ldots, w_p, v_1, \ldots, v_q \} \) is an orthogonal set.
   b. Explain why the set in part (a) spans \( \mathbb{R}^n \).
   c. Show that \( \dim W + \dim W^\perp = n \).

25. [M] Let \( U \) be the \( 8 \times 4 \) matrix in Exercise 36 of Section 6.2. Find the closest point to \( y = (1, 1, 1, 1, 1, 1, 1, 1) \) in Col \( U \). Write the keystrokes or commands you use to solve this problem.

26. [M] Let \( U \) be the matrix in Exercise 25. Find the distance from \( b = (1, 1, 1, -1, -1, -1, -1, -1) \) to Col \( U \).