Chapter 11

Diversification and Risky Asset Allocation
Diversification

• Intuitively, we all know that if you hold many investments
  • Through time, some will increase in value
  • Through time, some will decrease in value
  • It is unlikely that their values will all change in the same way

• Diversification has a profound effect on portfolio return and portfolio risk.

• But, exactly how does diversification work?
Diversification and Asset Allocation

• Our goal in this chapter is to examine the role of diversification and asset allocation in investing.

• In the early 1950s, professor Harry Markowitz was the first to examine the role and impact of diversification.

• Based on his work, we will see how diversification works, and we can be sure that we have “efficiently diversified portfolios.”
  – An efficiently diversified portfolio is one that has the highest expected return, given its risk.
  – You must be aware that diversification concerns expected returns.
• **Expected return** is the “weighted average” return on a risky asset, from today to some future date. The formula is:

\[
\text{expected return}_i = \sum_{s=1}^{n} [p_s \times \text{return}_{i,s}]
\]

• To calculate an expected return, you must first:
  – Decide on the number of possible economic scenarios that might occur.
  – Estimate how well the security will perform in each scenario, and
  – Assign a probability to each scenario
  – (BTW, finance professors call these economic scenarios, “states.”)

• The next slide shows how the expected return formula is used when there are two states.
  – Note that the “states” are equally likely to occur in this example.
  – BUT! They do not have to be equally likely--they can have different probabilities of occurring.
Expected Return, II.

- Suppose:
  - There are two stocks:
    - Starcents
    - Jpod
  - We are looking at a period of one year.

- Investors agree that the expected return:
  - for Starcents is 25 percent
  - for Jpod is 20 percent

- Why would anyone want to hold Jpod shares when Starcents is expected to have a higher return?
Expected Return, III.

• The answer depends on risk

• Starcents is expected to return 25 percent

• But the realized return on Starcents could be significantly higher or lower than 25 percent

• Similarly, the realized return on Jpod could be significantly higher or lower than 20 percent.
Calculating Expected Returns

**TABLE 11.2 Calculating Expected Returns**

<table>
<thead>
<tr>
<th>(1) State of Economy</th>
<th>(2) Probability of State of Economy</th>
<th>(3) Return If State Occurs</th>
<th>(4) Product (2) x (3)</th>
<th>(5) Return If State Occurs</th>
<th>(6) Product (2) x (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>.50</td>
<td>–20%</td>
<td>–10%</td>
<td>30%</td>
<td>15%</td>
</tr>
<tr>
<td>Boom</td>
<td>.50</td>
<td>70</td>
<td>35</td>
<td>10</td>
<td>05</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ E(R_S) = 25\% \]

\[ E(R) = 20\% \]
Expected Risk Premium

• Recall:

\[ \text{expected risk premium} = \text{expected return} - \text{riskfree rate} \]

• Suppose riskfree investments have an 8% return. If so,
  – the expected risk premium on Jpod is 12%
  – The expected risk premium on Starcents is 17%

• This expected risk premium is simply the difference between
  – the expected return on the risky asset in question and
  – the certain return on a risk-free investment
Calculating the Variance of Expected Returns

• The variance of expected returns is calculated using this formula:

\[
\text{Variance} = \sigma^2 = \sum_{s=1}^{n} \left[ p_s \times (\text{return}_s - \text{expected return})^2 \right]
\]

• This formula is not as difficult as it appears.

• This formula says is to add up the squared deviations of each return from its expected return after it has been multiplied by the probability of observing a particular economic state (denoted by “s”).

• The standard deviation is simply the square root of the variance.

\[
\text{Standard Deviation} = \sigma = \sqrt{\text{Variance}}
\]
Example: Calculating Expected Returns and Variances: Equal State Probabilities

Calculating Expected Returns:

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State Occurs</th>
<th>Product: (2) x (3)</th>
<th>State Product: (2) x (5)</th>
<th>E(Ret):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.50</td>
<td>-0.20</td>
<td>-0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>Boom</td>
<td>0.50</td>
<td>0.70</td>
<td>0.35</td>
<td>0.10</td>
</tr>
<tr>
<td>Sum:</td>
<td>1.00</td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
</tbody>
</table>

Calculating Variance of Expected Returns:

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Probability of State Occurs</th>
<th>Expected Return: (3)</th>
<th>Difference: (3) - (4)</th>
<th>Squared: (5) x (4)</th>
<th>Product: (2) x (6)</th>
<th>Sum = the Variance:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.50</td>
<td>-0.20</td>
<td>-0.45</td>
<td>0.2025</td>
<td>0.10125</td>
<td>0.2025</td>
</tr>
<tr>
<td>Boom</td>
<td>0.50</td>
<td>0.70</td>
<td>0.45</td>
<td>0.2025</td>
<td>0.10125</td>
<td>0.2025</td>
</tr>
<tr>
<td>Sum:</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2025</td>
</tr>
</tbody>
</table>

Note that the second spreadsheet is only for Starcents. What would you get for Jpod?
# Expected Returns and Variances, Starcents and Jpod

<table>
<thead>
<tr>
<th></th>
<th>Starcents</th>
<th>Jpod</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected return, $E(R)$</td>
<td>.25, or 25%</td>
<td>.20, or 20%</td>
</tr>
<tr>
<td>Variance, $\sigma^2$</td>
<td>.2025</td>
<td>.0100</td>
</tr>
<tr>
<td>Standard deviation, $\sigma$</td>
<td>.45, or 45%</td>
<td>.10, or 10%</td>
</tr>
</tbody>
</table>
Portfolios

- Portfolios are groups of assets, such as stocks and bonds, that are held by an investor.

- One convenient way to describe a portfolio is by listing the proportion of the total value of the portfolio that is invested into each asset.

- These proportions are called **portfolio weights**.
  - Portfolio weights are sometimes expressed in percentages.
  - However, in calculations, make sure you use proportions (i.e., decimals).
Portfolios: Expected Returns

• The expected return on a portfolio is a linear combination, or weighted average, of the expected returns on the assets in that portfolio.

• The formula, for “n” assets, is:

\[ E(R_P) = \sum_{i=1}^{n} [w_i \times E(R_i)] \]

In the formula:  
- \( E(R_P) \) = expected portfolio return  
- \( w_i \) = portfolio weight in portfolio asset \( i \)  
- \( E(R_i) \) = expected return for portfolio asset \( i \)
Note that the portfolio weight in Jpod = 1 – portfolio weight in Starcents.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.50</td>
<td>-0.20</td>
<td>0.50</td>
<td>-0.10</td>
<td>0.30</td>
<td>0.50</td>
<td>0.15</td>
<td>0.05</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>Boom</td>
<td>0.50</td>
<td>0.70</td>
<td>0.50</td>
<td>0.35</td>
<td>0.10</td>
<td>0.50</td>
<td>0.05</td>
<td>0.40</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>Sum:</td>
<td>1.00</td>
<td></td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculating Expected Portfolio Returns:

Sum is Expected Portfolio Return: 0.225
Variance of Portfolio Expected Returns

- Note: Unlike returns, portfolio variance is generally not a simple weighted average of the variances of the assets in the portfolio.

- If there are “n” states, the formula is:

\[
\text{VAR} \left( R_P \right) = \sum_{s=1}^{n} \left[ \rho_s \times \left( \text{E}(R_{p,s}) - \text{E}(R_P) \right)^2 \right]
\]

- In the formula, \( \text{VAR}(R_P) = \text{variance of portfolio expected return} \)
  - \( \rho_s = \text{probability of state of economy, s} \)
  - \( \text{E}(R_{p,s}) = \text{expected portfolio return in state s} \)
  - \( \text{E}(R_P) = \text{portfolio expected return} \)

- Note that the formula is like the formula for the variance of the expected return of a single asset.
Example: Calculating Variance of Portfolio Expected Returns

- It is possible to construct a portfolio of risky assets with zero portfolio variance! What? How? (Open this spreadsheet, scroll up, and set the weight in Starcents to 2/11ths.)

- What happens when you use .40 as the weight in Starcents?

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Prob. of State</th>
<th>State Occurs</th>
<th>Expected Return</th>
<th>Difference: (3) - (4)</th>
<th>Squared: (5) x (5)</th>
<th>Product: (2) x (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.50</td>
<td>0.209</td>
<td>0.209</td>
<td>0.00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Boom</td>
<td>0.50</td>
<td>0.209</td>
<td>0.209</td>
<td>0.00</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sum:</td>
<td>1.00</td>
<td></td>
<td></td>
<td>Sum is Variance:</td>
<td></td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Calculating Variance of Expected Portfolio Returns:

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return if State of State Occurs: Expected Return: Difference: (3) - (4)</td>
<td>Squared: (5) x (5)</td>
<td>Product: (2) x (6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recession</td>
<td>0.50</td>
<td>0.209</td>
<td>0.209</td>
<td>0.00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Boom</td>
<td>0.50</td>
<td>0.209</td>
<td>0.209</td>
<td>0.00</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sum:</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Diversification and Risk, I.

### TABLE 11.7 Portfolio Standard Deviations

<table>
<thead>
<tr>
<th>(1) Number of Stocks in Portfolio</th>
<th>(2) Average Standard Deviation of Annual Portfolio Returns</th>
<th>(3) Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.74%</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>37.36</td>
<td>.76</td>
</tr>
<tr>
<td>4</td>
<td>29.69</td>
<td>.60</td>
</tr>
<tr>
<td>6</td>
<td>26.64</td>
<td>.54</td>
</tr>
<tr>
<td>8</td>
<td>24.98</td>
<td>.51</td>
</tr>
<tr>
<td>10</td>
<td>23.93</td>
<td>.49</td>
</tr>
<tr>
<td>20</td>
<td>21.68</td>
<td>.44</td>
</tr>
<tr>
<td>30</td>
<td>20.87</td>
<td>.42</td>
</tr>
<tr>
<td>40</td>
<td>20.46</td>
<td>.42</td>
</tr>
<tr>
<td>50</td>
<td>20.20</td>
<td>.41</td>
</tr>
<tr>
<td>100</td>
<td>19.69</td>
<td>.40</td>
</tr>
<tr>
<td>200</td>
<td>19.42</td>
<td>.39</td>
</tr>
<tr>
<td>300</td>
<td>19.34</td>
<td>.39</td>
</tr>
<tr>
<td>400</td>
<td>19.29</td>
<td>.39</td>
</tr>
<tr>
<td>500</td>
<td>19.27</td>
<td>.39</td>
</tr>
<tr>
<td>1,000</td>
<td>19.21</td>
<td>.39</td>
</tr>
</tbody>
</table>

Source: These figures are from Table 1 in Meir Statman, “How Many Stocks Make a Diversified Portfolio?” *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–64. They were derived from E. J. Elton and M. J. Gruber, “Risk Reduction and Portfolio Size: An Analytic Solution,” *Journal of Business* 50 (October 1977), pp. 415–37.
FIGURE 11.1 Portfolio Diversification

Average annual standard deviation (%)

- Diversifiable risk
- Nondiversifiable risk

Number of stocks in portfolio
Why Diversification Works, I.

• **Correlation**: The tendency of the returns on two assets to move together. Imperfect correlation is the key reason why diversification reduces portfolio risk as measured by the portfolio standard deviation.

  • *Positively* correlated assets tend to move up and down together.
  • *Negatively* correlated assets tend to move in opposite directions.

• Imperfect correlation, positive or negative, is why diversification reduces portfolio risk.
Why Diversification Works, II.

- The *correlation coefficient* is denoted by $\text{Corr}(R_A, R_B)$ or simply, $\rho_{A,B}$.

- The correlation coefficient measures correlation and ranges from:

  From:  -1  (perfect negative correlation)

  Through:  0  (uncorrelated)

  To:  +1  (perfect positive correlation)
Why Diversification Works, III.

**FIGURE 11.2 Correlations**

- **Perfect positive correlation**
  \[ \text{Corr} (R_A, R_B) = +1 \]
  - Both the return on Security A and the return on Security B are higher than average at the same time.

- **Perfect negative correlation**
  \[ \text{Corr} (R_A, R_B) = -1 \]
  - Security A has a higher-than-average return when Security B has a lower-than-average return, and vice versa.

- **Zero correlation**
  \[ \text{Corr} (R_A, R_B) = 0 \]
  - The return on Security A is completely unrelated to the return on Security B.
### Why Diversification Works, IV.

**TABLE 11.8 Annual Returns on Stocks A and B**

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock A</th>
<th>Stock B</th>
<th>Portfolio AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>10%</td>
<td>15%</td>
<td>12.5%</td>
</tr>
<tr>
<td>2002</td>
<td>30</td>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>2003</td>
<td>-10</td>
<td>25</td>
<td>7.5</td>
</tr>
<tr>
<td>2004</td>
<td>5</td>
<td>20</td>
<td>12.5</td>
</tr>
<tr>
<td>2005</td>
<td>10</td>
<td>15</td>
<td>12.5</td>
</tr>
<tr>
<td>Average returns</td>
<td>9</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>Standard deviations</td>
<td>14.3</td>
<td>13.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>
Why Diversification Works, V.

Figure 11.3: Impact of Diversification

- **Stock A Annual Returns**: 2001–2005
- **Stock B Annual Returns**: 2001–2005
- **Portfolio AB Annual Returns**: 2001–2005

**Annual return (%)**

Calculating Portfolio Risk

For a portfolio of two assets, A and B, the variance of the return on the portfolio is:

\[
\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \text{COV}(A,B)
\]

\[
\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \sigma_A \sigma_B \text{CORR}(R_A R_B)
\]

Where:  
- \(x_A\) = portfolio weight of asset A
- \(x_B\) = portfolio weight of asset B
  - such that \(x_A + x_B = 1\).

(Important: Recall Correlation Definition!)
The Importance of Asset Allocation, Part 1.

• Suppose that as a very conservative, risk-averse investor, you decide to invest all of your money in a bond mutual fund. Very conservative, indeed?

\[\text{Uh, is this decision a wise one?}\]
Correlation and Diversification, I.

**FIGURE 11.4** Risk and Return with Stocks and Bonds

- **Expected return (%)**
- **Standard deviation (%)**
- Minimum variance portfolio
- 100% Bonds
- 100% Stocks
Correlation and Diversification, II.

**FIGURE 11.3** Impact of Diversification

- **Stock A Annual Returns**
  - 2001–2005
- **Stock B Annual Returns**
  - 2001–2005
- **Portfolio AB Annual Returns**
  - 2001–2005
The various combinations of risk and return available all fall on a smooth curve.

This curve is called an investment opportunity set, because it shows the possible combinations of risk and return available from portfolios of these two assets.

A portfolio that offers the highest return for its level of risk is said to be an efficient portfolio.

The undesirable portfolios are said to be dominated or inefficient.
More on Correlation & the Risk-Return Trade-Off (The Next Slide is an Excel Example)

**FIGURE 11.5** Risk and Return with Two Assets

- **Correlation coefficients**
  - Green line: $\text{Corr} = -1$
  - Pink line: $\text{Corr} = 0$
  - Red line: $\text{Corr} = +1$

- Expected return (%) vs. Standard deviation (%)

- Stock A
- Stock B

- $\rho = -1$
- $\rho = 0$
- $\rho = +1$
Example: Correlation and the Risk-Return Trade-Off, Two Risky Assets

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky Asset 1</td>
<td>14.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>Risky Asset 2</td>
<td>8.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td>30.0%</td>
</tr>
</tbody>
</table>

Efficient Set--Two Asset Portfolio
The Importance of Asset Allocation, Part 2.

- We can illustrate the importance of asset allocation with 3 assets.

- How? Suppose we invest in three mutual funds:
  - One that contains Foreign Stocks, F
  - One that contains U.S. Stocks, S
  - One that contains U.S. Bonds, B

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign Stocks, F</td>
<td>18%</td>
<td>35%</td>
</tr>
<tr>
<td>U.S. Stocks, S</td>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>U.S. Bonds, B</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>

- Figure 11.6 shows the results of calculating various expected returns and portfolio standard deviations with these three assets.
Risk and Return with Multiple Assets, I.
Risk and Return with Multiple Assets, II.

• Figure 11.6 used these formulas for portfolio return and variance:

\[ r_p = x_F R_F + x_s R_S + x_B R_B \]

\[ \sigma_p^2 = x_F^2 \sigma_F^2 + x_s^2 \sigma_S^2 + x_B^2 \sigma_B^2 \]

\[ + 2x_F x_s \sigma_F \sigma_S \text{CORR}(R_F R_S) \]

\[ + 2x_F x_B \sigma_F \sigma_B \text{CORR}(R_F R_B) \]

\[ + 2x_s x_B \sigma_S \sigma_B \text{CORR}(R_S R_B) \]

• But, we made a simplifying assumption. We assumed that the assets are all uncorrelated.

• If so, the portfolio variance becomes:

\[ \sigma_p^2 = x_F^2 \sigma_F^2 + x_s^2 \sigma_S^2 + x_B^2 \sigma_B^2 \]
The Markowitz Efficient Frontier

• The Markowitz Efficient frontier is the set of portfolios with the maximum return for a given risk AND the minimum risk given a return.

• For the plot, the upper left-hand boundary is the Markowitz efficient frontier.

• All the other possible combinations are inefficient. That is, investors would not hold these portfolios because they could get either
  – more return for a given level of risk, or
  – less risk for a given level of return.
Example: Web-Based Markowitz Frontiers

www.finportfolio.com
Useful Internet Sites

• www.411stocks.com (to find expected returns)
• www.investopedia.com (for more on risk measures)
• www.teachmefinance.com (also contains more on risk measure)
• www.morningstar.com (measure diversification using “instant x-ray”)
• www.moneychimp.com (review modern portfolio theory)
• www.efficentfrontier.com (check out the online journal)
Chapter Review, I.

• Expected Returns and Variances
  – Expected returns
  – Calculating the variance

• Portfolios
  – Portfolio weights
  – Portfolio expected returns
  – Portfolio variance
Chapter Review, II.

• Diversification and Portfolio Risk
  – The Effect of diversification: Another lesson from market history
  – The principle of diversification

• Correlation and Diversification
  – Why diversification works
  – Calculating portfolio risk
  – More on correlation and the risk-return trade-off

• The Markowitz Efficient Frontier
  – Risk and return with multiple assets