Chapter

10

Bond Prices and Yields

McGraw-Hill/Irwin

Bond Prices and Yields

- Our goal in this chapter is to understand the relationship between bond prices and yields.
- In addition, we will examine some fundamental tools that fixed-income portfolio managers use when they assess bond risk.

Bond Basics, I.

- A **Straight bond** is an IOU that obligates the issuer of the bond to pay the holder of the bond:
  - A fixed sum of money (called the principal, par value, or face value) at the bond's maturity, and sometimes
  - Constant, periodic interest payments (called coupons) during the life of the bond
- **U.S. Treasury bonds are straight bonds**.
- Special features may be attached
  - Convertible bonds
  - Callable bonds
  - Putable bonds

Bond Basics, II.

- Two basic yield measures for a bond are its *coupon rate* and its *current yield*.

$$\text{Coupon rate} = \frac{\text{Annual coupon}}{\text{Par value}}$$

$$\text{Current yield} = \frac{\text{Annual coupon}}{\text{Bond price}}$$

Straight Bond Prices and Yield to Maturity

- ***The price of a bond is found by adding together the present value of the bond's coupon payments and the present value of the bond's face value.***
- The **Yield to maturity (YTM)** of a bond is the discount rate that equates the today's bond price with the present value of the future cash flows of the bond.

The Bond Pricing Formula

- ***The price of a bond is found by adding together the present value of the bond's coupon payments and the present value of the bond's face value.***

- The formula is:

$$\text{Bond Price} = \frac{C}{YTM}\left[1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}}\right] + \frac{FV}{\left(1 + \frac{YTM}{2}\right)^{2M}}$$

- In the formula, C represents the **annual** coupon payments (in $), FV is the face value of the bond (in $), and M is the maturity of the bond, measured in years.

Example: Using the Bond Pricing Formula

- What is the price of a straight bond with: $1,000 face value, coupon rate of 8%, YTM of 9%, and a maturity of 20 years?

$$\text{Bond Price} = \frac{C}{YTM}\left[1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}}\right] + \frac{FV}{\left(1 + \frac{YTM}{2}\right)^{2M}}$$

$$\text{Bond Price} = \frac{80}{0.09}\left[1 - \frac{1}{\left(1 + \frac{0.09}{2}\right)^{2\times 20}}\right] + \frac{1000}{\left(1 + \frac{0.09}{2}\right)^{2\times 20}}$$

$$= (888.89 \times 0.82807) + 171.93$$

$$= \$907.99.$$

Example: Calculating the Price of this Straight Bond Using *Excel*

- *Excel* has a function that allows you to price straight bonds, and it is called PRICE.

=PRICE(“Today”,“Maturity”,Coupon Rate,YTM,100,2,3)

- Enter “Today” and “Maturity” in quotes, using mm/dd/yyyy format.
- Enter the Coupon Rate and the YTM as a decimal.
- The "100" tells *Excel* to us $100 as the par value.
- The "2" tells *Excel* to use semi-annual coupons.
- The "3" tells *Excel* to use an actual day count with 365 days per year.

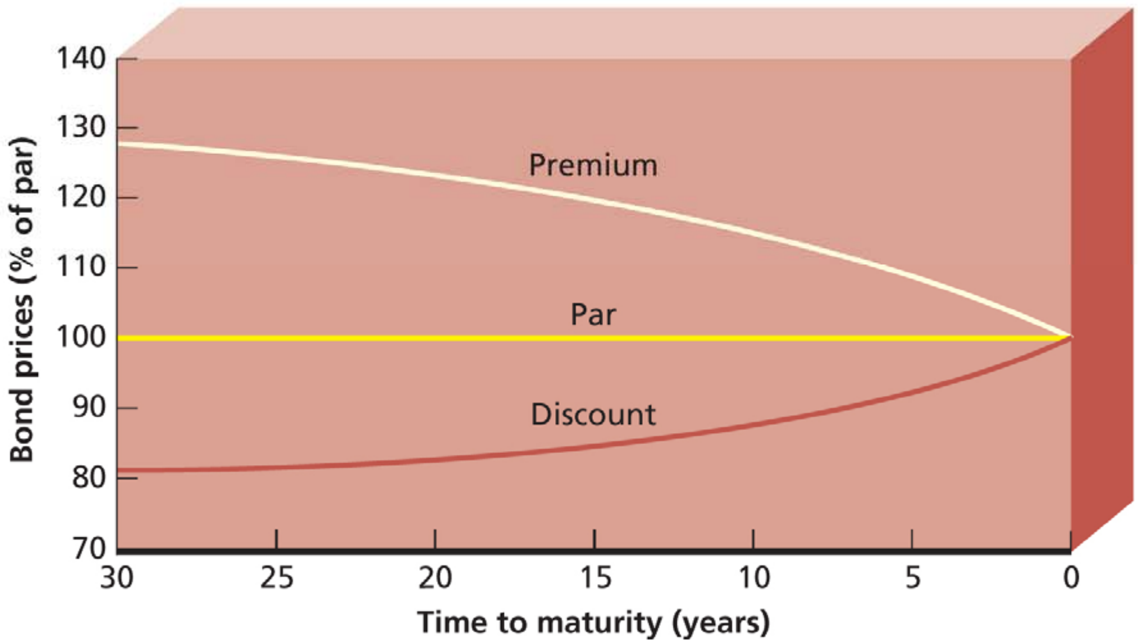
Note: *Excel* returns a price per $100 face.

Premium and Discount Bonds, I.

- Bonds are given names according to the relationship between the bond’s selling price and its par value.
- ***Premium bonds***: price > par value
YTM < coupon rate
- ***Discount bonds***: price < par value
YTM > coupon rate
- ***Par bonds***: price = par value
YTM = coupon rate

Premium and Discount Bonds, II.

FIGURE 10.1 Premium, Par, and Discount Bond Prices



Premium and Discount Bonds, III.

- In general, when the coupon rate and YTM are held constant:

  for premium bonds: the longer the term to maturity, the greater the premium over par value.

  for discount bonds: the longer the term to maturity, the greater the discount from par value.

Relationships among Yield Measures

for premium bonds:

coupon rate > current yield > YTM

for discount bonds:

coupon rate < current yield < YTM

for par value bonds:

coupon rate = current yield = YTM

Calculating Yield to Maturity, I.

- Suppose we know the current price of a bond, its coupon rate, and its time to maturity. How do we calculate the YTM?
- We can use the straight bond formula, trying different yields until we come across the one that produces the current price of the bond.

$$\$907.99 = \frac{\$80}{\text{YTM}}\left[1 - \frac{1}{\left(1 + \text{YTM}/2\right)^{2\times 20}}\right] + \frac{\$1,000}{\left(1 + \text{YTM}/2\right)^{2\times 20}}$$

- This is tedious. So, to speed up the calculation, financial calculators and spreadsheets are often used.

Calculating Yield to Maturity, II.

- We can use the YIELD function in *Excel:*

=YIELD(“Today”,“Maturity”,Coupon Rate,Price,100,2,3)

- Enter “Today” and “Maturity” in quotes, using mm/dd/yyyy format.
- Enter the Coupon Rate as a decimal.
- Enter the Price as per hundred dollars of face value.
- Note: As before,
  - **The "100" tells *Excel* to us $100 as the par value.**
  - **The "2" tells *Excel* to use semi-annual coupons.**
  - **The "3" tells *Excel* to use an actual day count with 365 days per year.**

- Using dates 20 years apart, a coupon rate of 8%, a price (per hundred) of $90.80, give a YTM of 0.089999, or 9%.

A Quick Note on Bond Quotations, I.

- We have seen how bond prices are quoted in the financial press, and how to calculate bond prices.
- Note: If you buy a bond between coupon dates, you will receive the next coupon payment (and might have to pay taxes on it).
- However, when you buy the bond between coupon payments, you must compensate the seller for any **accrued interest**.

A Quick Note on Bond Quotations, II.

- The convention in bond price quotes is to ignore accrued interest.
  - This results in what is commonly called a **clean price** (i.e., a quoted price net of accrued interest).
  - Sometimes, this price is also known as a **flat price.**

- The price the buyer actually pays is called the **dirty price**
  - This is because accrued interest is added to the **clean price.**
  - Note: The price the buyer actually pays is sometimes known as the **full price**, or **invoice price.**

Callable Bonds

- Thus far, we have calculated bond prices assuming that the actual bond maturity is the original stated maturity.
- However, most bonds are **callable bonds.**
- A **callable bond** gives the issuer the option to buy back the bond at a specified ***call price*** anytime **after** an initial ***call protection period.***
- Therefore, for callable bonds, YTM may not be useful.

Yield to Call

- *Yield to call (YTC)* is a yield measure that assumes a bond will be called at its earliest possible call date.

- The formula to price a callable bond is:

$$\text{Callable Bond Price} = \frac{C}{YTC}\left[1 - \frac{1}{\left(1 + \frac{YTC}{2}\right)^{2T}}\right] + \frac{CP}{\left(1 + \frac{YTC}{2}\right)^{2T}}$$

- In the formula, C is the **annual** coupon (in $), CP is the call price of the bond, T is the time (in years) to the earliest possible call date, and YTC is the yield to call, with semi-annual coupons.

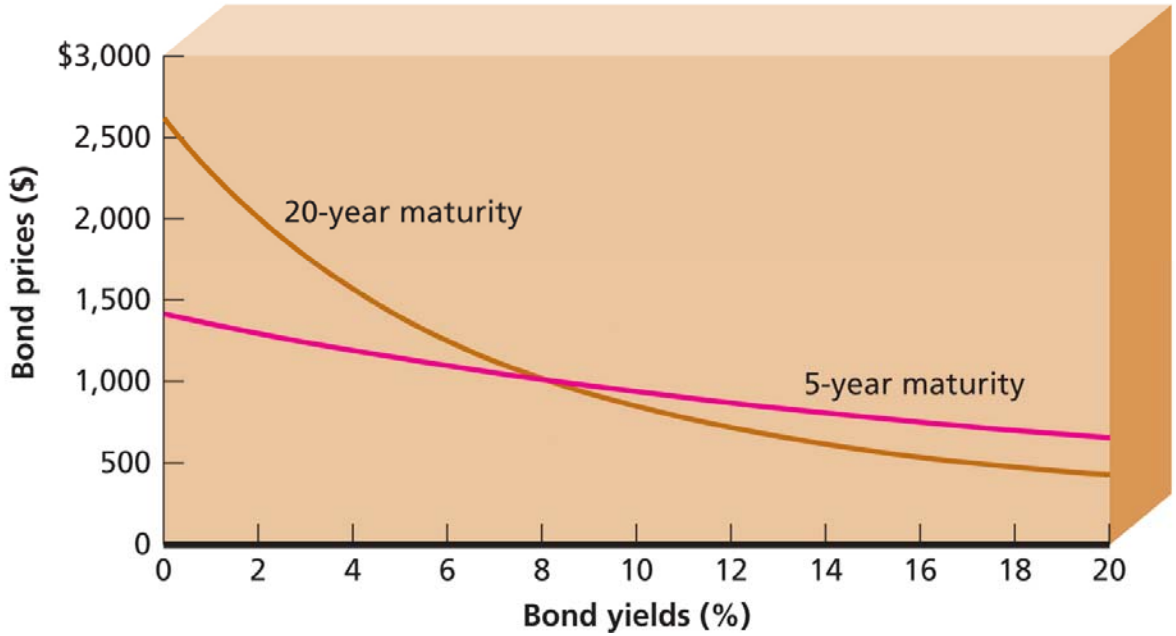
- As with straight bonds, we can solve for the YTC, if we know the price of a callable bond.

Interest Rate Risk

- Holders of bonds face **Interest Rate Risk.**
- **Interest Rate Risk** is the possibility that changes in interest rates will result in losses in the bond's value.
- The yield actually earned or "realized" on a bond is called the ***realized yield.***
- Realized yield is almost never exactly equal to the ***yield to maturity***, or ***promised yield*****.**

Interest Rate Risk and Maturity

FIGURE 10.2 Bond Prices and Yields



Malkiel's Theorems, I.

① Bond prices and bond yields move in opposite directions.
- As a bond's yield increases, its price decreases.
- Conversely, as a bond's yield decreases, its price increases.

② For a given change in a bond's YTM, the longer the term to maturity of the bond, the greater the magnitude of the change in the bond's price.

Malkiel's Theorems, II.

③ For a given change in a bond's YTM, the size of the change in the bond's price increases at a diminishing rate as the bond's term to maturity lengthens.

④ For a given change in a bond's YTM, the absolute magnitude of the resulting change in the bond's price is inversely related to the bond's coupon rate.

⑤ For a given absolute change in a bond's YTM, the magnitude of the price increase caused by a decrease in yield is greater than the price decrease caused by an increase in yield.

Bond Prices and Yields

SPREADSHEET ANALYSIS

	A	B	C	D	E	F	G	H
1								
2		Calculating the Price of a Coupon Bond						
3								
4	A Treasury bond traded on March 30, 2006 matures in 20 years on March 30, 2026.							
5	Assuming an 8 percent coupon rate and a 7 percent yield to maturity, what is the							
6	price of this bond?							
7	Hint: Use the Excel function PRICE.							
8								
9		$110.6775	= PRICE("3/30/2006","3/30/2026",0.08,0.07,100,2,3)					
10								
11	For a bond with $1,000 face value, multiply the price by 10 to get $1,106.78.							
12								
13	This function uses the following arguments:							
14								
15		=PRICE("Now","Maturity", Coupon,Yield,100,2,3)						
16								
17	The 100 indicates redemption value as a percent of face value.							
18	The 2 indicates semi-annual coupons.							
19	The 3 specifies an actual day count with 365 days per year.							
20								
21								

SPREADSHEET ANALYSIS

	A	B	C	D	E	F	G	H
1								
2		Calculating the Yield to Maturity of a Coupon Bond						
3								
4	A Treasury bond traded on March 30, 2006, matures in 8 years on March 30, 2014.							
5	Assuming an 8 percent coupon rate and a price of 110, what is this bond's yield							
6	to maturity?							
7	Hint: Use the Excel function YIELD.							
8								
9		6.3843%	= YIELD("3/30/2006","3/30/2014",0.08,110,100,2,3)					
10								
11	This function uses the following arguments:							
12								
13		= YIELD("Now","Maturity",Coupon,Price,100,2,3)						
14								
15	Price is entered as a percent of face value.							
16	The 100 indicates redemption value as a percent of face value.							
17	The 2 indicates semi-annual coupons.							
18	The 3 specifies an actual day count with 365 days per year.							
19								
20								

Duration

- Bondholders know that the price of their bonds change when interest rates change. But,
  - **How big is this change?**
  - **How is this change in price estimated?**

- **Macaulay Duration, or Duration,** is the name of concept that helps bondholders measure the sensitivity of a bond price to changes in bond yields. That is:

$$\text{Pct. Change in Bond Price} \approx -\text{Duration} \times \frac{\text{Change in YTM}}{\left(1+\text{YTM}/2\right)}$$

➔ ***Two bonds with the same duration**, but not necessarily the same maturity, **will have approximately the same price sensitivity to a (small) change in bond yields.***

Example: Using Duration

- Example: Suppose a bond has a **Macaulay Duration** of 11 years, and a current yield to maturity of 8%.

- If the yield to maturity increases to 8.50%, what is the resulting percentage change in the price of the bond?

$$\text{Pct. Change in Bond Price} \approx -11 \times \frac{[(0.085 - 0.08)]}{\left(1 + 0.08/2\right)}$$

$$\approx -5.29\%.$$

Modified Duration

- Some analysts prefer to use a variation of Macaulay's Duration, known as Modified Duration.

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{\left(1 + \frac{\text{YTM}}{2}\right)}$$

- The relationship between percentage changes in bond prices and changes in bond yields is approximately:

$$\text{Pct. Change in Bond Price} \approx -\text{Modified Duration} \times \text{Change in YTM}$$

Calculating Macaulay’s Duration

- Macaulay’s duration values are stated in years, and are often described as a bond’s *effective maturity*.
- *For a zero-coupon bond,* duration = maturity.
- *For a coupon bond,* duration = a weighted average of individual maturities of all the bond’s separate cash flows, where the weights are proportionate to the present values of each cash flow.

Calculating Macaulay's Duration

- In general, for a bond paying constant semiannual coupons, the formula for Macaulay's Duration is:

$$\text{Duration} = \frac{1+\frac{YTM}{2}}{YTM} - \frac{1+\frac{YTM}{2}+M(C-YTM)}{YTM+C\left[\left(1+\frac{YTM}{2}\right)^{2M}-1\right]}$$

- In the formula, C is the **annual** coupon rate, M is the bond maturity (in years), and YTM is the yield to maturity, assuming semiannual coupons.

Calculating Macaulay's Duration for Par Bonds

- If a bond is selling for par value, the duration formula can be simplified to:

$$\text{Par Value Bond Duration} = \frac{1 + \frac{YTM}{2}}{YTM}\left[1 - \frac{1}{\left(1 + \frac{YTM}{2}\right)^{2M}}\right]$$

Calculating Duration Using Excel

- We can use the **DURATION** and **MDURATION** functions in *Excel* to calculate **Macaulay Duration** and **Modified Duration**.

- The Excel functions use arguments like we saw before:

=**DURATION**("Today","Maturity",Coupon Rate,YTM,2,3)

- You can verify that a 5-year bond, with a 9% coupon and a 7% YTM has a Duration of **4.17** and a Modified Duration of **4.03**.

Calculating Macaulay's Duration

SPREADSHEET ANALYSIS

	A	B	C	D	E	F	G	H
1								
2		**Calculating Macaulay and Modified Durations**						
3								
4	A Treasury bond traded on March 30, 2006, matures in 12 years on March 30, 2018.							
5	Assuming a 6 percent coupon rate and a 7 percent yield to maturity, what are the							
6	Macaulay and Modified durations of this bond?							
7	Hint: Use the Excel functions DURATION and MDURATION.							
8								
9		8.561	= DURATION("3/30/2006","3/30/2018",0.06,0.07,2,3)					
10								
11		8.272	= MDURATION("3/30/2006","3/30/2018",0.06,0.07,2,3)					
12								
13	These functions use the following arguments convention:							
14								
15		= DURATION("Now","Maturity",Coupon,Yield,2,3)						
16								
17	The 2 indicates semi-annual coupons.							
18	The 3 specifies an actual day count with 365 days per year.							
19								
20								

Duration Properties

① All else the same, the longer a bond's maturity, the longer is its duration.

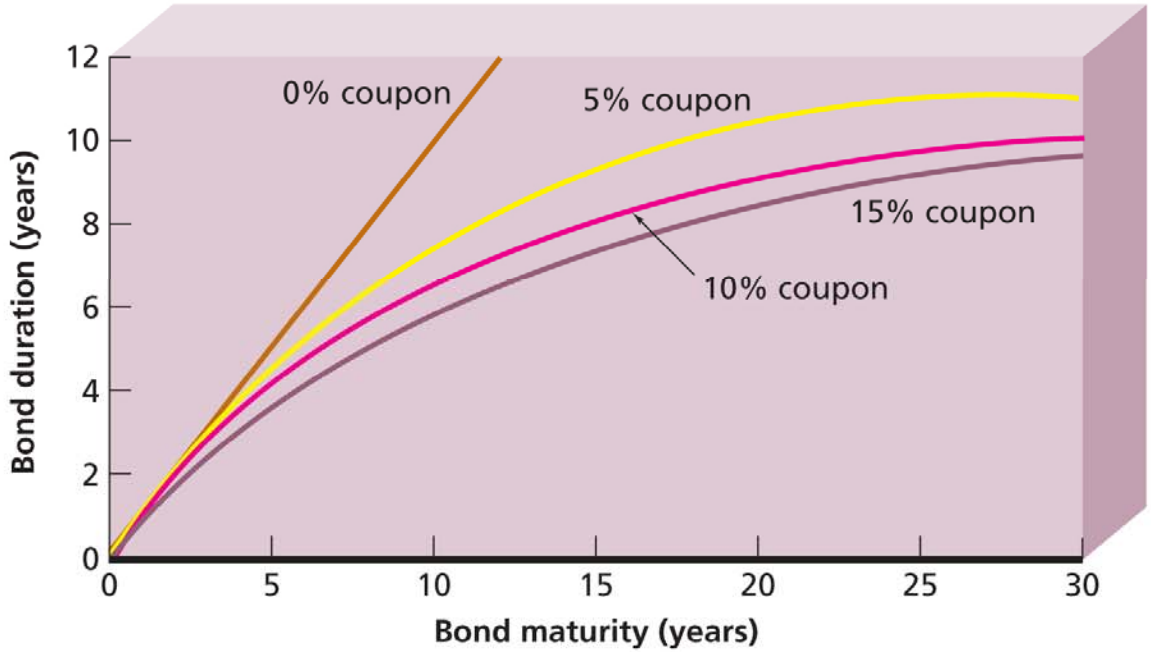
② All else the same, a bond's duration increases at a decreasing rate as maturity lengthens.

③ All else the same, the higher a bond's coupon, the shorter is its duration.

④ All else the same, a higher yield to maturity implies a shorter duration, and a lower yield to maturity implies a longer duration.

Properties of Duration

FIGURE 10.3 Bond Duration and Maturity



Bond Risk Measures Based on Duration, I.

- Dollar Value of an 01: Measures the change in bond price from a one basis point change in yield.

$$\text{Dollar Value of an 01} \approx -\text{Modified Duration} \times \text{Bond Price} \times 0.01$$

- Yield Value of a 32^{nd}: Measures the change in yield that would lead to a $1/32^{nd}$ change in the bond price.

$$\text{Yield Value of a 32nd} \approx \frac{1}{32 \times \text{Dollar Value of an 01}}$$

In both cases, the bond price is per $100 face value.

Bond Risk Measures Based on Duration, II.

- Suppose a bond has a modified duration of 8.27 years.
  - What is the dollar value of an 01 for this bond (per $100 face value)?
  - What is the yield value of a 32nd (per $100 face value)?

- First, we need the price of the bond, which is $91.97. Verify using:
  - YTM = 7%
  - Coupon = 6%
  - Maturity = 12 Years.

- The **Dollar Value of an 01** is **$0.07606**, which says that if the YTM changes one basis point, the bond price changes by **7.6 cents**.

- The **Yield Value of a 32nd** is **.41086**, which says that a yield change of .41 basis points changes the bond price by 1/32nd (**3.125 cents**).

Dedicated Portfolios

- A **Dedicated Portfolio** is a bond portfolio created to prepare for a future cash payment, e.g. pension funds.
- The date the payment is due is commonly called the portfolio's ***target date***.

Reinvestment Risk

- **Reinvestment Rate Risk** is the uncertainty about the value of the portfolio on the target date.
- Reinvestment Rate Risk stems from the need to reinvest bond coupons at yields not known in advance.
- **Simple Solution**: purchase zero coupon bonds.
- **Problem with Simple Solution**:
  - U.S. Treasury STRIPS are the only zero coupon bonds issued in sufficiently large quantities.
  - STRIPS have lower yields than even the highest quality corporate bonds.

Price Risk

- **Price Risk** is the risk that bond prices will decrease.

- Price risk arises in dedicated portfolios when the target date value of a bond is not known with certainty.

Price Risk versus Reinvestment Rate Risk

- For a dedicated portfolio, interest rate **increases** have two effects:
  - Increases in interest rates **decrease** bond prices, but
  - Increases in interest rates **increase** the future value of reinvested coupons
- For a dedicated portfolio, interest rate **decreases** have two effects:
  - Decreases in interest rates **increase** bond prices, but
  - Decreases in interest rates **decrease** the future value of reinvested coupons

Immunization

- **Immunization** is the term for constructing a dedicated portfolio such that the uncertainty surrounding the target date value is minimized.

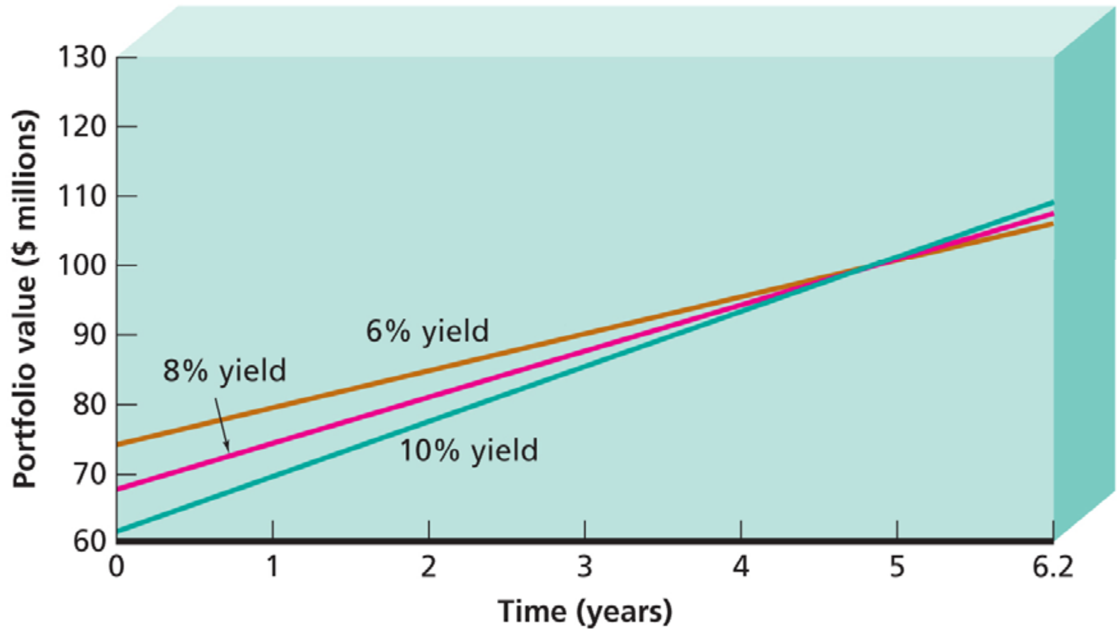
- It is possible to engineer a portfolio such that price risk and reinvestment rate risk offset each other (just about entirely).

Immunization by Duration Matching

- A dedicated portfolio can be immunized by ***duration matching*** - matching the duration of the portfolio to its target date.
- Then, the impacts of price and reinvestment rate risk will almost exactly offset.
- This means that interest rate changes will have a minimal impact on the target date value of the portfolio.

Immunization by Duration Matching

FIGURE 10.4 Bond Price and Reinvestment Rate Risk



Dynamic Immunization

- **Dynamic immunization** is a periodic rebalancing of a dedicated bond portfolio for the purpose of maintaining a duration that matches the target maturity date.
- The advantage is that the reinvestment risk caused by continually changing bond yields is greatly reduced.
- The drawback is that each rebalancing incurs management and transaction costs.

Useful Internet Sites

- www.bondmarkets.com (check out the bonds section)
- www.bondpage.com (treasury bond prices and yields search tool)
- www.jamesbaker.com (a practical view of bond portfolio management)
- www.bondsonline.com (bond basics and current market data)
- www.investinginbonds.com (bond basics and current market data)
- www.bloomberg.com (for information on government bonds)

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  - Yield to Call

- Interest Rate Risk and Malkiel's Theorems
  - Promised Yield and Realized Yield
  - Interest Rate Risk and Maturity
  - Malkiel's Theorems

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  - Dedicated Portfolios
  - Reinvestment Risk

- Immunization
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