Sample Answers for Problem Set 4


a) To find the steady-state output level, start with the capital accumulation per worker equation,

\[
\frac{K_{t+1}}{N} - \frac{K_t}{N} = s \frac{K_t^{1/3} N^{2/3}}{N} - \delta \frac{K_t}{N}
\]

and set \( \frac{K_{t+1}}{N} = \frac{K_t}{N} \). You solve

\[
s \frac{K_t^{1/3} N^{2/3}}{N} = \delta \frac{K_t}{N} \Rightarrow \left( \frac{K_t}{N} \right)^{1/3} = \frac{\delta}{s} \frac{K_t}{N}
\]

For \( \left( \frac{K_t}{N} \right)^{2/3} = \frac{s}{\delta} \).

For \( s = \delta \), \( \frac{K_t}{N} = 1 \).

b) Output per worker is \( \frac{Y}{N} = \left( \frac{K}{N} \right)^{1/3} = 1 \).

c) For \( s = 0.1 \) and \( \delta = 0.2 \), \( \left( \frac{K}{N} \right)^{2/3} = \frac{1}{2} \) so that \( \frac{K}{N} = \frac{1}{2\sqrt{2}} \). Output per worker is \( \frac{Y}{N} = \left( \frac{K}{N} \right)^{1/3} = \frac{1}{\sqrt{2}} \).

d) The calculations use

\[
\frac{K_{t+1}}{N} - \frac{K_t}{N} = s \left( \frac{K_t}{N} \right)^{1/3} - \delta \frac{K_t}{N}
\]

starting with \( \frac{K_t}{N} = 1 \), \( s = 0.1 \) and \( \delta = 0.2 \). For \( t+1 \), this is

\[
\frac{K_{t+1}}{N} = 0.1 - 0.2 = -0.1 \Rightarrow \frac{K_{t+1}}{N} = 0.9
\]

and \( \frac{Y_{t+1}}{N} = \left( \frac{K_{t+1}}{N} \right)^{1/3} = 0.9^{1/3} \). The next two values for capital per worker and output per worker are found by substituting again:

\[
\frac{K_{t+2}}{N} - \frac{K_{t+1}}{N} = s \left( \frac{K_{t+1}}{N} \right)^{1/3} - \delta \frac{K_{t+1}}{N} \Rightarrow \frac{K_{t+2}}{N} = 0.9 + 0.1(0.9)^{1/3} - 0.2 \times 0.9.
\]
2. Chapter 12, problem 8

a) The quantity \( g_Y - g_N \) is the growth rate of the ratio, \( \frac{Y}{N} \). The growth rate of a ratio is the difference between the growth rate of the numerator and the growth rate of the denominator. To see this just take the logarithm of \( \frac{Y}{N} \) and differentiate with respect to time. \( g_K - g_N \) is the growth rate of the ratio, \( \frac{K}{N} \).

b) You rearrange \( \frac{2}{3} g_A = (g_Y - g_N) - \frac{1}{3} (g_K - g_N) \)

to get \( 2g_A = 3(g_Y - g_N) - (g_K - g_N) \) and then

\[ g_K - g_N = 3(g_Y - g_N) - 2g_A. \]

c) Substituting in for the US, you get

\[ g_K - g_N = 3(1.8) - 2(2.0) = 1.4. \]


a) The price of the consol is given by \( V = \frac{\$z}{i} = \frac{100}{0.10} = $1000 \).

b) Use the exact sum for the fixed maturity bonds that begin paying \( \$z \) next year for \( n \) years:

\[
V = z \left[ \frac{1}{1+i} + \ldots + \left( \frac{1}{1+i} \right)^n \right] = z \left[ 1 - \left( \frac{1}{1+i} \right)^n \right] \cdot \frac{1}{1+i} = \frac{z}{i} \left[ 1 - \left( \frac{1}{1+i} \right)^n \right]
\]

The price for the 10-year bond is

\[
V = 1000 \left[ 1 - \left( \frac{1}{1+0.10} \right)^{10} \right] = $610
\]

and the prices for the 20-year and 30-year bonds are

\( V = $850 \) and \( V = $940 \).

c) By repeating these for interest rates equal to 2% and 5%, you can see that the difference between the consol price and long-maturity bond price decreases as the interest rate increases because \( 1 - \left( \frac{1}{1+i} \right)^n \) increases as \( i \) increases (and as \( n \) increases).

4. Chapter 15, problems 5, 6 and 7
1) For problem 5, you will show that the nominal interest rate falls up to the one year then rises toward its new medium-run level of \( i = g_m - g_y + r_n \). The yield curve averages these rates over the maturity of the bond. Short-term interest rates will fall (for example, the one-year bond rate falls) but long-term interest rates will rise (for example, 10-year bonds).

2) Problem 6, part a is a typo (its also problem 7 part a). For part b: a steep yield curve indicates that nominal interest rates are expected to rise. This is consistent with an expansionary future monetary policy.

3) Problem 7, part a: the current price of a stock equals \( Q_t = \frac{D_{t+1}}{0.05} \) if the real rate of interest is 5% and equals \( Q_t = \frac{D_{t+1}}{0.08} \) if the real rate of interest is 8%. An expected rise in the real rate of interest reduces the value of the stock by 5/8. For part b, you add the risk premium to the real rate of interest. For example, \( Q_t = \frac{D_{t+1}}{0.05 + 0.08} = \frac{D_{t+1}}{0.13} \) if the real rate of interest is 5% and the risk premium is 8%.