Problem Set 2: Answers to Algebraic Problems

1. a. In this part, I is autonomous. Aggregate demand is given by

\[ Z = c_0 + c_1(Y - T) + \bar{I} + G \]

and the multiplier is \( \frac{1}{1-c_1} \).

b. Aggregate demand is now given by

\[ Z = (c_0 + c_1(Y - T)) + (b_0 + b_1Y - b_2i) + G \]

and the multiplier is \( \frac{1}{1-(c_1 + b_1)} \). The multiplier is larger for part b than for part a because both consumption and investment demand rise as income rises in response to an increase in autonomous demand.

c. Rearrange the LM relation as

\[ i = \frac{d_1}{d_2}Y - \frac{1}{d_2} \frac{M}{P} \]

and the IS relation as

\[ i = \frac{1}{b_2} (c_0 - c_1T + b_0 + G) - \frac{1}{b_2} (1 - c_1 - b_1)Y \]

Solving for the common equilibrium by eliminating \( i \),

\[ \frac{d_1}{d_2}Y - \frac{1}{d_2} \frac{M}{P} = \frac{1}{b_2} (c_0 - c_1T + b_0 + G) - \frac{1}{b_2} (1 - c_1 - b_1)Y \]

and rearranging so that \( Y \) is a function of the parameters and policy variables leads to

\[ Y = \frac{(c_0 - c_1T + b_0 + G) + \frac{b_2}{d_2} \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1b_2}{d_2}} \]

The multiplier is given by \( \frac{1}{1 - c_1 - b_1 + \frac{d_1b_2}{d_2}} \). This gives the increase in \( Y \) caused by an increase in autonomous demand.
d. It is smaller due to the additional positive term, \( \frac{d_1 b_2}{d_2} \), in the denominator which reflects the impact of an increase in autonomous demand on the interest rate in financial market equilibrium, \( \frac{d_1}{d_2} \), times the sensitivity of investment demand on the interest rate, \( b_2 \).

2. a. You will draw an inward shift of the IS curve holding the LM curve fixed.

b. This is done in problem 1.

c. First, write down the solution for \( Y \) from problem 1,

\[
Y = \frac{(c_0 - c_1 T + b_0 + G) + \frac{b_2 M}{d_2 P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}}
\]

and substitute into the LM relation from problem 1,

\[
i = \frac{d_1 Y}{d_2} - \frac{1}{d_2} \frac{M}{P}
\]

to get

\[
i = \frac{d_1}{d_2} \left( \frac{c_0 - c_1 T + b_0 + G + \frac{b_2 M}{d_2 P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \right) - \frac{1}{d_2} \frac{M}{P}
\]

\[
= \frac{1}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \left( \frac{d_1}{d_2} \left( c_0 - c_1 T + b_0 + G \right) + \frac{1 - c_1 - b_1 M}{d_2 P} \right)
\]

d. To solve for investment, you substitute the solutions for \( Y \) and \( i \) into the equation,

\[
I = b_0 + b_Y Y - b_i i.
\]

This is not as ugly as you might think:
\[ I = b_0 + b_1 Y - b_2 i \]

\[ = b_0 + b_1 \left( \frac{(c_0 - c_1 T + b_0 + G) + b_2 \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \right) - b_2 \left( \frac{\frac{d_1}{d_2} (c_0 - c_1 T + b_0 + G) - \frac{1 - c_1 - b_1}{d_2} \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \right) \]

\[ = b_0 + \left( b_1 - \frac{d_1 b_2}{d_2} \right) \frac{c_0 - c_1 T + b_0 + G}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} + \frac{\frac{b_2}{d_2} (1 - c_1) \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \]

d. Look at the effect of G on I by taking the derivative:

\[ \frac{\partial I}{\partial G} = \left( b_1 - \frac{d_1 b_2}{d_2} \right) \frac{1}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \]

We already required that \( c_1 + b_1 \) was less than one, so the multiplier is positive. If the term, \( b_1 - \frac{d_1 b_2}{d_2} \), is positive, then investment rises when G rises. If it is negative, then investment falls as G rises. The condition can be rewritten as

\[ \frac{\partial I}{\partial G} > 0 \quad \text{if} \quad \frac{b_1}{b_2} - \frac{d_1}{d_2} > 0 \]

and

\[ \frac{\partial I}{\partial G} < 0 \quad \text{if} \quad \frac{b_1}{b_2} - \frac{d_1}{d_2} < 0. \]

e. The ratio, \( \frac{b_1}{b_2} \), gives the relative importance of output to the interest rate on investment. It tells you how much the interest rate needs to rise when output rises by one unit to keep investment constant. The ratio, \( \frac{d_1}{d_2} \), gives the relative importance of output to the interest rate on money demand. It tells you how much the interest rate rises when output rises by one unit to keep money demand constant. When \( \frac{b_1}{b_2} > \frac{d_1}{d_2} \), as Y goes up with G, the interest rate rises but not enough to keep investment from going up.

3. a. The IS relation is given in the answer to 1c as
\[ i = \frac{1}{b_2} \left( c_0 - c_1 T + b_0 + G \right) - \frac{1}{b_2} \left( 1 - c_1 - b_1 \right) Y \]

The slope of the IS curve is given by the coefficient on \( Y \) as

\[ \frac{di}{dY} = -\frac{1}{b_2} \left( 1 - c_1 - b_1 \right) \]

This slope is negative and an increase in \( \frac{b_1}{b_2} \) increases the slope. The IS curve becomes flatter.

b. Use the solution for equilibrium output,

\[ Y = \frac{\left( c_0 - c_1 T + b_0 + G \right) + \frac{b_2}{d_2} \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \]

to find that

\[ \frac{\partial Y}{\partial \left( M / P \right)} = \frac{b_2}{d_2} = \frac{1}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}} \]

An increase in \( \frac{b_1}{b_2} \) reduces the denominator and increases the effect of a rise in real balances on output. That is, output rises by more for a given increase in \( M/P \) when \( \frac{b_1}{b_2} \) is larger.

c. A larger value of \( b_1 \) means that investment is more sensitive to changes in output, and a smaller value of \( b_2 \) that investment is less sensitive to interest rate changes. An increase in real balances raises income by more if \( \frac{b_1}{b_2} \) is larger because the multiplier is larger.

d. From problem 1c, the LM curve is given by

\[ i = \frac{d_1}{d_2} Y - \frac{1}{d_2} \frac{M}{P} \]

and its slope by

\[ \frac{\partial i}{\partial Y} = \frac{d_1}{d_2} \cdot \frac{1}{d_2} \]

A rise in \( d_2 \) means that money demand is more sensitive to changes in the interest rate; a fall in
$d_1$ means that money demand is less sensitive to changes in output. If $\frac{d_1}{d_2}$ is larger, then the interest rate must rise by more when output increases to keep money demand equal to a constant real balance supply. The increase in output raises money demand by more if $d_1$ is larger, and the interest rate must rise by even more to offset this increase in money demand if $d_2$ is smaller.

e. Again, use the solution for equilibrium output,

$$Y = \frac{(c_0 - c_1 T + b_0 + G) + \frac{b_2}{d_2} \frac{M}{P}}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}}$$

to get the change in $Y$ for an increase in $G$,

$$\frac{\partial Y}{\partial G} = \frac{1}{1 - c_1 - b_1 + \frac{d_1 b_2}{d_2}}.$$

You can see that an increase in $\frac{d_1}{d_2}$ reduces the multiplier.

f. Letting $d_1$ be zero increases the multiplier to

$$\frac{\partial Y}{\partial G} = \frac{1}{1 - c_1 - b_1}.$$

This is the horizontal shift in the IS curve when government expenditure rises. When $d_1$ is zero, an increase in output has no effect on money demand so the interest rate does not rise (if it did, money demand would fall and money would be in excess supply). Therefore, the interest rate does not rise when $G$ rises if $d_1$ is zero and there is no crowding out of investment by government expenditure.