Sample Answers for Problem Set 1

Chapter 3, Problem 5

a. Begin by writing out aggregate demand as

\[ Z = C + I + G \]
\[ = \left( c_0 + c_1 Y_D \right) + \bar{I} + G \]
\[ = \left( c_0 + c_1 \left( Y - t_0 - t_1 Y \right) \right) + \bar{I} + G \]

and set \( Y = Z \) to find equilibrium output:

\[ Y = \left( c_0 - c_1 t_0 + c_1 \left( 1 - t_1 \right) Y \right) + \bar{I} + G. \]

Solving for \( Y \), you get

\[ Y = \frac{c_0 - c_1 t_0 + \bar{I} + G}{1 - c_1 \left( 1 - t_1 \right)}. \]

b. The multiplier is \( \frac{1}{1 - c_1 \left( 1 - t_1 \right)} \). GDP responds less to an increase in autonomous spending when \( t_1 \) is positive. The multiplier is smaller if \( t_1 \) is positive because an increase in output leads to a smaller increase in disposable income by the amount \( \frac{dY_D}{dY} = t_1 \) so that consumption rises by \( \frac{dC}{dY} = c_1 \left( 1 - t_1 \right) \) rather than by \( \frac{dC}{dY} = c_1 \) when autonomous demand increases.

c. Fiscal policy is called an automatic stabilizer in this case because disturbances to autonomous demand lead to smaller disturbances to GDP when taxes are proportional to income at the margin. (In this case, the marginal tax rate, \( t_1 \), is constant). For example, taxes fall as \( Y \) falls and rise as \( Y \) rises reducing the fluctuations in GDP through the multiplier.

Chapter 3, Problem 8

a. Begin again by writing out aggregate demand as

\[ Z = C + I + G \]
\[ = \left( c_0 + c_1 Y_D \right) + \left( b_0 + b_1 Y \right) + G \]
\[ = \left( c_0 + c_1 \left( Y - T \right) \right) + \left( b_0 + b_1 Y \right) + G \]
and set $Y=Z$ to find equilibrium output:

$$Y = (c_0 + c_1(Y - T)) + (b_0 + b_1Y) + G.$$ 

Solve for $Y$ to get

$$Y = \frac{c_0 - c_1T + b_0 + G}{1 - c_1 - b_1}$$

b. The multiplier is

$$\frac{1}{1 - (c_1 + b_1)}.$$ 

The dependence of investment on the demand for output is given by the coefficient $b_1$. An increase in output raises investment so that aggregate demand increases with income through both consumption and investment. The total effect of an increase in $Y$ on aggregate demand is given by the sum, $c_1 + b_1$. This larger marginal increase in aggregate demand from an increase in income increases the multiplier. The sum, $c_1 + b_1$, must be less than one for the multiplier to be positive. This restriction makes sense because it means that the increase in aggregate demand caused by an increase in output is not bigger than the increase in output.

c. $b_0$ is autonomous investment (or, slightly more exactly, the autonomous component of investment). An increase in $b_0$ is an increase in autonomous demand which increases output which in turn raises investment further through $b_1$. The effects of an increase in $b_0$ on output and investment are given by

$$\frac{dY}{db_0} = \frac{1}{1 - c_1 - b_1}$$

and

$$\frac{dI}{db_0} = 1 + b_1 \frac{dY}{db_0} = 1 + \frac{b_1}{1 - c_1 - b_1} = \frac{1 - c_1}{1 - c_1 - b_1}.$$ 

The increase in national savings is exactly equal to the increase in investment because we have the identity,

$$S = I + G - T$$

and $T$ and $G$ do not change. As investment rises, savings rises because

$$S = Y_d - C = Y - T - (c_0 + c_1(Y - T)) = (1 - c_1)(Y - T) - c_0$$

and $Y$ rises with $b_0$.

Chapter 4, problem 5

a. Wealth is divided between money and bonds. The person’s total wealth equals $W$ so that

$$W = M^d + B^d.$$
Subtracting and substituting in the money demand function, you get the demand for bonds:

\[ B^d = W - M^d = \$50,000 - \$Y(0.35 - i). \]

The change in the demand for bonds when the interest rate rises by 10% (that is, by 0.1), is

\[ \Delta B^d = -\$Y(\Delta i) = \$Y(0.1). \]

b. An increase in wealth has no effect on the demand for money. Money demand is determined by the value of transactions in the economy (proportional to $Y$) and by the opportunity cost of holding money (i). Wealth does not appear in the money demand function.

Bond demand equals wealth minus money demand. An increase in wealth causes a one-to-one increase in bond demand from our equation,

\[ B^d = W - M^d = \$50,000 - \$Y(0.35 - i). \]

c. An increase in income proportionally raises money demand because people will hold money in proportion to the dollar value of transactions in the economy. When money demand rises with income, bond demand falls by exactly the same amount. An increase in income does not increase current wealth. As $Y$ rises, $W$ is constant, but $M^d$ rises so that $B^d$ falls.

d. This statement is answered in part c. “People earn more money” means nominal income rises. As nominal income rises, the dollar value of transactions rises and money demand rises. People want to hold more money at a constant interest rate to buy more goods and services. Bond demand falls when money demand rises because money demand and bond demand add up to the total stock of wealth.

Chapter 4, problem 8

a. The stock of central bank money equals reserves in this example. The demand for central bank money is given by

\[ H^d = \theta M^d = 0.1\$Y(0.8 - 4i). \]

b. To find the equilibrium interest rate, set the demand for central bank money equal to the supply of central bank money as

\[ H = H^d = \theta M^d \]

\$100b = 0.1($5000b)(0.8 - 4i)

and solve for i which is 0.15 or 15%.

c. The overall supply of money must equal the overall demand for money at this interest rate because
\[ \frac{1}{\theta} H = M = M^d \]
\[ \frac{100b}{0.1} = (5000b)(0.8 - 4(0.15)) \]

d. If \( H \) rises from $100b to $300b, the interest rate falls from 15\% to 5\%. This is found by using the equilibrium condition again:

\[ H = H^d = \theta M^d \]
\[ 300b = 0.1(5000b)(0.8 - 4i) \]

e. i would become -13\%. 