Notes on the derivation of the expectations-augmented Phillips curve from the Aggregate Supply curve

The aggregate supply curve is found by combining the Wage Setting relationship

\[ W = P^e F(u, z) \]

with the Price Setting relationship

\[ P = W(1 + \mu). \]

Eliminating the nominal wage rate, \( W \), leads to the AS relationship:

\[ P^e F(u, z) = \frac{P}{1 + \mu} \]

\[ \Rightarrow P = P^e (1 + \mu)F(u, z) \]

Now, we substitute a linear function for \( F(u, z) \), given by \( F(u, z) = 1 - \alpha u + z \). This makes sense for comparing this condition to data and is mathematically convenient for converting from the price level, \( P \), (and expected future price level, \( P^e \)) to the rate of change of the price level, the inflation rate.

For this example, the AS relation is

\[ P = P^e (1 + \mu)(1 - \alpha u + z) \]

We begin by taking the natural logarithm of both sides of the AS equation:

\[ (1) \quad \log P = \log[P^e (1 + \mu)(1 - \alpha u + z)] \]

\[ \Rightarrow \log P = \log P^e + \log(1 + \mu) + \log(1 - \alpha u + z) \]

Now, we use the approximation,

\[ \log(1 + x) \approx x \]

for small \( x \), to get

\[ \log(1 + \mu) = \mu \]

\[ and \]

\[ \log(1 - \alpha u + z) = -\alpha u + z \]
Putting these approximations into equation (1), we get

\[(2) \quad \log P = \log P^e + \mu + z - \alpha u\]

When \( P = P^e \), the unemployment rate equals the natural rate of unemployment. Using equation (2), the natural rate of unemployment is given by

\[u_n = \frac{\mu + z}{\alpha}\]

so that equation (2) can be rewritten as

\[(3) \quad \log P_t = \log P_t^e - \alpha(u_t - u_n),\]

where the time subscripts are added (for example, \( P_t \) means this month’s price level).

Now, subtract \( \log P_{t-1} \), the price level for the previous month, from both sides of equation (3) to get

\[(4) \quad \log P_t - \log P_{t-1} = \left( \log P_t^e - \log P_{t-1} \right) - \alpha(u_t - u_n)\]

The difference, \( \log P_t - \log P_{t-1} \), is the inflation rate and \( \log P_t^e - \log P_{t-1} \) is the expected inflation rate.

To see this, remember that

\[
\frac{d \log x}{dt} = \frac{1}{t} \frac{dx}{dt}
\]

is the growth rate of \( x \), so that the growth rate of the price level is \( \log P_t - \log P_{t-1} \) which is approximately equal to \( \frac{P_t - P_{t-1}}{P_{t-1}} \).

The expectations-augmented Phillips curve is

\[(5) \quad \pi_t = \pi_t^e - \alpha(u_t - u_n).\]