RESEARCH STATEMENT

KASIA JANKIEWICZ

I am a topologist studying infinite discrete groups, by exploring the connection between the algebraic properties of the groups and the topology and geometry of spaces that they act on. In my work I use combinatorial techniques, tools from algebraic topology, hyperbolic geometry, study of mapping class groups and the dynamics of the automorphisms of free groups, theory of computation, and probabilistic methods. My research has connections with the study of 3- and 4-manifolds, which provide examples and tools. My work also has applications to these fields.

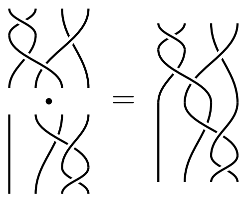
Below I briefly introduce the three main (overlapping) themes of my research program. In the following sections I discuss my past and future work in more detail.

Artin groups. They were introduced by Tits in the 60s, and they are a generalization of braid groups. Artin groups are closely related to Coxeter groups and arise as fundamental groups of complexified hyperplane arrangement. Spherical Artin groups became famous due to the work of Bieskorn, Saito and Deligne in the 70s. Despite simple presentations, general Artin groups are poorly understood. Artin groups can be studied via their actions on non-positively curved spaces, following Davis, Charney and Salvetti. Combinatorial techniques, such as Garside methods, can also be utilized in understanding Artin groups. In my work, I am interested in the existence and non-existence of actions of Artin groups on nonpositively curved spaces ([HJP16]), their "nonpositive-like" features ([Jan22a, Jan22b, Jan23, JS23b]), and their subgroup structure ([JS22, JS23a]).

CAT(0) cube complexes and spaces. In the late 80s, in a foundational paper for geometric group theory, Gromov introduced CAT(0) spaces, as a notion of "non-positive curvature" for metric spaces. CAT(0) spaces are in a precise sense at least as nonpositively curved as the Euclidean plane, and the notion has far reaching algebraic consequences for groups acting on them. CAT(0) cube complexes are easily constructible CAT(0) spaces with additional structure and even stronger algebraic features of their symmetry groups. A famous application of CAT(0) cube complexes to the theory of 3-manifolds led to the resolution of Waldhausen's virtual Haken conjecture and Thurston's virtual fibering conjecture by Agol and Wise. In my work, I both construct ([Jan17, JW22, AJW23]) and obstruct ([HJP16, Jan20, JW21, GJN23]) actions of groups on CAT(0) cube complexes, and general CAT(0) spaces [JKRS21].

Algebraic fibering and coherence of groups. These properties can be thought of as group theoretic analogues of properties of 3-manifolds, decomposing as a fiber bundle over a circle, and the compact core property of 3-manifolds. Algebraic fibration can be constructed using discrete methods, such as Bestvina-Brady Morse theory. The notion of coherence has been studied since the 70s, as it arose from the work of Scott, Shalen, and Stalings. My work with Norin and Wise [JNW21] on fibering right-angled Coxeter groups, was recently used by Italiano-Martelli-Migliorni to construct an example of a hyperbolic group with a finite type subgroup that is not hyperbolic, answering one of the most famous open questions in geometric group theory. I am also interested in the notion of coherence ([JW16]), and I would like to understand whether coherence is a "geometric" property, in a precise sense.

1. Artin groups



A *braid on n strands* can be thought of as n pieces of intertwined strings joining n points at the top of the diagram with n points at the bottom. A *braid group on n strands B_n* has isotopy classes of braids as its group elements, and concatenation of braids as a group operation (see left). The group B_4 admits a group presentation $\langle a, b, c \mid aba = bab, bcb = cbc, ac = ca\rangle$ where each generator a, b, c corresponds to a single interchange of two consecutive strands. *Artin groups* are a class of discrete infinite groups that generalize braid groups. Artin groups admit presentations defined by a finite graph Γ, called the *defining graph*, whose edges are labeled by integers ≥ 2. Vertices correspond to generators and an edge labeled by m joining vertices a and b corresponds to the relation $\underbrace{aba\cdots}_{m} = \underbrace{bab\cdots}_{m}$ where on each side there is an alternating word in a, b of length m.

Spherical Artin groups are those whose quotient Coxeter groups, obtained by imposing that all generators have order 2, are finite. Spherical Artin groups, like finite Coxeter groups, are classified by their Dynkin-Coxeter diagrams.

1.1. Actions on CAT(0) cube complexes

A CAT(0) cube complex is a CW complex obtained from a collection of Euclidean cubes by identifying their sides via isometries that satisfies a combinatorial condition guaranteeing that the complex admits a CAT(0) metric in which the cubes are Euclidean unit cubes. CAT(0) cube complexes were first introduced by Gromov as easily constructible examples of CAT(0) spaces.

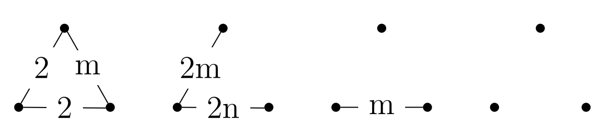
A motivating question for my research described in this section is the following question posed e.g. in [Cha, Prob 5].

Question 1. Do Artin groups act properly on CAT(0) cube complexes?

Together with Huang and Przytycki we gave a partial answer to the above questions in the case of proper and cocompact actions by proving that many Artin groups do not admit such actions. This is particularly interesting, since unlike proving the existence of actions on CAT(0) cube complexes, there is no general procedure of disproving the existence of such actions. We gave a complete characterization of 2-dimensional or 3-generated Artin groups that admit such actions, in terms of the combinatorics of its defining graph.

Theorem 2 (Huang-Jankiewicz-Przytycki [HJP16])**.** Let A be a three-generator Artin group. Then the following are equivalent.

(1) A is cocompactly cubulated,
(2) A has a finite index subgroup which is cocompactly cubulated,
(3) the defining graph of A is one of the diagrams on the right.



This theorem gives a partial answer to Question 1, under the assumption that the action is cocompact. The following is a noteworthy corollary, which was very surprising to the experts in the field, since B_4 is known to be CAT(0) by Brady-McCammond [BM10].

Corollary 3 (Huang-Jankiewicz-Przytycki [HJP16])**.** The braid group B_4 on 4 strands does not have a finite index subgroup that acts properly and cocompactly on a CAT(0) cube complex.

We also obtained a a complete classification of 2-dimensional Artin groups acting properly and cocompactly on CAT(0) cube complexes in terms of the combinatorics of their defining graphs (see [HJP16] for precise statement). Independently, the above results were also obtained by Haettel [Hae21]. The following question remains open, and I would like to answer it negatively, by classifying all the codimension-one subgroups of the braid group on 4 strands.

Question 4. Do braid groups act properly on CAT(0) cube complexes?

1.2. Profinite properties of Artin groups

The non-existence of well-behaved actions of certain Artin groups on CAT(0) cube complexes (as discussed in previous section) poses limitations on the tools accessible to study the geometry of those groups. In particular, the powerful theory of special cube complexes cannot be applied. This led me to a question of what properties of virtually special groups Artin groups have.

A group G is *residually finite* if for every $g \in G - \{1\}$ there exists a finite quotient $\phi : G \to \bar{G}$ such that $\phi(g) \neq 1$. All *linear* groups, i.e. groups with faithful representations into $GL_n(\mathbb{R})$ for some n, are residually finite by a classical theorem of Malcev. Virtually special groups are linear [HW08].

Question 5. Are Artin groups residually finite? More generally, are they linear?

The question of linearity was posed e.g. in [Bes, Q 2.15], and is known to hold for spherical Artin groups [Kra00, Big01, CW02, Dig03].

Very few other families of Artin groups are known to be residually finite. It can be deduced from [Squ87] that affine Artin groups on three generators are also residually finite. Some examples were provided in [BGMPP19, BGJP18]. In the latter paper, a question whether all 3-generator Artin groups are residually finite was posed. The question of residual finiteness of Artin groups was also raised in [HW99]. In a series of solo papers, I gave a partial answer to these questions.

Theorem 6 (Jankiewicz [Jan22a, Jan22b])**.** The three generators Artin group A_{MNP} splits as an algebraically clean graph of free groups provided that either all M, N, P are ≥ 5 or they are all even. In particular, they are residually finite by [Wis02], good in the sense of Serre, and for every prime p virtually residually-p by [JS23b].

The above results hold for a more general family of Artin groups, whose defining graph satisfies certain combinatorial criterion.

Residual finiteness can be thought of as the property that the "geometry" of the group can be approximated by its finite quotients. Indeed, a group G is residually finite if and only if every ball in the Cayley graph of G is isometric to a ball in the Cayley graph of some finite quotient of G.

Theorem 7 (Jankiewicz-Schreve [JS23b])**.** Algebraically clean graphs of free groups:

- are good in the sense of Serre,
- for every prime p are virtually residually p-finite.

In particular, this applies to the Artin groups considered in Theorem 6, as noted above.

Our exploration of profinite properties from Theorem 7 is partially motivated by the following questions that Scherve and I plan to tackle using some of the techniques developed by Schreve [Sch14].

Question 8. Do algebraically clean graphs of free groups satisfy the Atiyah conjecture? More generally, do Artin groups satisfy the Atiyah conjecture?

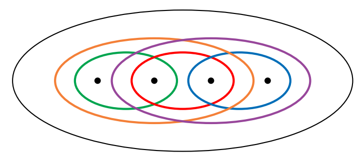
More recently, I have proven that more Artin groups are residually finite, by establishing *finite stature* in the sense of Huang-Wise [HW19], which is a property closely related to the notion of *finite height* introduced in [GMRS98].

Theorem 9 (Jankiewicz [Jan23])**.** An Artin group A_{MNP} splits as free products with amalgamation of finite rank free groups, provided that either $M > 2$ or $N > 3$, where $M \leq N \leq P$. In particular, A_{MNP} is residually finite.

1.3. Right-angled Artin subgroups of Artin groups

The Tits Conjecture, proved by Crisp and Paris in [CP01], asserts that for every Artin group A with the standard set of generators S, the subgroup $\langle s^2 : s \in S\rangle$ is an obvious right-angled Artin group, i.e. where $[s^2, t^2] = 1$ if and only if $[s, t] = 1$.

With Schreve, we proposed the following generalization of the Tits Conjecture. Given an irreducible, spherical subset $T \subset S$, let Δ_T^2 denote the corresponding center of the pure Artin subgroup PA_T. In the braid group, viewed as a mapping class group of a punctured disc, those new elements correspond to the Dehn twists about the curves enclosing more than two consecutive punctures.



Question 10 (Generalized Tits Conjecture)**.** Given $N \geq 1$, is the subgroup $\langle \Delta_T^{2N} : T \subset S\rangle$ of an Artin group A the obvious right-angled Artin group?

The above problem, with $N = 1$, was conjectured to have a positive answer for every Artin group in [DLS19]. However, Schreve and I disproved it for $N = 1$, and we gave positive answers for certain families of Artin groups for larger N.

Theorem 11 (Jankiewicz-Schreve [JS22])**.**

- Every locally reducible (in the sense of [Cha00]) Artin group A satisfies the Generalized Tits Conjecture with $N = 2$.
- Let A be a spherical Artin group of any type other than E_6, E_7, E_8. Then A satisfies the Generalized Tits Conjecture with sufficiently large N.

As a corollary, we answered the questions of Gordon-Long-Reid about the surface subgroups of Artin groups posed in [GLR04], and completed the classification of spherical Artin groups with surface subgroups. Almeida-Lima [AL21] applied our result, and the classification of subgroup separable right-angled Artin groups [MR08], to give a characterization of subgroup separability of general Artin groups in terms of the defining graph. We expect that Question E has a positive answer for sufficiently large N for all Artin groups.

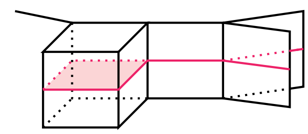
In another joint work with Schreve, similar techniques let us establish an unexpected connection between two famous conjectures about Artin groups: the $K(\pi, 1)$-conjecture, and the center conjecture. Specifically, we showed that the former conjecture implies the latter.

Theorem 12 (Jankiewicz-Schreve [JS23a])**.** Every Artin group without a spherical factor that satisfies the $K(\pi, 1)$-conjecture has trivial center.

Schreve and I are currently investigating the Tits conjecture in the context of graph braid groups. A graph braid group $B_n(\Gamma)$ is the fundamental group of the configuration space of n particles in the graph Γ. They are known to be the fundamental groups of special cube complexes [Abr00, CW04]. Generating sets for graph braid groups were described in e.g. [FS12].

Question 13. Do graph braid groups satisfy the Tits conjecture with respect to their natural generating set?

2. CAT(0) cube complexes



We recall that CAT(0) cube complexes were defined in Section 1.1. The combinatorial nature of CAT(0) cube complexes provided by their *hyperplanes* makes them natural generalizations of trees and makes them very accessible to work with. There are strong relations between actions on CAT(0) cube complexes and algebraic properties of the group. The most fundamental such correspondence comes from the *Sageev's construction* relating essential actions on CAT(0) cube complexes with codimension-1 subgroups. Other examples are the Tits alternative or the rank rigidity conjecture of Ballman-Buyalo, which are known to hold for groups acting properly and cocompactly on CAT(0) cube complexes, but remain unknown for general CAT(0) groups.

2.1. Cubical dimension and uniform exponential growth

In [Jan20], by analogy to the *CAT(0) dimension* of a group, I introduced the notion of *cubical dimension* cub-dim G, defined as the minimum n such that G acts properly on an n-dimensional CAT(0) cube complex. We have

$$\operatorname{cd} G \leq \text{CAT(0)-dim}\, G \leq \text{cub-dim}\, G$$

where $\operatorname{cd} G$ is the cohomological dimension of G. Examples of a group G with gaps between $\operatorname{cd} G$ and CAT(0)-dim G were provided in [Bri01, BC02]. Using small cancellation techniques, I constructed the first examples of groups with arbitrarily large finite gaps between their cubical dimension and their cohomological and CAT(0) dimensions.

Theorem 14 (Jankiewicz [Jan20])**.** For every n there exist hyperbolic groups with $\operatorname{cd} G =$ CAT(0)-dim $G = 2$, and cub-dim $G > n$.

In a subsequent work, joint with Wise, we use my result to construct the following example.

Theorem 15 (Jankiewicz-Wise [JW21])**.** There exists a finitely generated group of cohomological dimension 2 that acts freely (and thus metrically properly) on a locally-finite CAT(0) cube complex, but does not act properly on any finite dimensional CAT(0) cube complex.

Partially building on my work [Jan20], in a joint work with Gupta and Ng, we study uniform exponential growth CAT(0) cube complexes. A group G has *uniform exponential growth* if there exists $k > 1$ such that $w(G, S) > k$ for every finite generating set S, where

$$w(G, S) := \lim_{n \to \infty} |B(n, S)|^{1/n}$$

is the *exponential growth rate of G with respect to a finite generating set S*.

Kar-Sageev proved that a group acting freely on a CAT(0) square complex has uniform exponential growth, unless it is virtually abelian [KS19]. We extended their result in two directions.

Theorem 16 (Gupta-Jankiewicz-Ng [GJN23])**.**

- Every non virtually abelian group acting properly on a CAT(0) square complex has uniform exponential growth.
- Let G be a non virtually abelian group acting freely on a CAT(0) cube complex of dimension d with isolated flats that admits a geometric group action. Then G has uniform exponential growth depending on d only.

I believe that our methods can be pushed further to establish uniform exponential growth for all cubical groups.

Question 17. Does every non virtually abelian group acting properly on a finite dimensional CAT(0) cube complex has uniform exponential growth?

2.2. Cubical Small Cancellation Theory

The theory of small cancellation was initiated by Tartakovskii [Tar49] and developed by Greendlinger [Gre60] where they provide the solution to the word problem for a broad family of groups generalizing the solution for surface groups. Later, Gromov introduced the notion of word hyperbolic groups [Gro87], as a common generalization of small cancellation groups and the fundamental groups of negatively curved manifolds.

The cubical version of small cancellation, due to Wise [Wis21], involves *cubical presentation* of a group G which consists of a nonpositively curved cube complex X and a collection of local isometries $Y_i \to X$, where $G = \pi_1 X / \langle\langle \pi_1 Y_1, \pi_1 Y_2, \ldots \rangle\rangle$. The group G is also the fundamental group of the complex X with the subcomplexes Y_i coned-off, denoted by X^*. If X is a wedge of circles, and each Y_i is a cycle, the cubical presentation becomes a standard group presentation.

In a solo paper [Jan17], I strengthen and simplify the proof of the Wise's cubical version of the Greendlinger's Lemma, the fundamental theorem of small cancellation theory [Wis21]. A precise statement of the following theorem can be found in [Jan17].

Theorem 18 (Jankiewicz [Jan17])**.** Let $\langle X, \{Y_i\} \rangle$ satisfy C(9). There is a classification of all possible minimal disc diagrams $(D, \partial D) \to (X^*, X)$, and in each case D has an *exposed* cell.

With Wise, we used cubical small cancellation theory to give a new, simplified proof of the result of Martin and Steenbock on cubulation of small cancellation quotients of a free product of cubulated groups [MS16].

Theorem 19 (Jankiewicz-Wise [JW22])**.** Suppose $G = \langle G_1, \ldots, G_r \mid R_1, \ldots, R_s \rangle$ is $C'(1/20)$ small cancellation over the free products $G_1 * \cdots * G_r$. If each G_i is the fundamental group of a compact nonpositively curved cube complex, then G is the fundamental group of a compact nonpositively curved cube complex.

As an application, combined with an idea of Pride, we give the following example of a compact nonpositively curved cube complex X with non-trivial fundamental group $\pi_1 X$ such that no finite subgroup of $\pi_1 X$ splits as an amalgamated product or HNN extension. This answers a question posed by Chatterjee.

With Arenas and Wise we used Theorem 18 to prove hyperbolicity and relative hyperbolicity of *non-metric* cubical small cancellation groups.

Theorem 20 (Arenas-Jankiewicz-Wise [AJW23])**.** Let $X^* = \langle X \mid Y_1, \ldots, Y_s \rangle$ be a cubical presentation satisfying the $C(9)$ cubical small cancellation condition where X is hyperbolic and $X, Y_1, \ldots, Y_s$ are compact. Then $\pi_1 X^*$ is hyperbolic.

If instead, $\pi_1 X$ is hyperbolic relative to the collection $\{\pi_1 X_i\}_{i=1}^n$ where for each i $X_i \looparrowright X$ is a local isometry, $\langle X \mid X_1, \ldots, X_n, Y_1, \ldots, Y_s \rangle$ is $C(9)$ cubical small cancellation, and there is a bound on the size of pieces in X^*, then $\pi_1 X^*$ is hyperbolic relative to the images of $\{\pi_1 X_i\}_{i=1}^n$ in $\pi_1 X^*$.

3. Algebraic fibering and coherence

A group G *algebraically fibers*, if it admits an epimorphism onto $\mathbb{Z}$ with finitely generated kernel. Stallings' fibration theorem states that a compact irreducible 3-manifold M fibers as a surface bundle over a circle if and only if $\pi_1 M$ algebraically fibers. A group is *coherent* if every finitely generated subgroup is finitely presented, and otherwise it is *incoherent*. Examples of coherent groups include free groups, surface groups, fundamental groups of 3-manifolds, free-by-cyclic groups. A classical example of a group that is incoherent is $F_2 \times F_2$, which is an example of a reducible lattice in product of trees. The notions of algebraic fibering and coherence are related. They both can be viewed as group theoretic analogues of geometric properties of 3-manifolds. Another connections is via the result of Bieri, which states that every group G of cohomological dimension 2 that algebraically fibers is incoherent, unless G is free-by-cyclic [Bie81].

3.1. Coxeter groups

Using the Bestvina-Brady Morse theory [BB97], with Norin and Wise we give a sufficient condition for a right-angled Coxeter group W to virtually algebraically fiber.

Theorem 21 (Jankiewicz-Norin-Wise [JNW21])**.** There is a combinatorial criterion in terms of coloring of the defining graph of W that ensures a virtual algebraic fibering of W.

Among the most noteworthy examples of Γ with legal systems, are the 24-cell and the 600-cell, which define 4-dimensional hyperbolic reflection groups with fundamental domain respectively right-angled ideal hyperbolic 24-cell, and right-angled hyperbolic 120-cell. This yields the following. We note that no hyperbolic 4-manifolds can fiber over a circle, because of the Gauss-Bonnet theorem, so algebraic fibering is a natural candidate for generalizing virtual fibering of 3-manifolds to the higher dimensions.

Our work inspired a number of groups of other mathematicians, see e.g. [PGK17], [MZ21], [BM21], [IMM21b], [SZ21]. Most remarkably, Italiano-Martelli-Migliorni used our methods to construct finite-volume cusped hyperbolic 5-manifolds that fiber over the circle, and as a consequence obtained an example of a finite type subgroup (i.e. admitting a finite $K(\pi, 1)$-complex) of a hyperbolic group that is not hyperbolic [IMM21a]. This answered one of the most famous open questions in geometric group theory.

Previously, Wise and I have also considered algebraic fibering of Coxeter groups $W_{(r,m)}$ of rank r and a constant exponent m (i.e. $(ab)^m = 1$ for all pairs of generators a, b).

Theorem 22 (Jankiewicz-Wise [JW16])**.** For each $m \geq 3$ for all sufficiently large r the group $W_{(r,m)}$ virtually algebraically fibers, and in particular is incoherent.

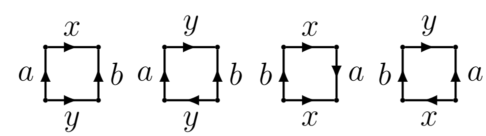
With Arenas, Goffer, Lopez de Gamiz, and Walsh, I am currently working an a classification of coherent right angled-Coxeter groups, in terms of their defining graphs. Jaikin-Zapirain – Linton that a Coxeter group W_Γ is coherent provided that for every subgraph Λ on at least three vertices of Γ, we have $\chi(W_\Lambda) \leq 0$, where χ is the Euler characteristic understood as a rescaled Euler characteristic of a torsion-free subgroup of W_Λ.

Question 23. Classify coherent right-angled Artin groups.

3.2. Lattices in product of trees

Let T, T' be regular, locally finite trees of degree ≥ 3. The product $T \times T'$ has a natural structure of a CAT(0) square complex. A *(uniform) lattice* in $\mathrm{Aut}(T \times T')$ is a discrete group acting properly and cocompactly on $T \times T'$. The simplest example of such group is $F_2 \times F_2$ which acts on a product of two copies of a regular valence 4 tree.

A lattice that does not virtually split as a direct product of free group is called *irreducible* and can be characterized as having non-discrete projections to each of the factors $\mathrm{Aut}(T)$. Famous examples of irreducible lattices in product of trees are due to Burger-Mozes, who constructed examples that are virtually simple [BM97]. Wise independently constructed non-residually finite examples [Wis96]. An example of an irreducible lattice is a group generated by a, b, x, y with the relations below.



Genevieve Walsh and I are investigating the following question.

Question 24. Are all lattices in a product of trees incoherent?

A negative answer would imply that coherence is not a quasi-isometry invariant. A positive answer would be an indication that coherence might be a "geometric" property, i.e. if two groups that act geometrically on the same space are either both coherent, or both incoherent. There are many collections of coherent groups that share geometry (such as acting on a tree, hyperbolic plane, hyperbolic 3-space), and most proofs of coherence are geometric in nature. On the other hand most of the known proofs of incoherence rely on the existence of an incoherent subgroup such as $F_2 \times F_2$ or the kernel of an algebraic fibering in a group of dimension 2. Thus, it would be very interesting to find geometric reasons for incoherence.

We have so far focused on understanding some small examples, and proved incoherence of some of them by finding subgroups that algebraically fiber. We are working on finding geometric or algorithmic methods of constructing subgroups of lattices in products of trees.

I have previously studied lattices in products of trees with Karrer, Ruane, Sathaye, in the context of the *boundary rigidity.* A *visual boundary* $\partial_\infty X$ of a proper CAT(0) space X is a well-defined compactification of X. Unlike for hyperbolic groups, a visual boundary of a CAT(0) group does not need to be unique. Croke-Kleiner constructed an example of a CAT(0) group acting geometrically on two CAT(0) spaces X_1, X_2 whose visual boundaries $\partial X, \partial X'$ are non-homeomorphic.

Theorem 25 (Jankiewicz-Karrer-Ruane-Sathaye [JKRS21])**.** Every torsion-free lattice $G < \mathrm{Aut}(T \times T')$ is *boundary rigid*, i.e. any two boundaries of CAT(0) spaces that G acts on geometrically are homeomorphic.

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