Chapter 32
The Social Organization of a Middle School Mathematics Group Discussion

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Abstract This study analyzes patterns of interaction among bilingual middle school students while they engaged in peer mathematical discussions. Using a sociocultural lens on learning the practice of school mathematics, this study addresses three questions: (1) What kind of mathematical discourse practices did the students engage in? (2) What discourse patterns emerged during these mathematics conversations? (3) What are the implicit "rules" that appear shape students' interactions, what are the pragmatic implications of these rules, and which students benefit from these rules? Using conversation analysis and discourse analysis we show that the students primarily engaged in "calculational" conversations, that mathematics conversations followed rules distinct from the rules of everyday conversations, and "intellectual authority" emerged as an important construct for understanding students' mathematical discourse practices.

32.1 Introduction

Learning to communicate about mathematics and through mathematics is emphasized as a key element of developing mathematical competence in schools (National Council of Teachers of Mathematics, 1989, 2000). One popular tool for teaching mathematical communication among peers in a mathematics classroom is the use of small group activities. Researchers have documented the benefits associated with small-group discussions among peers in terms of the development of problem-solving skills (Mercer, 2005) and also in terms of the development of participation in mathematical discourse practices (Yackel et al., 1991; Yackel et al.,

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This study analyzes patterns of interaction among a small group of bilingual middle school students while they engaged in peer mathematical discussions, and proposes a model for analyzing regularities in students' peer mathematics discussions. We conjecture that mathematics discussions in school are structured by implicit "rules," and that these rules are distinct from the norms of everyday, or casual conversations (Eggs and Slade, 1997). Sociolinguists have documented some of the implicit social norms that govern the multiple elements of a conversation including turn-taking (Sacks et al., 1974), norms of politeness (Lakoff, 1973), and cooperation (Grice, 1975). Researchers such as Linde situated in the Conversation Analysis (CA) tradition have shown that specialized conversations—such as cockpit conversations between two police officers in a police helicopter—have specialized rules governing interaction among participants, and these rules reveal institutional structures that frame the interactions (Linde, 1988). Through this study, we seek to broaden our understanding of the regularities in a school mathematics discussion among peers, and we show that a peer discussion in school mathematics is a hybrid conversation that draws upon norms for everyday conversations as well as the discourse of school mathematics. This, in turn will broaden our understanding of how students manage the task of communicating while solving mathematics problems in collaborative groups.

32.2 Literature Review & Theoretical Framework

Prior research in mathematics education has called attention to the benefits of mathematics discussions in both whole class (Goos, 2004; Lampert, 1990) and small group settings (Yackel et al., 1991). For small-group discussions in mathematics classrooms there is extensive research that analyzes both the content and form of students' peer discussions in dyads (Forman and Larreamendy-Joerns, 1995; Moschkovich, 1996, 1998; Sfard and Kieran, 2001; Yackel et al., 1991). These studies have greatly expanded our understanding of how peer discussions mediate students' learning of mathematics. However with the exception of a few studies (Jurow, 2005; Pirie, 1991), analyses of students' mathematics discussions in small groups with more than two students located in typical classroom interactions have not received much attention. Studies such as this one, which employ an ethnomethodological approach, will enhance our understanding of the nature of students' mathematical discussions within typical classroom settings. This study of students' interactions during small group learning is situated in a sociocultural theoretical framework. Within this tradition we treat learning as a process of appropriating (Rogoff, 1990) the meanings and perspectives enacted by members of the learning community (Moschkovich, 1996, 2004). Therefore, learning is taken to be an active process, and the corresponding emphasis in analysis is on activities such as "knowing" and "representing" rather than static entities such as "knowledge" or "representations" (Sfard and McClain, 2002). Furthermore this orientation highlights that roles (such as intellectual authority or "teacher") are dynamic and continually re-negotiated through discourse.

This study investigates the following questions: (1) What kind of mathematical discourse practices (Moschkovich, 2007) did the students engaged in?; (2) What discourse patterns did students use while engaging in a mathematics conversation, and how are these similar to or different from the discourses of casual conversation? (3) What are the implicit "rules" that shape students' interactions, what are the pragmatic implications of these rules, and who appears to benefit from these rules?

32.3 Participants and Setting

To answer these questions, we gathered data for this study by video-recording one group of seven students in a dual-immersion bilingual sixth grade classroom in California for approximately one month. The class used a standard textbook (Maletsky, 2002), and the teacher followed the prescribed curriculum and state standards for sixth grade mathematics. The number of students sitting in the focus group ranged from three to seven. For three class days in one week we videotaped all of the group's interactions. Given our orientation toward conversation analysis, our data collection reflected an ethnomethodological approach (Schiffrin, 1994). We sought to capture "everyday" student discourse in a naturalistic setting and we made efforts to not disrupt the "normal" flow of activities within the school setting (so, for example, we did not alter the type of prompts the students worked on). After taping, we made content logs for all 7 h of video we recorded, and then we selected the sections in which the students were engaging in sustained mathematical discussions (Pirie, 1991) for closer analysis. We then transcribed those sections of the video (approximately 56 min) in detail using conventions from conversation analysis. Besides the video and transcripts, our other data sources include ethnographic field notes, a semi-structured interview with the teacher, a focus-group interview with the students in the group, and a copy of the students' written work for the week.

32.4 Findings

Three preliminary findings have emerged from this study. First, the students' group discourse practices reflected the discourse practices commonly associated with school mathematics. Specifically, the students engaged in discussions that generally reflected the "calculation(al)" orientation of the prompts they discussed. Second, the students' discussions exhibited patterns of interaction distinct from interaction patterns that researchers have documented in casual conversations. Finally, "intellectual authority" in the group was a dynamic construct that appeared to mediate the
interaction. The construction of intellectual authority was subject to negotiation during a discussion, and the role of “intellectual authority” is reflected in sociolinguistic aspects of the students’ discussions. Below, we will elaborate on these findings and we will illustrate them with examples.

**Calculational Orientation.** The students engaged in mathematics discussions that reflect the patterns of “typical” school mathematics (Moschkovich, 2007). Further, the types of questions the students answered appeared to shape the type of mathematical discourse practices in which they engaged. A majority of the prompts for the students’ group work were “exercises” (Schoenfeld, 1992), and the students used a subset of school mathematics discourse (Moschkovich, 2007) reflecting a “calculational” orientation (Thompson et al., 1994) while they engaged in their discussions. For example, one prompt asked students to consider the problem, “If six pairs of socks cost $4.50, how much will 9 pairs cost?” The text then listed four answer choices: “F $6.75 G $7.25 H $9.00 J Not Here.” (p. 393 Exercise #4) Given this prompt, the students engaged in the following discussion:2

(1) [Monday, 45:05–45:31]

1. Lorenzo A: ((Amber walks around the table and stands to the right of Lorenzo A. who is writing on a shared paper)) How would I do?

   ...

4. Amber: Ah a um let me see

5. Lorenzo A: Number four

6. Amber: Number four (for the) (1) then you put (2) nine up xxx equals nine (3.5)=

   ...

8. Amber: =Now put a:: number-a letter (1) alright, and then, let’s see:: six

9. Francisco: [Oh my God, you guys have the problem up there ((points to blackboard))

10. Lorenzo A: Six divided by (four-fifty)?

In this excerpt, Amber walked Lorenzo through step-by-step instructions on how to set-up the problem using equivalent ratios, and these instructions mirror the steps for setting up a proportion:

\[
\frac{9}{x} = \frac{6}{4.50}
\]

2Transcripts follow Jefferson’s conventions as detailed in Schiffrin (1994) with minor revisions. The… symbol denotes one or more turns omitted from this presentation because those turns refer to a side conversation that occurred in parallel to the focal interaction. Also, English translations of utterances in Spanish are give in double parenthesis and quotations (" ").

In line 8 Francisco interrupted Amber’s step-by-step instructions to tell Lorenzo and Amber that the solution to the problem was displayed on a poster on the wall. This interruption illustrates that the students perceived the goal of this activity to be finding the answer. Prior to Francisco’s interruption, however, Amber responded to Lorenzo’s original question “How would I do [the problem]” (line 1) with detailed step-by-step instructions for setting up a proportion. As Amber gave directions to Lorenzo she did not elaborate a rationale for each step, and she demonstrated to Lorenzo the mechanical steps necessary to complete the problem.

Throughout the data set, the students’ mathematical discourse commonly focused on the computational steps necessary to arrive at an answer. The students frequently answered peers’ questions and requests for help with calculational responses (this is how you…), rather than conceptual answers (this is why you…). This observation is not meant to impugn the mathematical practices or level of competence of the students in the focus group. Rather, their use of a calculational orientation is simply a demonstration of the type of mathematical discourse practices in which the students engaged in this setting in response to the given prompts, all of which might be characterized as “exercises.” As Webb et al. (2006) argue, the students use the discourse practices that they have appropriated from their experiences of school mathematics. Furthermore, the prompt for this specific discussion, a multiple choice question, explicitly highlights the importance of finding the correct answer by doing the steps and then choosing the correct answer choice.

**Distinctions between mathematical discussions and everyday conversation.** A second finding is that several interactional regularities emerged in the students’ discussion, and these patterns are distinct from patterns of interaction that sociolinguists have documented in everyday conversations (Eggin and Slade, 1997). Three such patterns unique to these mathematical discussions are: (1) verbalized inner speech, (2) the frequent use of direct contradiction or correction, and (3) the joint construction of meaning by two or more people verbalizing calculations in fragments while focusing on a shared artifact. This third point is illustrated in following excerpt where four students collaboratively do the operation 6 divided by 8 using long division:

(2) [Wednesday, session 1, 25:20–25:55]

1. Francisco: ah- why six over eight seventy five percent?

2. Claudia: ((bobbing head slowly)) Divide it.

3. Amber: OK watch. Six goes inside divide by eight [xxx=

4. Claudia: [Por eso ‘ira ((‘that’s why, look’))

5. Amber: =and [that …

6. Claudia: goes in the casita ((‘little house’)), you take out eight you put a decimal porque no se puede ((‘because you cannot’)) [put a decimal you put a zero=

7. Amber: [Uh-huh]

8. Claudia: =och por siete ((‘eight times seven’)) put a seven here it’s fifty-six
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(3) [Monday class 44:27–44:56]

1. Francisco: But who ge-Do you get number eight at all?
...  
2. Amber: Seventeen minus six (.) Duh
3. Francisco: Oye ("listen" or an interjection), I found it already ...
4. Francisco: Look. Subtract ss seventeen minus negative four [xxx
5. Amber: [Thats what I said
8. Francisco: You said subtract thirteen!
9. Amber: No, I said seventeen!
10. Teacher: They’re almost done ((addressing class))
11. Amber: (minus) negative four
12. Francisco: Now I get it
13. Lorenzo A.: Ahhh
14. Amber: I got number eight ((looking up at teacher))
15. Francisco: I got number eight too, but (kinda) she’s helped me

The construction of intellectual authority within the group. A final regularity that emerged from this data is that students’ participation in collaborative mathematical discussions revealed a dynamic where the students continually constructed and negotiated who (or what) served as an “intellectual authority” (Lampert, 1990) for the group. One result of our theoretical orientation—which highlights the social construction of knowing—is that no individual student can “have” (or lack) intellectual authority. Rather, the students continually (re)construct intellectual authority based on their active interpretation of the goals and meanings within the activity setting.

At some times the negotiation of intellectual authority was explicit. For example, during the first session on Wednesday, prior to the interaction detailed in excerpt (1) above, Amber and Claudia had a conflict when they had different answers to the same question. Ultimately Claudia convinced her group-mates of the correctness of her solution (through an appeal to the teacher’s authority), and after that point, the students treated Claudia as an authority for most of this session. In lines 2–6 of excerpt (1) we see how a dispute over intellectual authority was enacted when Amber and Claudia both began to answer Francisco’s question from line 1. Though Amber began answering the question, Claudia interrupted her and talked through Amber’s explanation. By line 7, Amber ceded the “floor” to Claudia, and her back-channeling agreement ("uh-huh") appears to affirm both Claudia’s solution as well as her right to give a solution. Claudia also appears to guard her position as an authority in this exchange by ignoring Francisco and Dennis’s quick answers to 60-56, and she carries out the subtraction algorithm with all of its steps.

At other times the negotiation of intellectual authority occurred implicitly, and it is a difficult construct to identify precisely. One way to identify a source of intellectual authority may be to analyze who is asked questions about mathematics. (This presumes that the students are not asking each other “known-answer” questions). Table 32.1 shows that with the exception of the first Wednesday session, Amber
Table 32.1 Number of questions about mathematics asked to each student during a group discussion

<table>
<thead>
<tr>
<th></th>
<th>Mon.</th>
<th>Wed. 1</th>
<th>Wed. 2</th>
<th>Fri.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amber</td>
<td>16</td>
<td>4</td>
<td>22</td>
<td>5</td>
<td>47</td>
</tr>
<tr>
<td>Claudia</td>
<td>-</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Dennis</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Francisco</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Joaquin</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lorenzo A.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Lorenzo Y.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>Totals</td>
<td>26</td>
<td>21</td>
<td>31</td>
<td>21</td>
<td>99</td>
</tr>
</tbody>
</table>

Note there were two separate math sessions on Wednesday.

aClaudia did not participate in the group during Monday’s session.
bLorenzo Y. missed a significant portion of the Monday and Wednesday sessions.

The teacher only joined the group on an occasional basis.

was asked significantly more questions about mathematics than all of the other students combined. What is also striking is the extent to which the students tried to rely on the teacher as an intellectual authority as well. Though the teacher only checked in with the group occasionally, they tended to ask her many questions. Also, during the "W2" session, Amber asked questions of the teacher off-camera, and that is why the number of questions to the teacher is coded as 0 in the table.

In addition to being constructed through the students’ discourse, the role of intellectual authority also appears to have an impact on the sociolinguistic characteristics of the peer discussion. For example, during excerpt (3) above, Amber is positioned in the role of the intellectual authority, and the shape of her interaction with Francisco reflects her assumed role. In her responses to Francisco’s question, Amber constructs her authority first by providing Francisco with a quick procedure to arrive at the answer (line 3). Second, she passes a judgment on the quality of the question through her use of the tag “Duh!”. In both tone (with the elongated syllable) and content, this tag serves the purposes of making Francisco’s question seem silly, and reifies the “unassailable” status of her answer. Francisco’s response in line 6 is a challenge to Amber’s intellectual authority, but it is interrupted by Amber, and again she uses an elongated syllable to make her opinion of the quality of his contribution known. In line 8 Francisco contradicts Amber, but she forcefully reasserts her dominant position in lines 9 and 11. Perhaps the most revealing part of this exchange is in line 15, where Francis ceded authority to Amber, and he tells the teacher that he got the answer, but with Amber’s help (and all of this despite the fact that he had the answer in line 6).

32.5 Discussion

Below we will highlight three possible implications of this work: (1) the importance of shared artifacts to build shared focus of attention during a group discussion, (2) the need for teachers and students to explicitly recognize the differences between a mathematical discussion and an everyday conversation, and (3) the need to address the emergent role of intellectual authority as a mediator of students’ mathematical discussions.

This group of students’ discussions while they worked on the exercises in the book and from worksheets varied considerably in terms of their coherence and their focus. For example, contrast the coherence of the conversation in excerpts (2) and (3) above. When the students appeared to have a unified joint focus of attention, their conversations exhibited far more coherence, and there is more evidence of intersubjectivity among the conversational participants. When students did not share a focusing artifact, the conversation quickly degenerated into apparently nonsensical talk, or into several concurrent side conversations. Therefore both an effective pedagogical tool and methodological tool may be the use of a single shared paper for the recording of group responses to shared prompts. This configuration appeared to create the most coherent group discussions, and the students were forced to respond to one another’s contradictions as they attempt to coordinate the writing of responses.

Second, mathematics as a cultural practice and specifically the practices associated with traditional school mathematics (Lampert, 1990; Moschkovich, 2007) appeared to influence the conversational “rules of interaction” observed in this study. In everyday conversations, direct contradiction or the passing of judgment on a person’s contribution usually signals a breakdown in the flow of conversation, and a flagrant violation of the “rules of politeness” (Eggin and Slade, 1997; Lakoff, 1973). Within the discussions observed in this study, however, such directly confrontational moves appeared with some regularity, and the right to make such moves appeared to be mediated by “intellectual authority.” At times the use of confrontational language bordered on teasing or insult (see excerpt 3 above). Fortunately, as a cultural practice, school mathematics is also changeable, and it may be worth considering how the practices enacted in school mathematics may be transformed to make the conversations less directly confrontational. By shifting our conception of mathematics from a stable body of knowledge that must be acquired to a cultural practice that is appropriated by students, we believe that we can transform the practice of (school) mathematics and address issues of authority leading to unproductive (and at times potentially harmful) interactions. One possible further study may be to document how the use of open-ended modeling prompts results in shifts in students peer mathematics discussions.

Following on this point, giving explicit attention to how intellectual authority is constructed and negotiated within the practice of (school) mathematics may be a way to transform how relations of power are enacted among students, and between students and teachers during mathematical conversations in schools. Yackell and Cobb illustrated how “sociomathematical norms” developed in a lower grade classroom over the course of the year (Yackell and Cobb, 1996). Like the development of sociomathematical norms, if intellectual authority is explicitly negotiated and built in a collaborative classroom environment, then it may be possible to reconfigure the nature of school mathematical conversations.
32.6 Summary

This research has illustrated the characteristics of peer-group mathematics discussion among bilingual middle school students. Through conversation analysis and mathematical discourse analysis we have shown that the interactional norms in these discussions differed from the norms of everyday conversation. We also developed the construct of "intellectual authority" as a continually negotiated role in school mathematics conversations, and we demonstrated how intellectual authority mediated interactions between students. There are a number of avenues of inquiry that we have left unexplored in this study. Below we detail two possible follow-up inquiries. One group of question pertains to the use of national languages, and students' language choices. In an interview after the activity, all of the students expressed a preference for learning mathematics in Spanish, so it may be interesting to consider how the use of various national languages influences the type of mathematical discussions the students engage in. A second possible avenue of exploration is to consider how changing the type of prompts that students work on might alter the conversational norms of a peer-mathematics discussion. Our decision to leave the prompts for this study up to the teacher's discretion reflected the ethnomethodological approach to studying conversation. Future studies might consider how Model Eliciting Activities with multiple plausible answers change the norms of a peer mathematics discussion, and transform the position of intellectual authority. Ultimately, consideration of how the "rules" of a school mathematics conversation come into being will be a valuable tool for teachers who seek to implement cooperative learning in the mathematics classroom.

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