WHAT COUNTS AS MATHEMATICAL DISCOURSE?

Judit Moschkovich

University of California, Santa Cruz

Abstract: In this paper I use situated and socio-cultural perspective (Gee, 1996 & 1999) to examine descriptions of mathematical discourse and an example of student talk in a mathematics classroom. Using this example, I discuss how the distinction between everyday and mathematical discourse can help or hinder us in hearing the mathematical content in student talk.

The distinction between everyday and mathematical discourses can be useful for describing mathematics learning as moving from everyday to more mathematical ways of talking. However, this distinction has limited uses in the classroom. First, it is difficult to use this distinction to categorize student talk since it is not always possible to tell whether a student's competence in communicating mathematically originates in their everyday or school experience. And, while learning mathematics certainly involves learning to use more mathematical language, everyday discourse practices should not be seen only as obstacles to learning mathematics. During mathematical discussions students use multiple resources from student experiences both outside and inside school. Before we label student talk as everyday or mathematical, we need to seriously consider what we include or exclude in our definition of mathematical discourse practices. If we assume that mathematical discourse consists only of textbook definitions or those practices that mathematicians use in formal settings, we may miss the mathematical competence in student talk.

We can begin to characterize mathematical discourse using the mathematics register, as defined by Halliday (1978):

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the "mathematics register," in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

Halliday is not referring to technical vocabulary but to meanings, styles, and modes of argument: "We should not think of a mathematical register as consisting solely of terminology, or of the development of a register as simply a process of adding new words." (p. 195)

Forman (1996) describes some of the characteristics of mathematical discourse as its syntax and impersonal nature (sentences without subject or using the impersonal "you" as subject). She also describes the particular modes of argument valued in mathematical discourse: precision, brevity, and logical coherence. Research in mathematics classrooms has provided empirical evidence that learning mathematics includes sorting out multiple meanings between these two registers (Pimm 1987, Khisty, 1995; Moschkovich, 1996). This research has focused on the differences between the

In Proceedings of the 2003 Joint Meeting of the International Group for the Psychology of mathematics Education (PME) and the North American Chapter of PME (PME-NA), N. Pateman, B. Dougherty, and J. Zilliox (Eds.). CRDG, College of Education, University of Hawaii: Honolulu, HI., 2003.

everyday and mathematical registers. Since there are multiple meanings for the same term or phrase, as students learn mathematics they are learning to use these multiple meanings appropriately. Several examples of such multiple meanings have been described. For example, the word "set" and the phrase "any number" (meaning "all numbers") have different meanings in a mathematical context (Pimm, 1987).

These multiple meanings can create obstacles in mathematical conversations because students often use the colloquial meanings of terms, while teachers (or other students) may use the mathematical meaning of terms. Another difference between the everyday and the school mathematics registers is the meaning of relational terms such as "steeper" and "less steep" and phrases such as "moves up the y-axis" and "moves down the y-axis." Meanings for these terms and phrases that may be sufficiently precise for everyday purposes may prove to be ambiguous for describing lines in the context of a mathematical discussion (Moschkovich, 1996).

Learning mathematics involves, in part, a shift from everyday to a more mathematical and precise use of language. Studies have described how students' language can move closer to the mathematics register, becoming more precise and reflecting more conceptual knowledge. For example, students develop more restricted meanings for everyday terms (O'Connor, 1992) and refine the uses of everyday meanings so that they reflect more conceptual knowledge (Moschkovich, 1996,1998).

Learning the mathematical meanings of words describes one important aspect of learning mathematics. Contrasting everyday meanings with the more restricted meanings of the mathematics register points to these multiple meanings as possible sources of misunderstandings in classroom discussions. However, the relationship between the everyday and the mathematics registers and communication in the classroom is more complex. First, mathematical discourse involves more than word meanings. Second, everyday meanings are not only obstacles but also resources for developing mathematical competence. And lastly, as Forman (1996) points out, in the classroom everyday and mathematical discourses are not separate but interwoven in discussions.

Forman (1998) and Wertsch (1990) suggest moving from individual word meaning to more general discursive practices to "identify the forms of speech or discourse characteristic of particular sociocultural settings" (Wertsch, 1990). This shift can broaden the characterization of mathematical communication beyond the use of particular words and their meanings. We can begin by using a definition of discourse as more than speech or writing. Gee (1996) defines Discourses as:

> " A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts', of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network', or to signal (that one is playing) a socially meaningful role." (p. 131)

Mathematical Discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view. Participating in mathematical discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act when talking about mathematics. Gee's (1996) example of a biker bar illustrates the ways that any Discourse practice involves more than technical language. In order to look and act like one belongs in a biker bar, one has to learn much more than a vocabulary. While knowing the names of motorcycle parts or models may be helpful, it is clearly not enough. In the same way, knowing a list of technical mathematical terms is not sufficient for participating in mathematical Discourse.

Are there some general characteristics of mathematical Discourse? Being precise and explicit, searching for certainty, abstracting, and generalizing are highly valued practices in mathematically oriented Discourse communities. Generalizing is exemplified by common mathematical statements such as "the angles of any triangle add up to 180 degrees," "parallel lines never meet," or "a + b will always equal b + a." While generalizing is a valued practice, it is also important to make claims that are applicable only to a precisely and explicitly defined set of situations. For example, the statement "multiplication makes a number bigger" can be true or false depending on the set of numbers the claim refers to: "Multiplication makes a number bigger except when multiplying by a number smaller than 1."

Many times claims are also tied to mathematical representations such as graphs, tables, or diagrams. Although less often considered, imagining is also a valued mathematical practice. For example, mathematical work often involves talking and writing about imagined things---such as infinity, zero, infinite lines, or lines that never meet--as well as visualizing shapes, objects, and relationships that may not exist in front of our eyes.

Mathematical Discourse, however, is not a single set of homogeneous practices. Although we might agree that mathematical Discourse is reasoned discourse (Hoyrup, 1994), it varies across individuals, communities, time, settings, and purposes. Current inquiry into the practices of mathematicians' concludes that there is not one mathematics, one way of understanding mathematics, one way of thinking about mathematics, or one way of working in mathematics (Burton, 1999):

> Out of the interviews with research mathematicians, I have a clear image of how impossible it is to speak about mathematics as if it is one thing, mathematical practices as if they are uniform and mathematicians as if they are discrete from both of these. (p. 141)

How do mathematical Discourse practices vary socially, culturally, and historically? Mathematical Discourse varies across different communities, for example research mathematicians and statisticians, or between elementary and secondary school teachers. Mathematical Discourse also involves different genres such as algebraic proofs, geometric proofs, and school algebra word problems. Mathematical arguments can be presented for different purposes such as convincing, summarizing, or explaining.

Mathematical Discourse is also historically situated. For example, mathematical arguments have changed over time (Hoyrup, 1994):

> What was a good argument in the scientific environment of Euclid was no longer so to Hilbert; and what was nothing but heuristic to Archimedes became good and sufficient reasoning in the mathematics of infinitesimals of the seventeenth and eighteenth centuries. (p. 3)

Even mathematical definitions have changed over time. For example, the definition of a function has changed throughout history from the Dirichlet definition as a relation between real numbers to the Bourbaki definition as a mapping between two sets. Mathematical definitions can also differ across cultural contexts. For example, in Spanish "the word trapezoid is reserved for the quadrilateral without any parallel sides, whereas trapezium is used when there is one pair of parallel sides. This is the opposite of American English usage." (Hirigoyen, 1997).

Mathematical Discourse practices also vary depending on purposes. Richards (1991) describes four types of mathematical discourse. Research math is the spoken mathematics of the professional mathematician and scientist. Inquiry math is mathematics as used by mathematically literate adults. Journal math emphasizes formal communication and is the language of mathematical publications and papers. This type of mathematical discourse is seen as different from the oral discussions of the research community because written formal texts reconstruct the story of mathematical discoveries. Lastly, he defines school math as the discourse typical in the traditional math classroom, sharing with other classrooms the initiation-reply-evaluation structures of other school lessons (Mehan, 1979). Richards points out that school math has more in common with journal math than with research or inquiry math.

Hearing the Mathematical Content in Student Talk

Moving from individual word meaning to discursive practices complicates the distinction between everyday and mathematical Discourses. We may be able to identify whether a student is using the everyday or mathematical meaning for words such as prime, set, function, or steeper. However, it is more difficult to separate Discourse practices as belonging to one setting or another, and it may be impossible to identify the origins of Discourse practices that students use in the classroom. Students combine resources from multiple Discourse practices. Students use resources from <u>both</u> everyday and mathematical Discourses to communicate mathematically. As analysts or teachers, we cannot decide whether student talk reflects or originates in everyday or mathematical Discourse. It is also a challenge to hear not only one acceptable version of mathematical communication, but also multiple authentic mathematical Discourse practices

The example below illustrates how our views of authentic mathematical practices influence whether we hear students as participating in mathematical Discourse or not. The excerpt comes from a lesson in a third grade bilingual classroom in an urban California school. The students have been working on a unit on two-dimensional geometric figures. For several weeks, instruction has included technical vocabulary such as the names of different quadrilaterals. Students have been talking about shapes and have also been asked to point, touch and identify different instances. In this lesson, students where describing quadrilaterals as they folded and cut paper to form tangram pieces.

Towards the end of his lesson, there was a whole-class discussion of whether a trapezoid is or is not a parallelogram. The teacher had posed the following question:

Teacher: What do we know about a trapezoid. Is this a parallelogram, or not? I want you to take a minute, and I want you at your tables, right at your tables I want you to talk with each other and tell me when I call on you, tell me what your group decided. Is this a parallelogram or not.

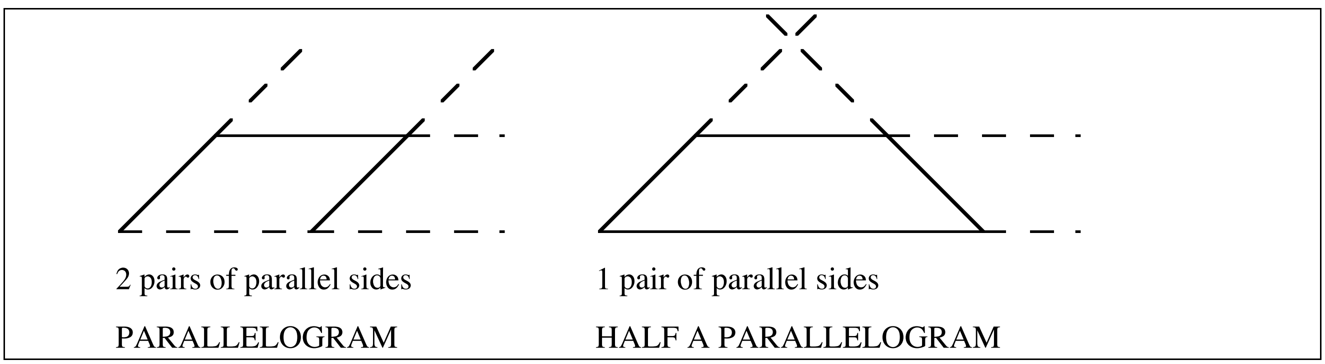
After the students had discuss this question in their groups, the following whole-class discussion ensued:

Teacher:	((To the whole class)) OK. Raise your hand. I want one of the groups to tell us what they do think. Is this ((holding up a trapezoid)) a parallelogram or not, and tell us why. I'm going to take this group right here.
Vincent:	These two sides will never meet, but these two will.
Teacher:	How many agree with that. So, is this a parallelogram or not?
Students:	Half.
Teacher:	OK. If it is half, it is, or it isn't?
Students:	Is.
Teacher:	Can we have a half of a parallelogram?
Students:	Yes.
Teacher:	Yes, but then, could we call it a parallelogram?
Students:	Yes.

The standard definition of a trapezoid is "a quadrilateral with one pair of parallel sides" and the standard definition of a parallelogram is "a quadrilateral with two pairs of parallel sides." The students' response to the question "Is this a parallelogram or not?" was "Half", implying that a trapezoid is half of a parallelogram.

First, let us consider how "a trapezoid is half a parallelogram" might be a reasonable response to the question. A parallelogram has two pairs of parallel sides and a trapezoid has one pair of parallel sides. A trapezoid can be seen as a half of a parallelogram because a trapezoid has half as many pairs of parallel sides as a parallelogram.

Figure 1: A trapezoid is half a parallelogram



How do the teacher's and the students' definitions compare? Students were focusing on whether and how these two figures possess the *property* of having pairs of parallel lines. The teacher was focusing on whether the figures belong to one of two *categories*: "figures with two pairs of parallel lines" or "figures with one or no pair of parallel lines."

While for the students "half a parallelogram" was an acceptable specification, this was not acceptable to the teacher. The teacher's initial question "Is this a parallelogram or not, and tell us why?" assumed that this was an either/or situation. The teacher's point of view was dichotomous: a given figure either is or is not a parallelogram. The teacher was using a formal dictionary definition of parallelogram. This definition is clearly binary: either this figure is a parallelogram or it is not a parallelogram. From this point of view, "half a parallelogram" is not an acceptable definition for a trapezoid.

We might conclude that because the teacher was using a dictionary definition, the teacher's point of view reflects mathematical Discourse practices. We might also conclude that because the students are not using a formal definition, their point of view reflects everyday Discourse practices. But there is another way to consider these two different points of view.

Does one definition necessarily reflect more or less authentic mathematical Discourse practices than the other one? That depends on how we define authentic mathematical practices. If using dictionary definitions is the only practice we imagine that mathematicians participate in, then the teacher's definition of a trapezoid is the only mathematical definition in this discussion. Instead, if we include "developing working definitions" as an authentic mathematical practice, then the students' definition is also mathematical.

O'Connor (1998) discusses different types of definitions. She includes stipulative, working, dictionary, and formal as different categories of mathematical definitions. Stipulative and working definitions are developed as part of an interaction or an exploratory activity; dictionary and formal are given by a text. Constructing shared definitions "is a signal example of what we mean by authentic intellectual practices of mathematics and science" (p. 42.)

From this view of mathematical practices, using dictionary definitions is not the only authentic mathematical practice and using formal definitions is not the only way to participate in mathematical Discourse practices. The definition students used can be described as a working and stipulative definition. These students are actually participating in an activity that may be closer to the practice of scientists and mathematicians than to school practices of using only dictionary definitions.

The example above points to the complexity of mathematical communication in the classroom. Whether or not student talk sounds mathematical depends on how we understand the distinction between everyday and mathematical Discourses. There are many authentic mathematical Discourse practices. We should not confuse "mathematical" definitions with "textbook" definitions; we should clarify the differences between mathematical ways of talking and formal ways of talking mathematically. Mathematical Discourse practices are varied.

Notes

This work was supported in part by a grant from NSF (REC-9896129 and REC-0096065).

References

Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics? Educational Studies in Mathematics, 37: 121-143.

Forman, E. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, and B. Greer (Eds.) Theories of mathematical learning (pp. 115-130). Mahwah, NJ: Lawrence Erlbaum Associates.

Forman, E., McCormick, D., & Donato, R. (1998). Learning what counts as a mathematical explanation. Linguistics and Education, 9(4), 313-339.

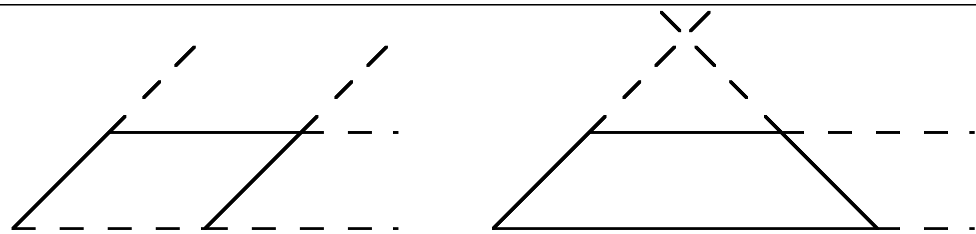
Gee, J. (1996) Social Linguistics and Literacies: Ideology in Discourses, Second Edition. London, Taylor &Francis.

Gee, J. (1999). An Introduction to Discourse Analysis: Theory and Method. NY: Routledge.

Halliday, M. A. K. (1978). Sociolinguistics aspects of mathematical education. In M. Halliday, The social interpretation of language and meaning, (pp. 194-204). London: University Park Press.

Hirigoyen, H. (1997). Dialectical variations in the language of mathematics: a source of multicultural experiences. In Trentacosta, J. and Kenney, M. J. (Eds), Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity-1997 Yearbook, Reston, VA, National Council of Teachers of Mathematics, pp. 164-168.
Høyrup, J. (1994). In measure, number, and weight: Studies in mathematics and culture. Albany: SUNY.
Khisty, L. L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), New directions for equity in mathematics education (pp. 279-297). New York: Cambridge University Press.
Mehan, H. (1979). Learning lessons: Social organization in the classroom. Cambridge, MA: Harvard University Press.
Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. The Journal of the Learning Sciences, 5(3), 239-277.
Moschkovich, J. N. (1998). Resources for refining conceptions: Case studies in the domain of linear functions. The Journal of the Learning Sciences, 7(2), 209-237.
Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. For the Learning of Mathematics, 19(1), 11-19.
O'Connor, M.C. (1998). Language socialization in the mathematics classroom.
O'Connor, M. C. (1992). Negotiated defining: The case of length and width. Unpublished manuscript, Boston University.
Pimm, D. (1987). Speaking mathematically: Communication in mathematics classrooms. London: Routledge.
Richards, J. (1991). Mathematical discussions. In E. von Glasersfeld (Ed.), Radical constructivism in mathematics education (pp. 13-51). The Netherlands: Kluwer.

Figure 1: A trapezoid is half a parallelogram



2 pairs of parallel sides — PARALLELOGRAM

1 pair of parallel sides — HALF A PARALLELOGRAM